MODIFIED WHITNEY-NUISMER CRITERIA FOR PREDICTION OF NOTCHED STRENGTH OF COMPOSITE LAMINATES

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Abstract

A novel fracture criterion for open hole tension on composite laminates is proposed. This criterion correlates quite well with experimental results for a wide range of materials, laminates and hole sizes without any correction factor. More important, these results were obtained by assuming the characteristic distances used in the criterion to be constant material properties. This feature drastically reduces the number of required tests for estimates using the proposed criterion.

1 Introduction

Laminated composites are well known to have notch sensitivity. Their open hole tension strength depends on the hole size. The material has a ductile behavior for small holes and a brittle one for large holes. Whitney and Nuismer [1] proposed criteria that are widely used in the aerospace companies for estimating the notched strength of composite laminates.

Their basic assumption for these criteria is the experimentally verified fact that composite laminates have the ability to withstand a load higher than their strength provided that this load acts over a limited region. They postulated two criteria for estimating the in-plane tensile fracture stress for a composite laminate with a center open hole with radius equal to R. They assumed a far field stress applied along the y-direction and the origin of the xy coordinate system at the center of the hole. The first criterion, named Point Stress Criterion (PSC), establishes that the laminate fracture stress is such that the laminate stress $\sigma_y(R+d_0,0)$ is equal to the tensile strength of the un-notched laminate.

The second criterion, named Average Stress Criterion (ASC), assumes that the laminate fracture occurs when the average laminate stress σ_y from $x = R$ to $x = R + a_0$ and $y = 0$ is equal to the tensile strength of the un-notched laminate. Both parameters, d_0 and a_0, were assumed to be constant material properties and can be estimated based on simple open hole tests. These criteria are very attractive for their simplicity.

Karlak [2] experimentally observed that, in fact, both parameters, d_0 and a_0, are not material constants: they vary with the hole size. His work dealt with this fact by using the standard Whitney-Nuismer criterion introducing a correction factor to estimate the characteristic length. After his work, many researchers proposed modifications of the Whitney-Nuismer criteria introducing new parameters to compute expressions for d_0 and a_0 to improve the accuracy of the criteria [3-9]. Despite being successful in improving the accuracy, these new criteria require extensive tests to characterize the new parameters considering all operational conditions that affect the open hole strength.

The open literature on the strength of composite laminates with unloaded holes is very vast. However, there are important factors affecting this strength that have not been fully explored, particularly in terms of experimental results. For example, papers on composite laminates with fabric layers are relatively scarce [10-11] and a single one dealing with open hole in braided composites was found [12]. Also, important aspects such as environmental conditions, multi-axial loadings and laminates combining unidirectional and fabric layers are virtually inexistent.

Another important factor in strength of composite laminates with unloaded holes are lamination angles. For example, it is well known that notched cross-ply laminates are brittle whereas a $[\pm 45]_{ns}$ angle ply laminate exhibits a ductile behavior. Moreover, they will have different fracture modes under tensile loading and have different values of toughness. As a consequence, the use of the original Whitney-Nuismer criteria [1] would lead to very different characteristic lengths for these two laminate types as observed by Soriano and Almeida [10]. This fact clearly indicates that the characteristic lengths originally proposed by Whitney-Nuismer cannot be considered as material properties. Using correction factors for the hole size increase the required amount of testing and does not solve the problem of the lamination angles.

This work proposes a different approach for modifying the Whitney-Nuismer criteria aiming at giving a contribution towards improving the estimate accuracy without increasing the required amount of testing. This goal is important in order to reduce certification costs and time.

2. Novel Fracture Criteria

Chen, Shen and Wang [13] proposed a criterion that is a modification of the Whitney-Nuismer ASC criterion:

$$\frac{1}{a_0}\int_{R}^{R+a_0}\sigma_y\left(0,x\right)\,dx = F_{1t} \qquad (1)$$

where σ_y is the stress along the load direction (y), a_0 is a characteristic length, assumed as a material property, R is the hole radius and F_{1t} is the maximum stress of a lamina for a tensile loading applied along the fiber direction.

Despite not using any correction factor for hole size, this criterion was shown to reduce the influence of the hole size and the lay-up angles on the optimum characteristic length a_0.

Therefore, this criterion was used to investigate the difference between the brittle behavior of a $[0/90]_s$ laminate and the ductile behavior of a $[45/\text{-}45]_s$ laminate under tensile load in the presence of an unloaded hole. The far field stress σ_y was set equal to 1. All the analyses performed in this work assume infinite plate and use complex variable analysis by Lekhniskii [14]. Typical carbon/epoxy properties used in [10] were assumed.

However, instead of computing the stresses only along the x direction as in [13], contour plots of $\sigma_y(x,y;0^o)$, the yy stress component at a 0° ply, were generated in a quarter of the plate due to symmetry. Since the laminate $[45/\text{-}45]_s$ does not have any layer at 0°, consistently, the in-plane shear stress component τ_s at the layers 45° was used and F_{1t} in the criteria are substituted by F_6, the maximum in-plane shear stress.

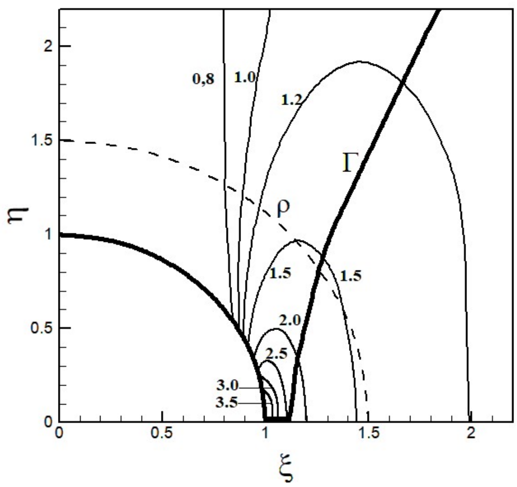

Fig.1. Contour plot for $\sigma_y(\xi,\eta;0^o)$ for laminate $[0/90]_s$

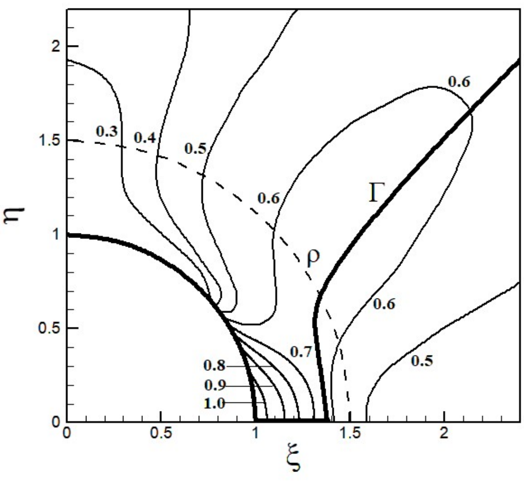

Fig.2. Contour plot for $\tau_s(\xi,\eta;45^o)$ for laminate $[45/\text{-}45]_s$

Also, a locus of the maximum of σ_y or τ_s, the critical load path (Γ), was plotted. Figures 1 and 2 depict the results for laminates $[0/90]_s$ and $[45/\text{-}45]_s$, respectively, using non dimensional coordinates, $\xi = x / R$ and $\eta = y / R$, where R is the hole radius.

Figures 1 and 2 clearly demonstrate that the critical load path, Γ, deviates from x-axis. It must be noticed that curve Γ starts at $(x,y) = (R,0)$ as proposed by Whitney and Nuismer [1], but at a short distance from this point, it deviates from the x–axis leading to a different estimate for the fracture stress. This is consistent with the fact that the failure mode of the notched laminates depends both of the laminate toughness and the hole size. Also, the contour plots are quite different. These two features, are consistent with the fact that laminates $[0/90]_s$ and $[45/-45]_s$ have different fracture modes.

The original Whitney-Nuismer criteria [1] and all the criteria based on their work assume that the critical load path coincides with x-axis which is certainly not correct. On the other hand, the estimates obtained with these criteria are in many cases correlate reasonably well with experimental results. However, assuming that the x-axis is the critical load path does not capture the effects of the different load distribution for different lay-ups.

The critical stress distribution over the actual critical load path, Γ, is different from the one along the x-axis. Since the length along Γ is important for the Whitney-Nuismer criteria, if the geometrical length along Γ is used, the correlation with experimental results would be very poor, particularly for $[45/-45]_s$ laminates. Several approaches were tested to solve this issue.

It must be pointed out that the far field stresses are constant. Therefore, far from the hole, the critical load path is not well determined since the stresses would be nearly constant. Also, as depicted in Fig.1, the laminate $[0/90]_s$ presents a large stress gradient along Γ, unlike laminate $[45/-45]_s$ that presents low gradients. In fact, for laminate $[45/-45]_s$, Γ deviates from x-axis, then follows a path that is nearly circular and finally follows a path that is nearly radial. Along the nearly circular part of Γ, the stresses are almost constant.

Analyzing these features, the length along Γ is defined as:

$$l(s) = \int_{R}^{R+s} \vec{t} \bullet \vec{r} \; d\Gamma \qquad (2)$$

where s is a geometrical length over Γ, $\vec{t}$ and $\vec{r}$ are the unit tangent vector to Γ and the unit radial vector at an arbitrary point on Γ, respectively. $l(s)$ corresponds to a point over Γ at a geometrical distance s of the point $(R,0)$ over the curve Γ according to Eq. (2).

Therefore, the modified point stress criterion (modified PSC) becomes:

$$\sigma_y \big|_{\Gamma(d_0)} = F_{1t} \qquad (3)$$

where the stress σ_y is computed over Γ at a distance d_0 according to Eq. (2) and F_{1t} is the maximum stress of a lamina for a tensile loading applied along the fiber direction.

The modified average stress criterion (modified ASC) becomes:

$$\frac{1}{a_o} \int_{R}^{R+a_0} \sigma_y \big|_{\Gamma} \; \vec{t} \bullet \vec{r} \; d\Gamma = F_{1t} \qquad (4)$$

where σ_y is computed at an arbitrary point over Γ.

For the ASC and PSC criteria, if the estimate is larger than the unnotched laminate strength (if available) then the estimate should be equal to the unnotched laminate strength. This situation might happen for small holes in tough laminates (dominated by ±45 plies).

These two criteria will be correlated with experimental results and compared to the Chen, Shen and Wang [13] criterion in the following section.

3. Results

3.1. Laminates with Fabric Plies

Soriano and Almeida [10] used carbon/epoxy plain balanced weave fabrics to produce five families of 5 layers laminates. They used orientation 0° and 45° and named the families *Li* where *i* is the number of layers at 45°. Therefore, *L0* and *L5* are a cross-ply and angle-ply laminates, respectively. The number of plies at 45° defines the toughness of the laminates.

Table 1 lists the lay-up angles for these laminates and Table 2 lists the material properties.

Four different hole diameters were tested for each family: 1/8 in, 1/4 in, 3/8 in and 1/2 in. Therefore, including the unnotched specimens, 30 different specimens were tested with 4 repetitions for each type of specimens.

The laminates were manufactured using pre-impregnated carbon/epoxy fabric Hexcel F584. This is a balanced eight harness satin weave fabric. The plates were cured in an autoclave. After all the plates have been ultrasonically inspected carbon/epoxy tabs were bonded and the specimens were cut in a milling machine with a diamond wheel. After the holes have been drilled, the notched specimens were subjected to another ultrasonic inspection in the region near the hole [10].

Table 1. Laminate lay-ups

Code	Lay-up
L0	$[(0,90)_5]$
L1	$[(0,90)_2/(45/-45)/(0,90)_2]$
L2	$[(0,90)/(45/-45)/(0,90)/(45/-45)/(0,90)]$
L3	$[(45/-45)/(0,90)/(45/-45)/(0,90)/(45/-45)]$
L4	$[(45/-45)_2/(0/90)/(45/-45)_2]$
L5	$[(45/-45)_5]$

Table 2. Material properties

Property	Value
E_1	73 GPa
E_2	73 GPa
G_{12}	6.6 GPa
ν_{12}	0.045
F_{1t}	943 MPa
F_6	278 MPa

In Table 2, E_1 and E_2 are the longitudinal and transverse modulus of elasticity, respectively, G_{12} is the in-plane shear modulus, ν_{12} is the in-plane Poisson's ratio, F_{1t} is the longitudinal maximum stress and F_6 is maximum in-plane shear.

In this work, the fracture stress was estimated as described in the previous section: (a) a detailed stress analysis was performed in the entire laminate and used for the criteria; and (b) instead of using the laminate stress σ_y, the stress along the fibers of the 0^o layers were used to estimate the fracture stress. This is consistent with the usual 10% rule; its use guarantees that there is at least one layer at 0^o in the laminate. Family *L5* does not have any layer at 0^o. Therefore, consistently, the in-plane shear stress component τ_6 at the layers 45^o was used.

Parameters d_0 and a_0 were adjusted using the experimental results for family *L1* (the hardest family that satisfies the 10% rule). The used approach was to minimize the percentage error for the four sizes of holes. With these values, the strength of all laminate families and hole sizes strength were computed. The values optimum values for d_0 and a_0 were 1.2 mm and 3.4 mm, respectively.

No discrepancy above 10% was obtained for both criteria a performance far better than the traditional Whitney-Nuismer criteria. Plots of the estimates for the modified PSC and ASC criteria, the Chen, Shen and Wang [13] criterion and the experimental points are shown in Figs. 3-8.

The plots demonstrate that the modified ASC criterion yielded excellent estimates for all laminates and hole sizes. The modified PSC and the Chen, Shen and Wang [13] criterion also provided good estimates. It must be emphasized that the worst estimates for this criterion were for the *L4* laminate that is has a strong dominance of ±45 plies. Also, this criterion was not used for laminate *L5* because this laminate does not have 0^o plies.

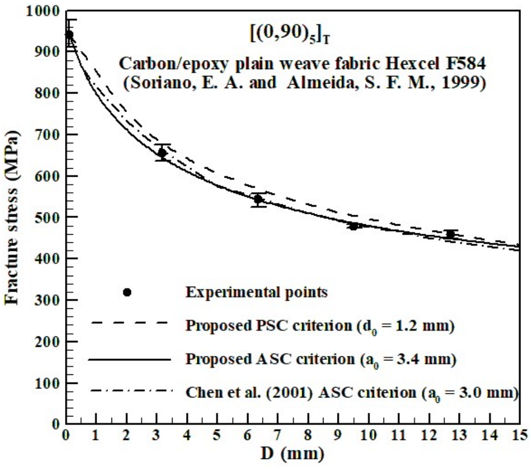

Fig. 3. Strength estimates for laminate *L0*

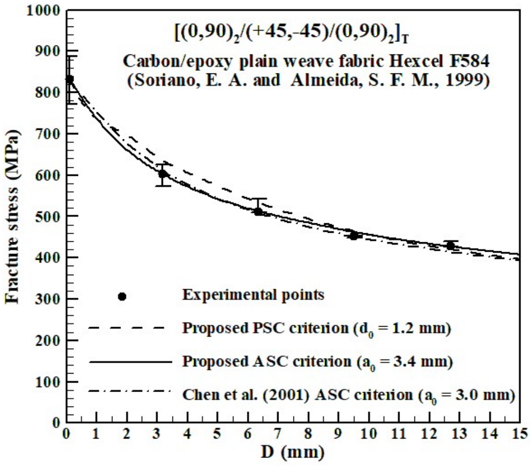

Fig. 4. Strength estimates for laminate *L1*

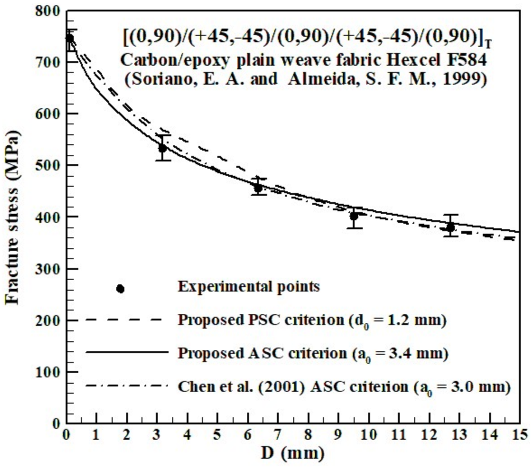

Fig. 5. Strength estimates for laminate *L2*

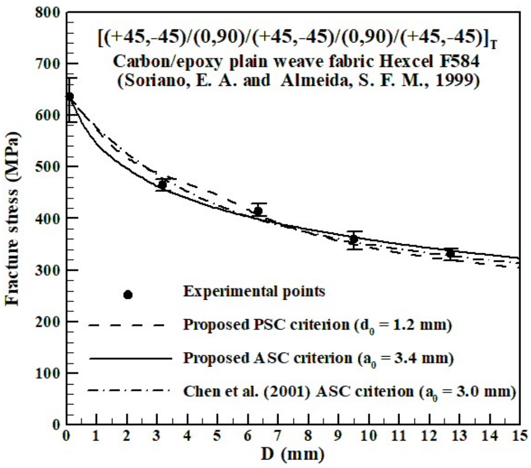

Fig. 6. Strength estimates for laminate *L3*

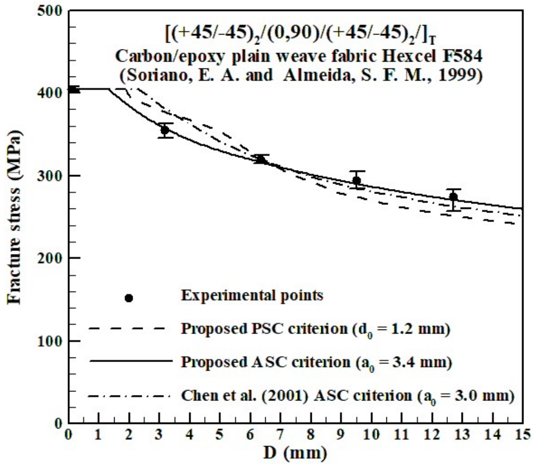

Fig. 7. Strength estimates for laminate *L4*

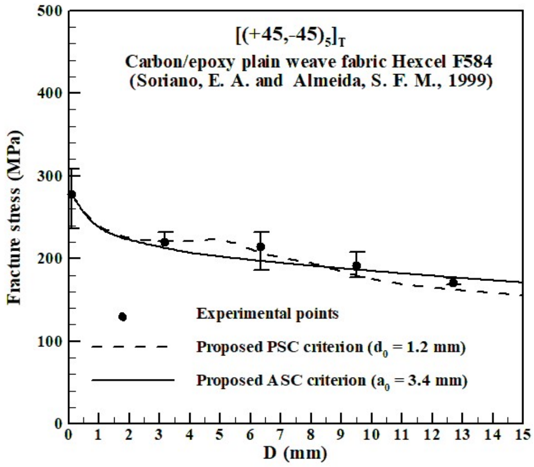

Fig. 8. Strength estimates for laminate *L5*

It is also important to notice that the modified PSC criterion estimates, despite being reasonably accurate, do not have always the expected monotonic behavior. This happens for laminates *L4* and *L5* that are dominated by ±45 plies. On the other hand, the modified ASC criterion does not suffer from this problem.

This is explained by the fact that the distribution of σ_y over the critical path curve (Γ) may present some peaks. To illustrate this, Fig. 9 depicts the master curve for the application of the modified PSC criterion for laminate *L5*. For a given d_0 and notch radius, *R*, one computes and d_0 / R and, using Fig. 9 obtains the estimate for the modified PSC criterion. However, this master curve has a local maximum near $d_0 / R = 0.4$. This local maximum causes the waviness observed in Figs. 7 and 8.

On the other hand, the master curve for the application of the modified ASC criterion for laminate *L5* is monotonic as expected. This happens because the average performed filters out the effect of this local maximum as depicted in Fig. 10.

As a conclusion for this set of experimental results, the modified ASC criterion gave excellent estimates for all hole sizes and all laminates. It clearly outperformed the modified PSC criterion and the Chen, Shen and Wang criterion [13].

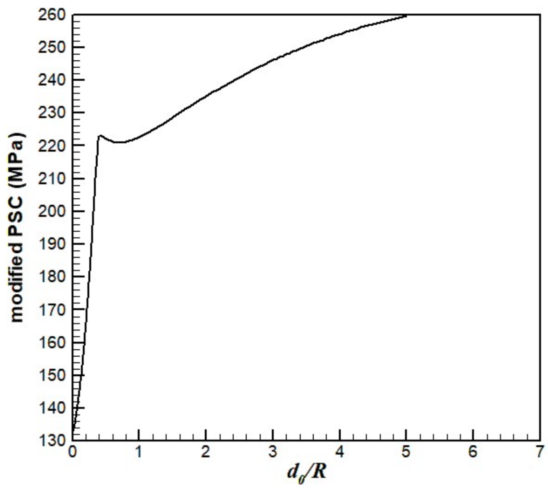

Fig. 9. Master curve for the modified PSC criterion

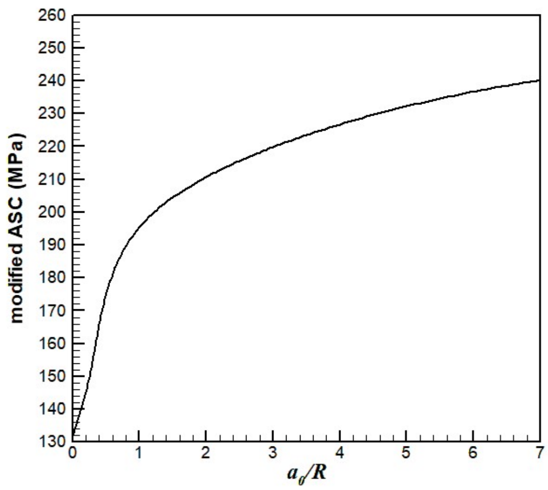

Fig. 10. Master curve for the modified ASC criterion

3.1. Laminates with Unidirectional Plies

In order to further validate the modified Whitney-Nuismer criterion, their estimates will be compared against experimental results of laminates with unidirectional plies. Some results from Chen, Shen and Wang [13] will be used.

The laminates were manufactured with carbon/epoxy T300/QY8911. The lay-ups and material properties are listed in Tables 3 and 4.

Table 3. Laminate lay-ups [13]

Code	Lay-up
L6	$[45/0/\text{-}45/0_2/90/0/90]_s$
L7	$[45/\text{-}45/0/45/90/\text{-}45/0/90]_s$
L8	$[45/0/\text{-}45/0/45/0/\text{-}45/90]_s$
L9	$[45/0/\text{-}45/90_2/0/90_2]_s$

Table 4. Material properties [13]

Property	Value
E_1	135 GPa
E_2	8.80 GPa
G_{12}	4.57 GPa
ν_{12}	0.33
F_{1t}	1860 MPa

The meaning of the properties is the same as in Table 2.

Figs. 11-14 depict the correlation between the experimental data and the studied fracture criterion. For the modified PSC and ASC criteria, the characteristic lengths were adjusted using laminate *L6* (the hardest one) and applied to all laminates. The results were: d_0 = 0.9 mm and a_0 = 2.6 mm.

Again, for this set of experimental data the modified ASC criterion provided excellent estimates for all laminates and hole sizes. The only exception is the 5 mm hole for the *L6* laminate that presented an error of -9.8%, still very reasonable. The modified ASC criterion also outperformed the modified PSC criterion and the Chen, Shen and Wang criterion [13].

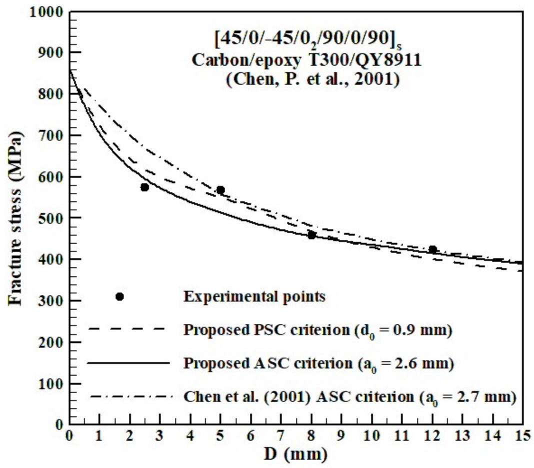

Fig.11. Strength estimates for laminate *L6*

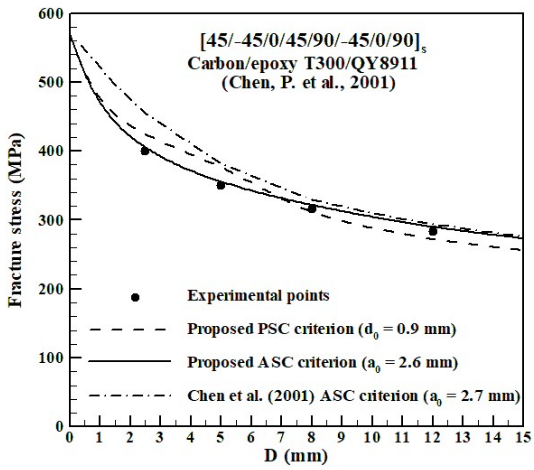

Fig. 12. Strength estimates for laminate *L7*

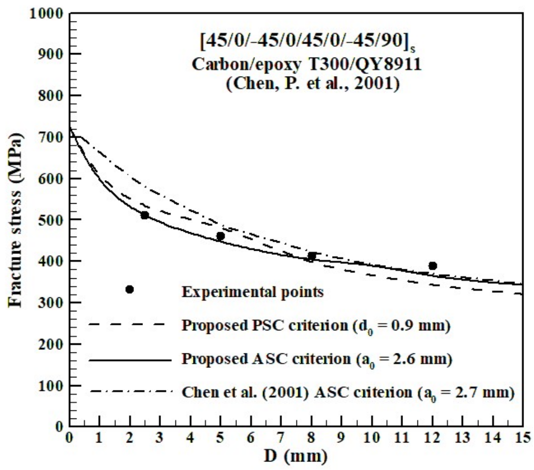

Fig. 13. Strength estimates for laminate *L8*

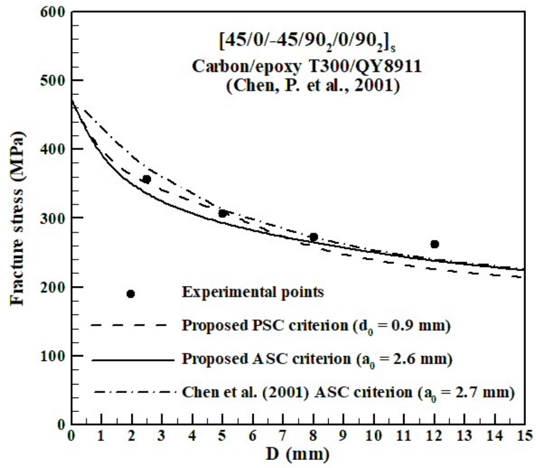

Fig. 14. Strength estimates for laminate *L9*

It must be pointed out that the optimum values of d_0 and a_0 for the laminates with unidirectional plies were smaller than the ones for the fabric plies. This is consistent with the well known fact that fabric plies are tougher than the unidirectional plies.

4. Comments on the Parameters for the Modified Criteria

The modified Whitney-Nuismer criteria consistently yielded accurate estimates for all validation tests. Another important feature is that the formulation depends only on lamina elastic properties, maximum stress along the fiber direction, the maximum shear stress and a characteristic length.

The elastic and strength properties are always experimentally available. Therefore, these criteria require only the experimental evaluation of the characteristic lengths. These can be achieved by standard test of open hole tension. However, the hole sizes should be judiciously chosen. It must be emphasized that either small or large holes compared to a_0 are not suitable for this purpose. This is so because: (a) small holes tend to be tougher and the value of a_0 has small sensitivity in this case as the fracture load will be very close to the unnotched strength; (b) for large holes compared to a_0 tend to be brittle and the fracture load will be primarily defined by the elastic stress concentration, therefore, the value of a_0 will have small sensitivity.

The value of a_0 is about 3 mm for most carbon/epoxy materials. Therefore, for carbon/epoxy, it is recommended to use hole sizes from 4 to 8 mm in diameter to experimentally determine a_0.

In principle, it suffices a single hole size to estimate d_0 and a_0. Of course, testing more hole sizes will improve the accuracy of the measurement. It is recommended to test at least five specimens for each hole size.

5. Conclusions

Based on the presented results, it was demonstrated that the proposed modifications of the Whitney-Nuismer criteria did result in better accuracy with a minimum number of tests and without using new parameters to account for different failure modes and hole sizes.

Particularly, the modified ASC criterion is a promising alternative for simple and reliable estimates of composite laminates with unloaded holes under tensile stress. This criterion was compared against several experimental data in the open literature and always provided good estimates regardless the hole size and laminate lay-up. From these results, the characteristic length a_0 apparently may be treated as a material property.

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