# FASTENING ANALYSIS USING LOW FIDELITY FINITE ELEMENT MODELS

**Rodrigo de Sá Martins*, Marco Túlio dos Santos**, Ernani Sales Palma***
***Universidade Federal de Minas Gerais, **Embraer**

**Keywords:** *Joints, finite elements, fastener stiffness, joint load distribution*

## Abstract

*There are several ways to create a finite element model of a fastener. The models are divided into two categories: low-fidelity models and high-fidelity models. The former is usually used in the sub-component analysis and the latter when dealing with details. Low-fidelity models are simpler models, using a static linear solution and one-dimensional elements for the fasteners. The purpose of this type of model is to obtain a load distribution in a joint. High-fidelity models use solutions with displacement and material nonlinearities, as well as contact, friction, etc. They are used when it is desired to verify the influence of different effects on the resistance of the joint (the interference, the friction, the size of the head of the fastener, etc.). High fidelity models are not always more accurate when compared to low fidelity models. This is because the non-linearities introduced in the model to simulate the associated physical phenomena may include undesirable numerical errors. To use high fidelity models, strict validation is required. The paper reviews the methods that are currently used to analyze fasteners using finite element analysis.*

## 1 Introduction

An aircraft structure is assembled from many components. These components can be made from thin skins and extrusions, as well as machined, cast and forged parts. When connected, these components form the major sub-assemblies of the airplane. Ideally, each component should be as monolithic as possible, to provide a continuous load distribution. Stress gradients are then avoided, making the structure less susceptible to failure. In real life, joints are required at the component level, to permit sub-assembly or to provide removable components. In the structural analysis and design of airplanes, a hierarchy is followed to achieve better results. It starts with global models representing large structures and refine then as we go down to smaller ones. Fig. 1 illustrates the hierarchy in modelling and analyzing.

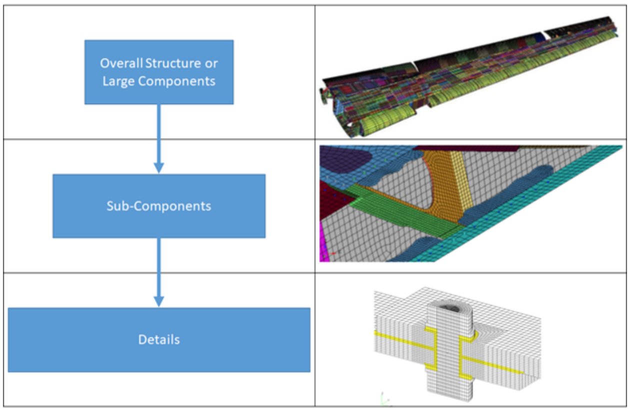


**Fig. 1** Finite element Modelling Hierarchy.

In the design of joints and splices, the engineer must consider the load transfer between components. This load depends on the fasteners flexibility. Thus, it has a great influence in the load distribution in a joint.

Since there are several ways to model a joint using 1D elements, the results can be very different and the designer is not always certain which method is more accurate.

## 1.1 Axial Stiffness

To define the axial stiffness of mechanically fastened joints with no pre-torque (such as riveted joints), can be approximated by a single rod[1]. The standard formula for a single rod is defined in equation 1.

$$k = \frac{E_f A}{t_1 + t_2} = \frac{E_f\left(\frac{\pi d^2}{4}\right)}{t_1 + t_2} \tag{1}$$

## 1.2 Transverse Stiffness

There are extra difficulties to obtain a general analytic formula to estimate transverse stiffness. Several empirical studies had been made and the final results of some of them are presented here. For more information refer to [1], [2], [3],[4], and [5].

- **Swift**

$$f = \frac{5}{E_f d} + \frac{0.8}{t_1 E_1} + \frac{0.8}{t_2 E_2} \tag{2}$$

- **Grumman**

$$f = \frac{(t_1 + t_2)^2}{E_f d^3} + 3.7\left(\frac{1}{t_1 E_1} + \frac{1}{t_2 E_2}\right) \tag{3}$$

- **Huth**

$$f = \left(\frac{t_1 + t_2}{2d}\right)\frac{b}{n}\left(\frac{1}{E_1 t_1} + \frac{1}{nE_2 t_2} + \frac{1}{nE_f t_1} + \frac{1}{2nE_f t_2}\right) \tag{4}$$

- **Tate and Rosenfeld**

$$f = \frac{1}{E_f t_1} + \frac{1}{E_f t_2} + \frac{1}{E_1 t_1} + + \frac{32}{9E_f \pi d^2}(1 + \nu_f)(t_1 + t_2) + + \frac{8}{5E_f \pi d^4} + \left(t_1^3 + +5t_1^2 t_2 + +5t_1 t_2^2 + t_2^3\right) \tag{5}$$

- **Morris**

$$f = \left\{\left(\frac{2845}{E_{L_1} t_1} + \frac{2845}{E_{L_2} t_2}\right) + c_f\left[\left(\frac{500}{E_f t_1} + \frac{1000}{E_{ST_1} t_1}\right)\left(\frac{t_1}{d}\right)^2 + \left(\frac{500}{E_f t_2} + \frac{1000}{E_{ST_2} t_2}\right) \times \left(\frac{t_2}{d}\right)^2 \left(\frac{d_{head}}{d}\right)^{-0.34} \left(\frac{s}{d}\right)^{-0.5} \left(\frac{p}{d}\right)^{0.34} e^{0.3r} \tag{6}$$

When the mesh is refined enough, a hole can be modeled with at least eight nodes[4]. When dealing with a coarser mesh, the union can be made by nodes or elements. Fig. 2 and Fig. 3 shows fasteners modelled as element do element and node to node conections respectively.

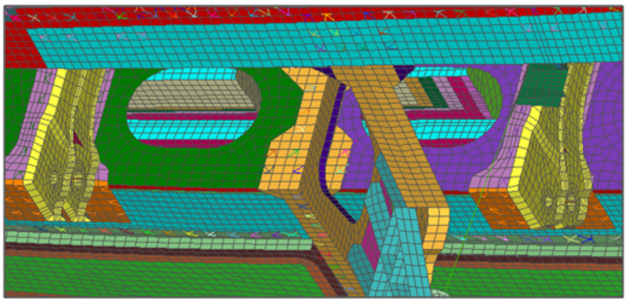

**Fig. 2** Detailed Model an elevator[4].

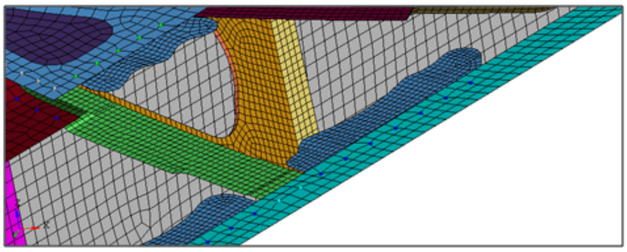

**Fig. 3** Detailed Model of a leading edge[4].

## 1.3 Multi Spring Models

The "multi-spring" modelling method, created by Rutman [6], is one of the most used in the aerospace industry and presents several advantages when compared with others. The most significant advantage comes from the fact that it can be used for any joint, with any number of elements joined together. Rutman's modelling technique differs from the traditional approach of a lap joint with a unique link element.

The traditional approach cannot be used for other joint configurations or joints with a larger number of connected components. The Rutman procedure is free from these limitations. However, Rutman did not validate his findings through experiments.

In the Rutman modelling technique described using the NASTRAN software, the fastener is represented by a CBAR (Beam) element and the contact between the plates and the fasteners is represented by a CBUSH (Spring) element.

The following components of rigidity are considered:

- Translational stiffness on plate bearing;
- Translational stiffness of the fastener bearing;
- Rotational stiffness on plate bearing;
- Rotational stiffness of the fastener bearing;
- Shear stiffness of the fastener;
- Bending stiffness of the fastener.

Fig. 4 illustrate the elements used in the multi-spring modelling approach.

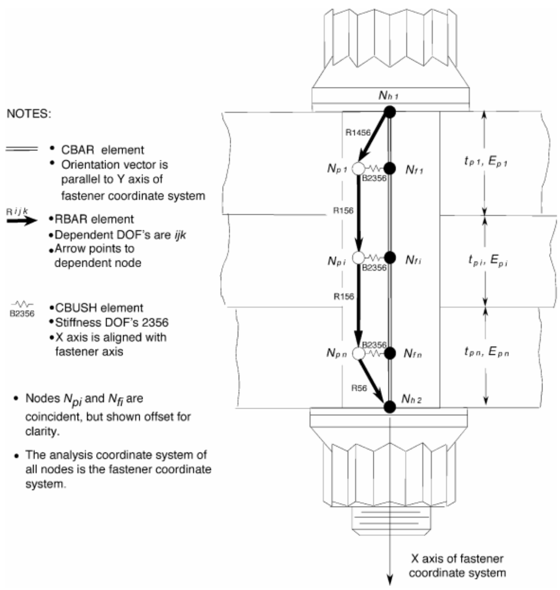


**Fig. 4** Multi Spring Model [6].

For more information please refer to reference [7] and [6].

## 2 Modelling Comparison Techniques

### 2.1 Flexibility equation comparison

For the following section, a series of models were built and the fastener stiffness(an input for the 1D element) was compared with the expected value. Since the sheet flexibility is known, the fastener flexibility can be indirectly estimated by[1]:

$$f = \left(\frac{\delta_{total}}{F_{total}} - \frac{l_0 + \frac{1}{2}l_1 + l_2}{EA}\right) \tag{7}$$

In this step, the models will be made in NASTRAN using different modelling techniques. The comparison will be carried out in the following way: The joint will be modeled, and it will be assigned a stiffness value according to the corresponding equation. The error of the modelling technique is computed as the difference between the expected stiffness (assigned to the spring using the empirical equation), and the measured joint stiffness using equation 7.

The test data used in this paper can be found in [1]. Since Morris equation was developed to fit in the data he collected, the comparison of different approaches will be made using his expression.

The mesh will be composed of square elements 2mm wide. The load was 0.4mm of enforced displacement. The models will be divided into four categories and for each of them, one model will be generated with the rigidity relative to each empirical equation. The categories are:

**1 - Model with gap:** Models with the plates modeled on the middle surface with a gap that is the result of the distance between the center lines of the plates, that is, equal to the sum of the thicknesses over two. The model used is shown in Fig. 5.

**Fig. 5** Model with gap between shell elements

**2 - Gap with Rigid Element Model:** Models with the plates modeled on the middle surface, exactly as before, however using the rigid element technique shown in Fig. 6.

A rigid element is created by replacing the link. It connects one of the plates to an auxiliary node created that matches the node on the next plate. The spring is then created between the two matching nodes.

Visually the model is exactly as in Fig. 5, but conceptually, it is eliminated (at least in theory) the secondary bending.

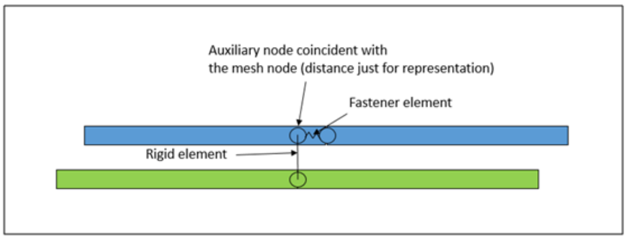


**Fig. 6** Technique of rigid element to model fasteners [8].

**3 - No gap model:** Models with the plates modeled in the plane of the connection, with the binding elements with infinitesimal length. See Fig. 7.

**Fig. 7** Model with no offset between shell elements

**4 - Element offset model:** Models with the plates modeled in the plane of the connection, with the offset applied to the shell element, maintaining the bending stiffness of the assembly. The technique is illustrated in Fig. 8.

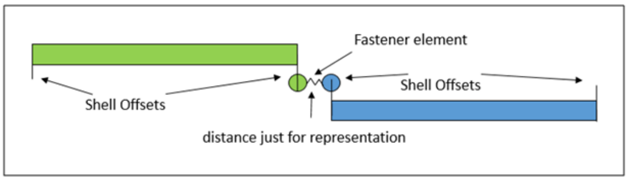


**Fig. 8** Model with no offset between shell elements and internal offset in the element property

**Table 1** Comparison between modelling techniques

| Model | Fx (daN) | f (mm/daN) | k (daN/mm) | $k_{expected}$ (daN/mm) | Difference |
|---|---|---|---|---|---|
| 1 | 192.3 | 0.00231111 | 432.69 | 1215 | 64% |
| 2 | 153.1 | 0.00337505 | 296.29 | | 76% |
| 3 | 283.0 | 0.00097805 | 1022.44 | | 16% |
| 4 | 254.1 | 0.00129978 | 769.36 | | 37% |

From the results obtained, shown in table 1, it can be seen that the numerical results are very poor. Even when the elimination of the sources of error, such as plates modeled in the same plane (type 3 modelling), the results still show significant differences.

When evaluating the results of the other models, it is noticed that the connection has become so flexible that it represents very little the actual structure. The following list will present some possible reasons for why this discrepancy may have occurred and then each hypothesis will be numerically evaluated.

**1)** The model can be very refined so that each element no longer meets the plate condition (two dimensions much larger than the other). According to [9], the ratio should be at least 1/20. This is hardly observed in refined models (for example, the thickness is equal to 1.27mm and the element size equal to 2mm). It is clear that the author refers in the overall analysis (dimensions of the specimen and not of the elements), but since there is a concentrated load, it may be that the model does not work well in the elements close to the connection, causing a large decrease in local rigidity.

Thus, it is suggested the evaluation of the relationship between mesh size and the calculated stiffness.

**2)** The next test would be to model the hole and create a rigid element to join the parts through a connector.

## 2.2 Study of the mesh size

In order to better understand the first hypothesis that pointed out for the differences between the stiffness that is used in the spring element in the model and the stiffness calculated through equation 7 , the calculation was redone by varying the size of the mesh. In this case, it was used as input the rigidity calculated with the Grumman equation and the plates modeled in the same plane (condition where the smallest error was found in the previous item). The result is shown in Fig. 9.

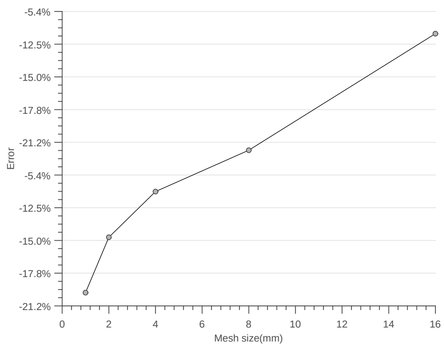


**Fig. 9** Influence of mesh size in the joint stiffness error.

As it was expected, the error tends to decrease when the mesh size increases. However, making a mesh as coarse as possible alone does not solve the problem of modelling fasteners, once, in general, the engineer needs to use models refined enough to represent the geometry. The model of the joint is simple enough to allow the use of elements 16mm wide. Even with the mesh this coarse mesh (Fig. 10), the error was not neglectable.

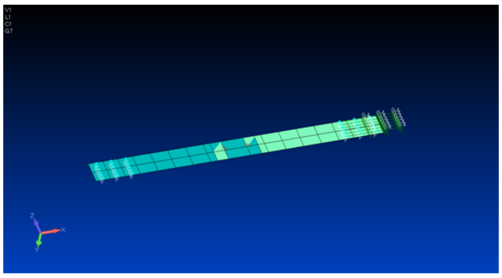

**Fig. 10** Joint model with element size equal to 16mm.

## 2.3 Study of additional modelling techniques

More ways of modelling are tested below. They are compared to the base model, with an element size equal to 2 millimeters. The base model will be denoted by "Model 1". The next configuration would be to create a RBE3[7] interpolation element in the area of influence of the fastener, i.e., all nodes that are at a distance of half the diameter. This model, shown in Fig. 11, will be denoted by "Model 2".

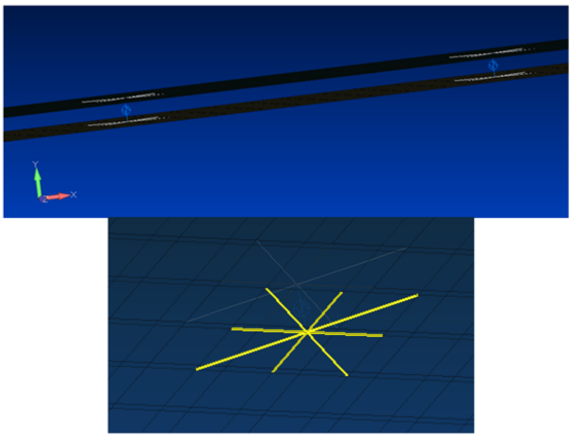

**Fig. 11** Model 2 details.

Next, a model with the same mesh size was built, but the hole was modeled as shown in Fig. 12. This model will be denoted by "Model 3".

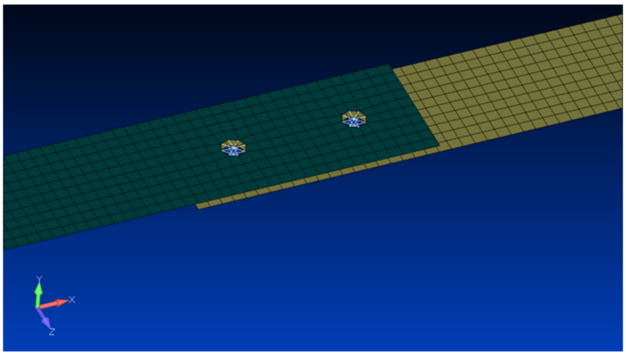

**Fig. 12** Model 3 details.

The three models were built with gap (plates modeled in the original center line) and without offset (plates modeled in the same plane). The results are shown in table 2.

**Table 2** Results of the improved models.

| Condition | Model | Force (daN) | f (mm/daN) | k (daN/mm) | k expected (daN/mm) | Error |
|---|---|---|---|---|---|---|
| With gap | 1 | 191.2 | 0.00233468 | 428.32 | 1214.66 | -65% |
| | 2 | 209.9 | 0.00196258 | 509.53 | | -58% |
| | 3 | 205.6 | 0.00204217 | 489.68 | | -60% |
| Without gap | 1 | 280.7 | 0.00100162 | 998.38 | | -18% |
| | 2 | 295.6 | 0.00085704 | 1166.81 | | -4% |
| | 3 | 292.1 | 0.00088966 | 1124.03 | | -7% |

It can be seen that the improvements are quite effective, especially when the error associated with the gap condition is removed. Model 2 got a relatively low error, even with a refined mesh.

## 3 Comparison with experimental results

In this section the experimental results shown in [1] are compared with finite element models. Models were created (one for each modelling approach) containing the four test specimens whose results are shown in section 3.2. One of the models is shown in Fig. 13.

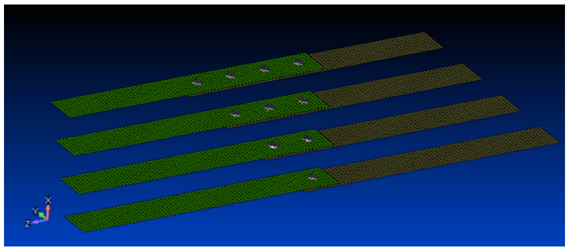

**Fig. 13** Finite Element Models for validation with test specimens.

It was noticed that the model of simple spring that obtained less numerical error was the model with the plates are modeled in the same plane and a rigid element is modeled in the nodes in the area of influence of the fastener. The Multi-Springs model will be used with the same characteristics, except that the plates will be modeled on their original average surfaces. To have a fair comparison, the simple spring model will be modeled also on the original surface.

Fig. 14 shows the comparison for the 1 row specimen.

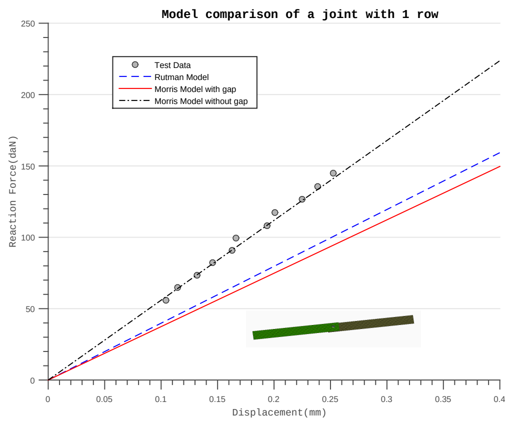


**Fig. 14** Comparison between numerical and experimental data for the 1 row specimen.

It can be seen that the best results for this case were for the model using Morris stiffness without gap. The Rutman and Morris models without gap obtained very similar poor results. This can be explained by the excessive secondary bending of the model that rotate the connection causing the model to move more than the specimen.

For models with two rows of rivets, the results were compared with the three test specimen results provided in [1]. The comparison is shown in Fig. 15.

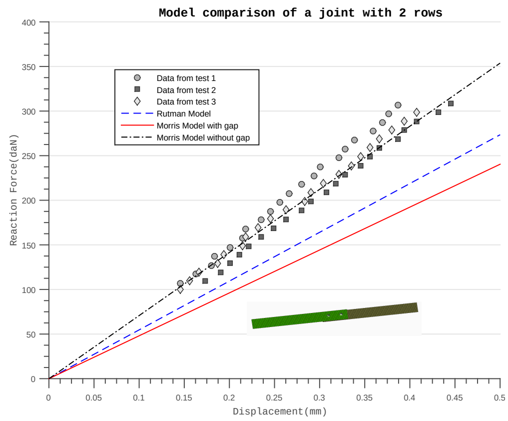


**Fig. 15** Comparison between numerical and experimental data for the 2 rows specimen.

Again, the Morris model with no gap is closer to that obtained in the tests, followed by the Rutman Model and the Morris model with gap. Fig. 16 shows the result for the three-row rivet model.

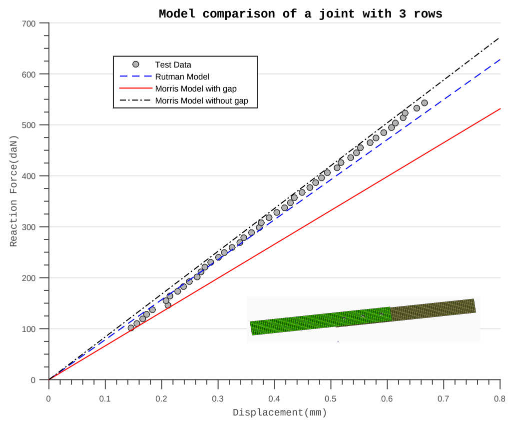


**Fig. 16** Comparison between numerical and experimental data for the 3 rows specimen.

In this case, the results were closer, and, the Rutman model behaved slightly better than the Morris model without gap. The Morris model with gap continued with more flexible results than the test. Next, in Fig. 17, the results are shown for four-row rivet models.

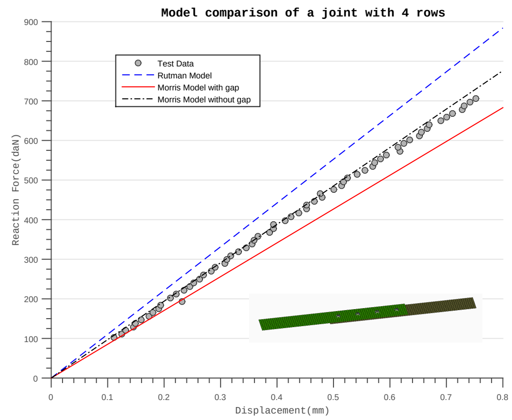


**Fig. 17** Comparison between numerical and experimental data for the 4 rows specimen.

In this case, all the results are reasonably good. The best approximation was the Morris model without gap.

Comparison with the tests gives a new perspective on the accuracy of the models. In general, the model using the Morris equation and without modelling the gap has the best result. As the number of rivets increases, the Rutman model tends to become stiffer and therefore closer to the test. In the four-row test, Rutman's model gets stiffer than the test and that can be an issue. This puts in check the trend of greater precision in a five-row test, for example.

Another point to consider is the fact that the Morris equation was empirically developed with the same data that was used. Therefore, all parameters were obtained by minimizing the error with the test, something that was not done with the Rutman model. Even so, when modelling gap structures (something extremely recommended for large assemblies), the error introduced by the model will significantly worsen the result.

Finally, it should be remembered that the study was performed for lap joints. In an aeronautical structure, there are several other types of riveted joints, such as butt joints, double shear and tension.

## 4 Use of high-fidelity models to compare the modelling techniques

Finally, to have a better understanding of the behavior and the accuracy of each modelling technique in other types of models, a high-fidelity model is built and its result is assumed to be correct. In order to make that assumption, the non-linear model parameters will be calibrated with the four specimen tested by Morris[1]. With the models calibrated, the same parameters will be used for all the other models.

### 4.1 Calibration of the high-fidelity models

The models were built in Altair Hypermesh [10] and the solver that was used were RADIOSS[11]. The baseline model were the one with two rows of fasteners. This choice was obvious, since it is the configuration which has the most data.

The model has 64898 nodes and 51814 elements. The elements are mostly eight-node bricks. The gap between the elements is 0.15mm and the friction coefficient is 0.3.

The test procedure was developed by Huth [2]. He tested several specimens with 6000 Falstaff loading cycles. The flexibility results after the cycles were the same of a quasi-static loading test at two thirds of the maximum load. The finite element analysis is non-linear static with a linear enforced displacement in one end of the plate. The curve was draw based in the Ramberg-Osgood 3 parameters)[12]. The material data was found in [13]. To comply with the test a penalty factor was applied equals to 2/3. The results for the 1 row specimen is shown in Fig. 18.

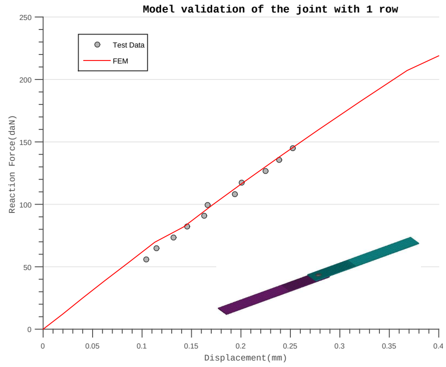


**Fig. 18** High-fidelity model results of the one row specimen.

The model accuracy was observed in the models with two, three and four row of fasteners too.This proves that the penalty factor of 2/3 is shown to be effective. Thus, the high-fidelity models can be then used to calibrate the low-fidelity ones. The next step consists in comparing the 1D approaches best (single and multi-spring) to model different joint configuration using the detailed model as a baseline.

### 4.2 Double Shear Joints

In this section the approaches of Rutman and Morris are compared with the detailed models using a single line of one to four fasteners. Both models use gap between plates. The results are shown in Fig. 19.

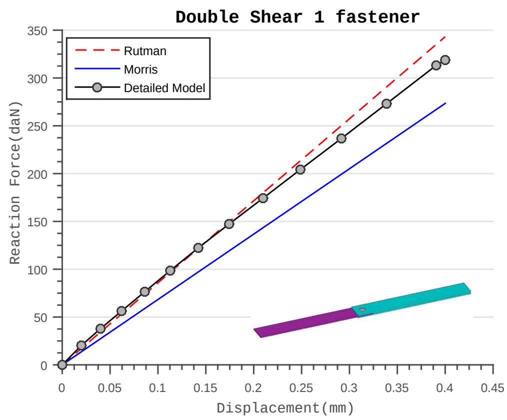


**Fig. 19** Comparison between 1D and 3D fasteners modelling of a 1 row double shear joint.

As can be seen from 19, the Rutman model is very close to the detailed one. The small difference occurs only in the nonlinear part of the Detailed Model.

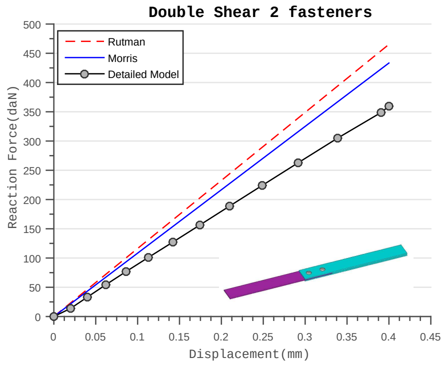


**Fig. 20** Comparison between 1D and 3D fasteners modelling of a 2 rows double shear joint.

Fig. 20 shows that both approaches tend to make the fastening more rigid when compared to the detailed model.

Using free-body diagrams it is possible to determine the load on each row of fasteners. The load on each fastener was obtained using the stresses before and after the fasteners. Fig. 21 shows the procedure to obtain the fastener load in the detailed model.

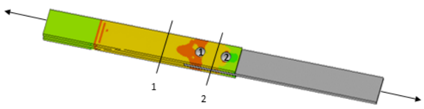


**Fig. 21** Free body diagram of a joint model.

Fig. 22 shows the obtained results.

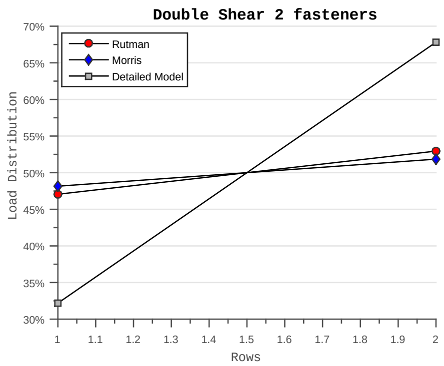


**Fig. 22** Load distribution in 2 rows double shear joint.

Although the average stiffness of the 1D joint models are higher that the detailed model, the load on the fasteners are non-conservative when compared to the detailed model (the detailed model presents a higher load concentration).

The tendency of the models to get stiffer and non-conservative increases with the number of fasteners in the joint. The difference between load distributions of single spring and multi-spring models are neglectable when compared to the detailed models.

## 4.3 Butt Joints

This section continue the study performed before. The main objective is to assert if the tendencies continue be observed for butt joints. Fig. 23 shows the results for the butt joint with one row.

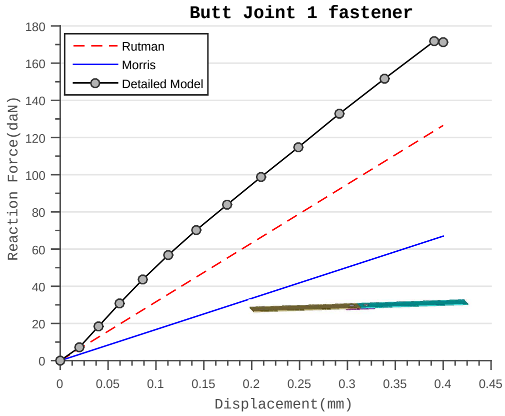


**Fig. 23** Comparison between 1D and 3D fasteners modelling of a 1 row butt joint.

The detailed model is much more rigid than the 1D models. The Rutman model provide a better representation, but still a poor representation of the joint behavior.

It was noticed that by using the same parameter as in the calibration, the detailed models of the 3x3 and 4x4 butt joint took several hours to run and presented a very unusual behavior. Thus, only the results from the 1x1 and 2x2 lap joints will be analyzed. In the case of a butt joint, the detailed model tends to be stiffer than the linear ones. On the other hand, the load distribution of the linear models tends to be conservative when compared to the detailed ones.

## 5 Conclusions

Based on the analyzes and results obtained, the following conclusions can be drawn:

- The results of the numerical experiments with the low-fidelity models show that the major sources of error in the modelling of fasteners is the secondary bending generated by the gap between the plates.

- Another source of error in fastener models is discrete load transfer. It has been observed that an efficient way of reducing error is by dividing the load in the area of influence of the fastener, being effective in all forms of modelling (single spring and multi-springs).

- The multi-spring model has been shown to be less sensitive to excessive secondary bending, however, as the number of rivets increases, it begins to become stiffer than the tested values.

- For the case of double and butt shear joints, none of the models presented satisfactory results. This shows that specific tests for these configurations are necessary for the development of more robust methodologies.

## References


[1] Geoff Morris. *Defining a standard formula and test-method for fastener flexibility in lap-joints*. PhD thesis, Technical University Delft, 2004.

[2] Heimo Huth. Influence of fastener flexibility on the prediction of load transfer and fatigue life for multiple-row joints. In ASTM, editor, *Fatigue in Mechanically Fastened Composite and Metallic Joints*, page 30, 1985.

[3] S. Rosenfeld M. B. Tate. Preliminary investigation on loads carried by individual joints - naca-tn-1051. Technical report, 1946.

[4] Rodrigo Martins. Investigação da precisão de modelos de juntas rebitadas utilizando elementos finitos, 2018.

[5] Swift T. Development of fail-safe design features of the dc-10. damage tolerance in aircraft structures. Technical report, 1971.

[6] Alexander Rutman. Fasteners modeling for msc.nastran finite element analysis. In SAE Internationa, editor, *World Aviation Conference.*, page 19, 2000.

[7] MD/MSC. Md/msc nastran quick reference guide. Technical report, MSC.Software Corporation., 2010.

[8] Rodrigo Martins. Influence of types of discrete modelling of fasteners in fem models. In NAFEMS, editor, *NAFEMS World Congress 2017*, page 26, 2017.

[9] A. Ugural. *Stresses In Plates and Shells*. McGraw-Hill, 1st edition edition, 1991.

[10] Altair. Hypermesh introduction pre-processing models for finite element analysis. Technical report, 2016.

[11] Altair. Radios user guide. Technical report, 2017.

[12] William R. Osgood Walter Ramberg. Description of stress-strain curves by three parameters. Technical report, 1943.

[13] FAA. Metallic materials properties development and standardization. Technical report, 2013.


## 6 Contact Author Email Address


For contacting the authors, please refer to:
rodrigomartins@demec.ufmg.br
marco.tulio@embraer.com.br
ernani@demec.ufmg.br


## Copyright Statement