STRUCTURAL OPTIMIZATION AND PROPOSITION OF PRE-SIZING PARAMETERS FOR UAVS WING SPARS

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Keywords: *UAV, structural optimization, spar, genetic algorithm, pre-sizing*

Abstract

UAVs are a growing segment in the aviation market and it has arisen the necessity of lighter and low cost design. Considering this, the present work's goal is to propose an optimized pre-sizing of an UAV wing spar using genetic algorithm optimization method followed by a finite elements analysis, which also provides low computational cost.

1 Introduction

Nowadays the unmanned aerial vehicles (UAVs) are responsible for the expansion of aerospace industry representing a growing segment of the aviation market. In this scenario, it is necessary lighter and low cost design to reduce the UAVs operational cost promoting their use and production not only in the universities but also in the industry.

This can be done by applying optimization techniques considering the suitable ones for each stage of the design process. The paper focus on the preliminary phase of the UAV wing structural design [1]. The methodology developed is based on the approximation concept discussed by [2] for a single-discipline optimization.

The methodology proposed was applied in the aircraft *Mach Crítico* designed at CEA (Centro de Estudos Aeronáuticos) - UFMG (*Universidade Federal de Minas Gerais*). The UAV competed in the the Regular Class of the SAE AERODESIGN BRASIL Competition 2017 (Society of Automotive Engineers). In this context, the aircraft must meet geometric restrictions and must not have a MTOW (Maximum Takeoff Weight) greater than 20kg [3]. Also, the project is evaluated through a score system, which is calculated based on the maximum payload and on the engineering solutions developed by the undergraduate students. The structural design must be the lightest possible to minimize the aircraft empty weight. In addition, the structure must be designed in a short time and with low-cost methods. This scenario is very similar to the current UAV industry and the methodology developed at CEA for the wing spar design is useful for similar aircrafts.

The *Mach Crítico* aircraft presented in Figure 1 received the "*Troféu Embraer de Excelência em Projeto*" (translation: Embraer Trophy of Design Excellence) by Embraer S.A. and the Award of Structure Optimization by Altair Engineering. These achievements show that the simple methodology developed for structural design gives satisfactory results for this type of aircraft.

Fig. 1 Mach Crítico UAV

The methodology is mainly divided in three parts: optimized pre-sizing, FEM analysis and compliance with aeroelastic criterion. Section

2.1 presents the first part, section 2.2 describes the second, and 2.3 presents the final one. Section 3 shows and discusses the results obtained for the *Mach Crítico* UAV. Sections 4 and 5 point out the results achieved and possible improvements in the methodology.

2 Methodology

2.1 Optimized structural pre-sizing

2.1.1 *Structure Modeling*

The wing structure must resist to all external loads and the most significant ones are the lift force and the aerodynamic moment for light UAV. The loads were previously calculated considering methodology suggested by Iscold [4] and Neves [5] which models the airplane as a rigid body. In the present work, the semi-wing spar is modeled as a cantilever beam and as a cantilever torsion shaft once it has been idealized as the structure supposed to resist to all loads. This decision was made to increase the wing structural efficiency. The final semi-spar is presented in Figure 2.

Fig. 2 Final semispar.

According to Niu [6], the section method is used to compute the internal loads, such as shear force, bending moment and torsion moment along the semi-span. The beam is sectioned off at multiple desired coordinates along the semi-span and the equilibrium equations are applied to the sectioned structure. The unknown variables will be the internal loads since the external loads are previously known. The shear force and bending moment due to aerodynamic loads can be determined regardless the shear center position. However, once the spar position along the chord is one of the optimization parameters for each section optimized, the resulting applied torsion moment at the shear center (which is a function of the spar position) has to be calculated during the optimization process, as shown below. In this work, the shear center was approximated as the cross-section center.

$$dT = dM + \delta dL \tag{1}$$

dT - applied torsion moment at the section of interest at the shear center;

dM - aerodynamic moment at the section of interest at the aerodynamic center;

dL - lift force at the section of interest at the aerodynamic center;

δ - distance between the shear center and the aerodynamic center.

Moreover, since the torsion moment at the root section depends on the torsion moment of the next sections, the optimization process starts from the tip to avoid unknown parameters. Therefore, it calculates the ensuing section up to the root using the known values of torsion moment from the previous sections starting from the tip.

The direct stresses in each section due to internal loads are determined according to Megson [7] using the Euler-Bernoulli Beam Theory as expressed by Equation 2.

$$\sigma_{yy} = E_{z,i} \cdot \frac{M_x \cdot (I'_{zz} \cdot z - I'_{xz} \cdot x)}{I'_{xx} \cdot I'_{zz} - Ixz'^2} \tag{2}$$

σ_{yy} - tension or compression stress;

$E_{z,i}$ - laminate elasticity modulus;

M_x - bending moment;

$I'_{xx} = \int E_{z,i} \cdot z^2 dA$- modified second moment of area about x axis;

$I'_{zz} = \int E_{z,i} \cdot x^2 dA$- modified second moment of area about z axis;

$I'_{xz} = \int E_{z,i} \cdot x \cdot z dA$ - modified second moment of area about the axis perpendicular to x and z axes;

x - x coordinate (from the centroid);

z - z coordinate (from the centroid).

In this work, every ply of laminate is directed with a 0°angle ply which means $E_{z,i}$ is constant along the cross-section area. This simplifies the latter equation to the Equation 3.

$$\sigma_{yy} = \frac{M_x \cdot (I_{zz} \cdot z - I_{xz} \cdot x)}{I_{xx} \cdot I_{zz} - Ixz^2} \quad (3)$$

σ_{yy} - tension or compression stress;
M_x - bending moment;
$I_{xx} = \int z^2 dA$- second moment of area about x axis;
$I_{zz} = \int x^2 dA$- second moment of area about z axis;
$I_{xz} = \int x \cdot z dA$ - second moment of area about the axis perpendicular to x and z axes;
x - x coordinate (from the centroid);
z - z coordinate (from the centroid).

The maximum shear stress in the webs due to the shear force in addition with the shear stress caused by the torsion moment is calculated as expressed in Equation 4. Here, it is already considered that all plies in the laminate are directed with a 0°angle ply.

$$\tau_{zy} = \frac{V_z \cdot Q_{x-max}}{I_{xx} \cdot 2 \cdot t_w} + \frac{T}{2 \cdot A \cdot t_w} \quad (4)$$

τ_{zy} - shear stress;
V_z - shear force;
Q_{x-max} - first moment of area about the cross-section neutral axis;
t_w - web thickness;
T - torsion moment;
A - area enclosed by the median contour.

The angles of twist are calculated, according to Megson [7], during the pre-sizing with the Bredt's Formula presented in Equation 5 already simplified for all plies in the laminate directed with a 0°angle ply.

$$\frac{d\theta}{dy} = \frac{1}{2 \cdot A \cdot G} \int \frac{q}{t} ds \quad (5)$$

s - coordinate along the contour;
y - coordinate along the span;
G - material's shear modulus;
A - area enclosed by the section median contour;
t - wall thickness;
T - torsion moment;
$q = \frac{T}{2 \cdot A}$ - shear flow;
θ - angle of twist.

Then, the stresses are compared with the allowable stresses of the material considering ultimate loads in order to determine either the structure is safe or not. Also, the angle of twist is compared to the allowable angle of twist as shown in section 2.1.3.

2.1.2 Parameterization and Strategies

As discussed earlier, the problem is to optimize the whole wing spar, so it consists of determining the materials used and the structure's dimensions along the span.

First, the wing spar materials play an important role in the spar design so it has to be decided which materials will be analyzed and their mechanical properties have to be known. The wing of *Mach Crítico* UAV has a complex geometry as shown in Figure 1. Therefore, in this case, the material chosen was carbon fiber laminate in order to meet aerodynamic requirements. Even though the results presented here are for carbon fiber laminate, the method presented can be easily adapted to other materials. The mechanical properties of the carbon fiber laminate were determined at CEA-UFMG via tensile, compression and shear tests according to ASTM D3039, ASTM D6641, and ASTM D7078. The PVC foam mechanical properties were determined by the foam manufacturer. All properties are presented in tables 1 and 2.

PVC foam mechanical properties	
Limit stress:	
Tension [MPa]	1.8
Compression [MPa]	0.9
Shear [MPa]	0.78
E [MPa]	72
G [MPa]	21
Density [kg/m^3]	60

Table 1 PVC foam mechanical properties.

Bidirectional carbon fiber mechanical properties		
	Parallel	Perpendicular
Limit stress:		
Tension $[MPa]$	682	682
Compression $[MPa]$	341	341
Shear $[MPa]$	72	72
E $[MPa]$	51000	48929
G $[MPa]$	2000	1980
ν	0.28	0.02
Density $[kg/m^3]$	1545	1545

Table 2 Bidirectional carbon fiber mechanical properties.

The second step is to brake the problem into a finite number of spar sections along the wing semi-span. For the *Mach Crítico* UAV, the wing semi-span is 1.30m and it was devided into 21 sections (Figure 3) in order to meet both manufacturing requirements and the structural theory hypothesis.

Fig. 3 Wing spar sections for the *Mach Crítico* UAV

With both material and number of sections defined, the next step is to choose which beam cross-sections will be analyzed. In order to resist torsion efficiently, the beam cross-section needed to be a closed one. Therefore, the beam cross-sections proposed and theirs suggested parameterization are presented in Figure 4 and Figure 5.

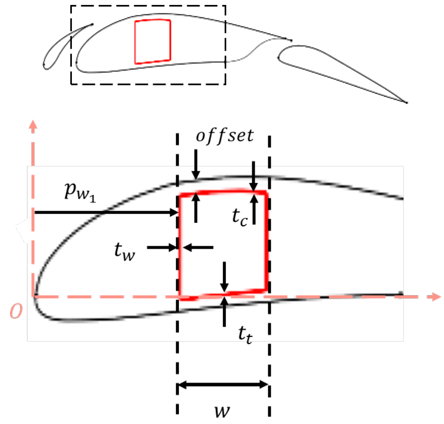

Fig. 4 Suggested cross-section 1

Number of parameters: 6

p_{w_1} - position of the first web;

w - width of the spar;

$offset$ - space between the upper camber and the upper cap;

t_w - web thickness;

t_c - upper cap thickness;

t_t - lower cap thickness.

The $offset$ was introduced because of manufacturing methods used to guarantee the aerodynamic profile.

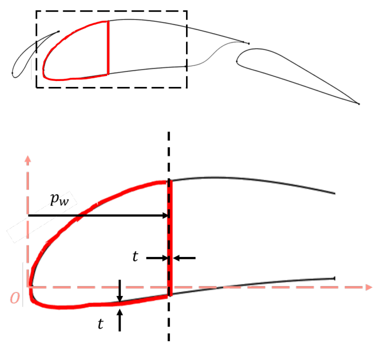

Fig. 5 Suggested cross-section 2

Number of parameters: 2

p_w - position of the web;

t - spar thickness.

2.1.3 The Optimization Problem

The optimization problem is to minimize the weight of the spar:

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & W_{spar}(x) \\ \text{subject to} \quad & b_{l_i} \leq x_i \leq b_{u_i},\ i = 1, \ldots, m. \end{aligned} \tag{6}$$

W_{spar} - spar weight i;
x_i - design variable i (See figures 4 and 5);
m - number of design variable;
b_{l_i} - lower bound for x_i;
b_{u_i} - upper bound for x_i.

This is a single-objective optimization and it is structured to be with no constraints since penalties are applied to the objective function when the structure is not suitable. Also, some of the design variables are discrete since their values depend on the number of carbon fiber plies, for example, the wall's thickness is a multiple of the thickness of the ply. Therefore, the single-objective optimization is a mixed discrete/continuous problem, unconstrained and bounded since the design variables have upper and lower limits according to geometric restrictions and manufacturing requirements. A genetic algorithm ('DE/rand/1/bin' [8]) was chosen due to the nature of the optimization problem [9] and it was implemented using MATLAB.

One optimization is performed for each panel in order to have an optimization process with a low number of parameters. Considering the *Mach Crítico* UAV, if the whole spar was optimized at once, the option presented in Figure 4 would end up with 126 parameters and the one presented in Figure 5, 42. These numbers of parameters may lead to difficulties while solving the optimization problem since it has multiple local minimums. It is not guaranteed that the global minimum is reached in a short time, however decreasing the number of parameters, the optimization leads to results faster. Therefore, it was decided to perform an optimization for each panel using the genetic algorithm since it is efficient and it can handle the characteristics of the optimization problem with the parameterization proposed. Furthermore, the genetic algorithm is simple to implement, to set up the optimization and it also can be easily parallelized. The goal with this methodology is to obtain satisfactory results with low-cost and in less time.

The penalty strategy is applied in the objective function. It calculates the mass of the panel whose section is being optimized by estimating its volume and multiplying it by the material's density. Thus, the stresses are calculated and each one is compared to its corresponding allowable stress. If the allowable stress is greater than the calculated one there is no penalty applied to the mass panel of the section tested in the optimization. On the other hand, if the allowable stress is overcome by the calculated one, the mass panel of the section tested is multiplied by a factor proportional to how much it fails by as illustrated in equations 7 and 8. The constant was iteratively defined in order to achieve the best optimization solution.

The wing's angle of twist is an important parameter during the design process and one of the most efficient ways of reducing the angle of twist is to increase the spar's closed section area. In addition, according to Barros [10], the total angle of twist should not be greater than 4° when 62.5% of the maximum torsion moment is applied. However, these 4° can be distributed along the semi-span in very different ways. For each distribution, there is an optimum choice of spar. Consequently, many distributions were generated and the optimization process was tested with them to identify which one of the distributions fitted best in the problem analyzed (which one resulted in the lighter spar).

For the *Mach Crítico* UAV, the best angle of twist distribution found was the one represented by the black curve shown in Figure 6. This distribution was adopted as the allowable angle of twist. On each section the incremental value of the allowable angle of twist is compared to the calculated one. If the calculated is greater than the allowable one, there is a penalty applied to the tested section as shown in Equation 9. The multiplying constant was chosen in order to obtain the lightest spar.

$$M_{final} = (P_{stress} + P_{twist\ angle}) \cdot M_{Estimated} \tag{7}$$

$$P_{stress} = C_{stress} \cdot \frac{\sigma - \sigma_{allowable}}{\sigma_{allowable}} \tag{8}$$

$$P_{twist\ angle} = C_{twist\ angle} \cdot \frac{\Delta\theta - \Delta\theta_{allowable}}{\Delta\theta_{allowable}} \tag{9}$$

M_{final} - panel mass considered during optimization;
$M_{Estimated}$ - estimated panel mass;
P_{stress} - penalty due to stress evaluation;
σ - calculated stress;
$\sigma_{allowable}$ - allowable stress;
C_{stress} - penalty constant of angle of twist;
$P_{twist\ angle}$ - penalty due to twist angle evaluation;
$\Delta\theta$ - calculated panel angle of twist;
$\Delta\theta_{allowable}$ - allowable panel angle of twist;
$C_{twist\ angle}$ - penalty constant of angle of twist.

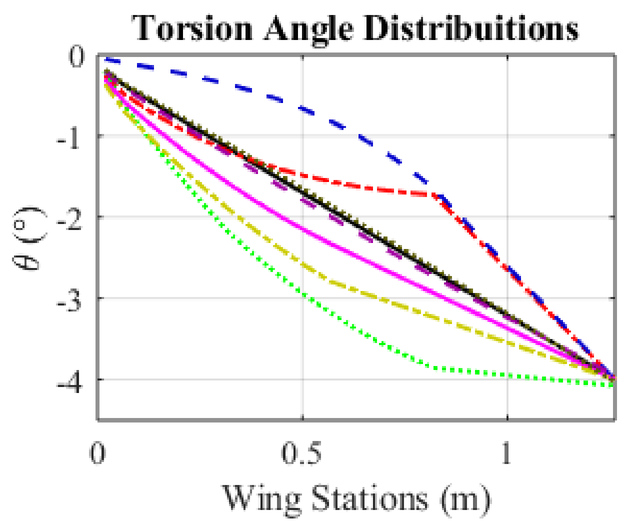

Fig. 6 Evaluated angle of twist distributions - *Mach Crítico* UAV

2.1.4 Post-Processing of the Pre-Sized Spar

As it was presented before, each section is pre-sized and positioned along the chord separately which does not guarantee a manufacturable and smooth spar. Therefore, it is necessary to analyze the first results and identify its tendency. Then, another whole optimization process may be performed with new upper and lower bounds for spar position based on the analysis made. The new results shall be analyzed and new bounds defined for another optimization until the final and manufacturable spar is defined. The spar position along the chord and its width deserve a special attention when it comes to the adjustments to ensure it is manufacturable. The Figure 7 shows the difference between the adjusted spar width and the optimized spar without modifications for the *Mach Crítico* UAV.

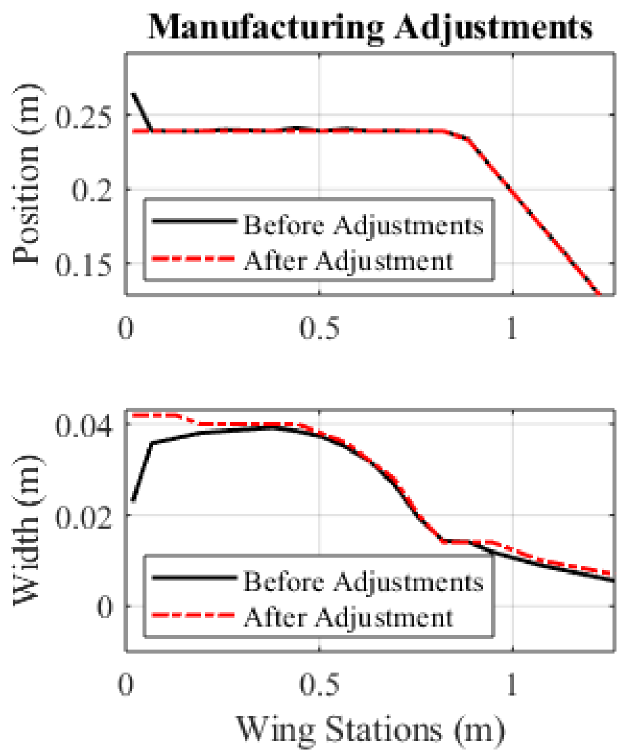

Fig. 7 Comparison between adjusted spar width and the optimized spar without modifications - *Mach Crítico* UAV

2.2 Buckling Analysis

It is known that the buckling problem is efficiently solved using a sandwich structure made of carbon fiber and PVC foam. However analyzing this kind of structure during an optimization process would increase its computational and time costs. These are the reasons why the spar buckling is supposed to be analyzed after the structure is pre-sized in order to reduce model complexity.

The pre-sized structure was analyzed in HyperWorks, an Altair FEM software. It was modeled with the element CQUAD4 and the property PCOMP, the constraints were applied to the holes of the ribs which connect the wing to the fuselage using the rigid element RBE2 and restricting 5 degrees of freedom. The ultimate loads were applied to the structure using the rigid element RBE3.

2.3 Aeroelastic criterion

Also, modifications may be necessary in order to the spar comply with aeroelastic criterion. For light UAV, it is suggested to use [11] as a requirement since its application is simple and conservative. The torsional flexibility of the wing should not be greater than a constant which is function of the dive speed [11]:

$$F = \int \theta_i C_i^2 ds \tag{10}$$

$$F \leq \frac{200}{V_D^2} \tag{11}$$

F - torsional flexibility of the wing;
θ_i - angle of twist of the wing at station i, per unit torsional moment applied at a wing station outboard of the end of the aileron ($\frac{rad}{ftlb}$);
C_i - wing chord length at station i (ft);
ds - increment of span (ft);
V_D - Design dive speed (IAS) of the airplane.

An increase in the cross-section area of the spar may be made so the torsional stiffness of the wing is increased and the torsional flexibility decreased.

3 Results

The final spar passed through three main process: the optimized pre-sizing, the FEM analysis and aeroelastic verification.

The optimized pre-sizing was performed for two kinds of beam cross-sections, as presented before. The lightest one was the suggested cross-section 1 with half of the suggested cross-section 2 mass. One of the reasons for this difference is that the second type of cross-section had its torsional axis closer to the leading edge than the first one, having to resist to a greater torsional moment. The angle of twist was the most demanding characteristic in the optimization. The optimization process was continued for the section 1 and after altering the parameter's bounds and optimizing 4 more times to meet manufacturing requirements, the final results of the pre-sized spar are shown in Figure 8.

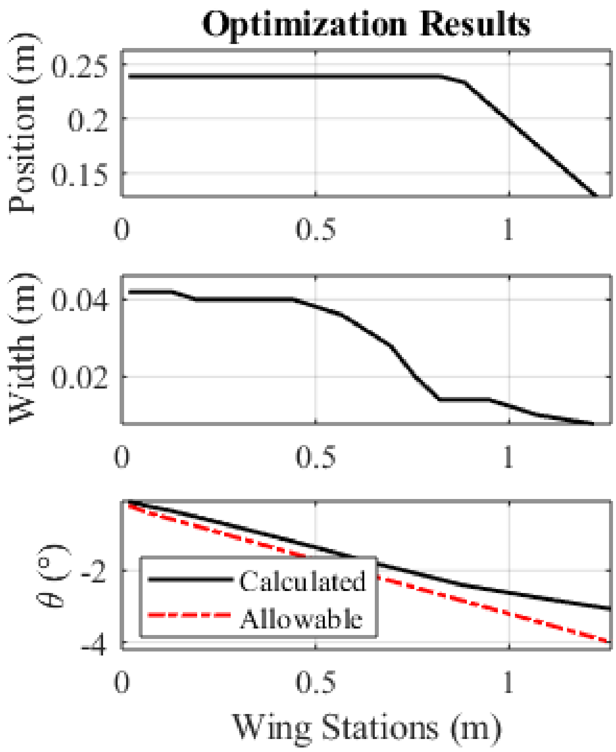

Fig. 8 Results of pre-sizing optimization after manufacturing adjustments.

Position(m) - first web position measured relatively to the wing trailing edge;
θ (°) - angle of twist;
Width(m) - distance between the webs.

Second, the FEM analysis were performed. The Linear Static Analysis was performed using the Tsai Wu failure criteria and the structure did not fail, as expected. Also, the Linear Buckling Analysis was performed revealing a severe buckling issue. The most critical regions were reinforced with a sandwich structure while others only needed another ply of carbon fiber. The adjustments added 92 g to the spar's weight. Although the pre-sized thickness was altered, its position along the chord and its geometric form was maintained. The final buckling eigenvalue of the spar was 1.025 for its ultimate load as shown in Figure 9. It was desired a small margin of safety in buckling at this moment of the design because the *Mach Crítico* UAV wing had another carbon fiber laminate structure, the slat, which was not being considered initially. Later, another model was built for the whole wing and the buckling FEM analysis showed that the spar final buckling eigenvalue was 1.231.

Fig. 9 Buckling analysis results after the spar was modified to meet the buckling requirement.

Third, the analysis of the aeroelastic criterion [11] showed that it was necessary to reduce the torsional flexibility. This was done by augmenting the spar width in some sections which added 22 g. The modifications allowed the spar to resist a greater torsional moment. Thus, it was decided to change the spar position along the chord bringing it closer to the leading edge. This choice was made because it would not add any weight to the structure and the torsional axis would be between the leading edge and the quarter chord, which is a desirable characteristic in the standpoint of aeroelasticity.

The design evolution during these processes are presented in the tables 3, 4 and 5 . The thickness is in number of carbon fiber plies of 0.25 mm each, and S stands for sandwich which means that there is a sandwich of 4 mm thickness PVC foam and two carbon fiber plies of 0.25 mm thickness. The spar's webs positions along the chord are measured relatively to the fourth of ninth rib section chord for the first nine sections while the others are measured relatively to there own fourth of chord, as shown in the Figure 10 below. All carbon fiber plies are at 0°angle.

Rib	1	4	7	8	12	16
Station [mm]	35	320	600	693	1021	1294
p_{w_1} [mm]	20	20	20	20	-5	-5
w [mm]	42	40	34	28	12	7
Number of Carbon Fiber Plies						
t_w	1	1	1	1	1	1
t_t	1	1	1	1	1	1
t_c	1	1	1	1	1	1

Table 3 After optimized pre-sizing - Mass of the spar: $125g$

Rib	1	4	7	8	12	16
Station [mm]	35	320	600	693	1021	1294
p_{w_1} [mm]	20	20	20	20	-5	-5
w [mm]	42	40	34	28	12	7
Number of Carbon Fiber Plies						
t_w	S	2	2	2	1	1
t_t	1	1	1	1	1	1
t_c	S	S	2	2	1	1

Table 4 After FEM Analysis - Mass of the spar: $217g$

Rib	1	4	7	8	12	16
Station [mm]	35	320	600	693	1021	1294
p_{w_1} [mm]	-30	-30	-30	-30	-15	-15
w [mm]	42	40	34	30	30	30
Number of Carbon Fiber Plies						
t_w	S	2	2	2	1	1
t_t	1	1	1	1	1	1
t_c	S	S	2	2	1	1

Table 5 After Aeroelastic Verification using [11] as criteria - Mass of the spar: $239g$

p_{w_1} - is the position of the first web measured relatively to the $\frac{1}{4}$ of the reference chord;

S - means that the section is a sandwich structure composed by carbon fiber and foam.

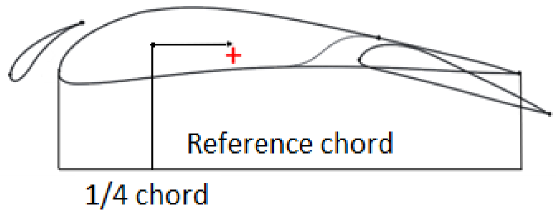

Fig. 10 Coordinate system for the first web position.

4 Discussion

The design evolution began with the optimized pre-sizing. In this phase the structural theories used were simple and the spar was pre-sized considering direct and shear stresses and the angle of twist. It was pre-sized and positioned along the chord and weighted 125g. After, in FEM analysis the Tsai Wu failure criterion was checked and the buckling problem was solved adding plies and a sandwich structure where it was fit to. The modifications added 92g. Then, the torsional flexibility had to be reduced, as the analysis using the aeroelastic criterion in [11] showed, augmenting the width of some sections. It added 22g. Also, the spar position was modified to reduce the distance between the leading edge and the torsional axis with no addition in weight. The last change, not mentioned before, was in the manufacturing method and it added 61g. Every increase in weight was justified and some were expected before hand.

As a way of verifying if the methodology proposed here was successful, its structure weight was compared to another wing structure weight made of composite materials and composed of a spar and a torsion box. Both aircrafts had similar MTOW's. The other structure was 27% heavier than the one presented here.

The design methodology presented here was suitable for the *Mach Crítico* UAV. However, there are ways of improving it. Adding the angle of ply to the optimization parameters may be profitable. Also, using a composite stress criterion during the pre-sizing is a suggestion. New types of beam cross-sections may be proposed and evaluated. Also, other kinds of algorithms for optimization may be tested. It is also recommended to repeat the optimization process considering the approximation concept [2]. In other words, after the modifications were made in the pre-sized spar, another optimized pre-sizing should be done with new bounds which consider the modifications. An example of how the modifications can feed the pre-sizing is the change in the spar position along the chord after the changes were made.

5 Conclusion

The wing spar design presented here had the purpose to design a light structure for the UAV in a short time. The methodology adopted had three main phases. The first one was the spar optimized pre-sizing using a genetic algorithm implemented in MATLAB along with the Euler-Bernoulli Beam Theory. Then, the spar was modified to meet aeroelastic criterion. Finally, the spar was analyzed in a finite elements software and the buckling problems were solved using a sandwich structure. The resulting structure was satisfactory and the design process was fast enough. The methodology developed here achieved its goals. It can be adapted to other UAVs designs and it can be improved as suggested before.

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