DIGITAL HYDRAULICS IN AEROSPACE APPLICATIONS: SLIDING MODE OF QUANTIZED CONTROL AND LMI APPROACH

Lie P.G. Pinto* , Petter Krus* , Victor J. De Negri**
*Linköping University, SE-581 83 Linköping, Sweden,
**Federal University of Santa Catarina, 88040-900, SC, Brazil

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Abstract

The use of digital hydraulics (DH) has many advantages for aerospace applications, for instance safety, due the intrinsic redundancy, and energy saving on the control surfaces actuators. The energy saving is obtained by having the hydraulic valves completely opened or closed, avoiding the throttling as it happens in a regular proportional valve. It is also important to reduce the amount of unnecessary switchings while controlling DH, which reduces the system lifespan and increases the power waste, cutting off the main reasons for using this technology. Since the available forces in digital hydraulics are quantized, when the system trajectory crosses the limit between two force levels, a sliding mode phenomena may occur, which is relevant for the control design. When the system trajectory reaches an attractive sliding surface, successive switchings will occur until the trajectory reaches the desired equilibrium point. For performing different maneuvering and flying under different conditions, the desired equilibrium points for the aircraft control surfaces must belong to a continuous set. As long the forces available for the DH belongs to a discrete set, the desired equilibrium point is only reached in sliding mode. In this paper it is proposed a linear matrix inequality (LMI) design approach for defining the control gain and reducing the amount of unnecessary valve switches using optimal control and sliding mode theory.

1 Introduction

Despite of the intense effort in using electric actuator in aircrafts, hydraulics actuators are vastly used in its control surfaces [3]. Digital fluid power is a technology focusing the use of low-cost on/off valves instead of a servo valves in order to reduce the power losses due the throttling, avoid internal leakage and recover the mechanical power. It can be done a correspondence between a power electronics and hydraulics, where a rheostat relates to the conventional proportional valve and the digital hydraulic are related to solutions based on thyristors [7]. For more information on digital hydraulics, please refer to [1, 8, 9]

In regular flight conditions, the reaction force on the aircraft's control surfaces acts like a spring, increasing the force as its angle is increased. As the angle of the control surface increases, the power is transferred from the system to the control surface. In other hand, when the control surfaces moves back to its neutral position, due the reaction force, the power is transferred from the control surface back to the system, see Figure 1, and if it is not used in any other device, it must be wasted. With digital hydraulics, this power may be transferred from different supply lines through the cylinders, as when the control surface returns to its neutral position, the cylinder acts like a pump, recovering part of the energy spent on the control [7].

The hydraulic power is the product of flow

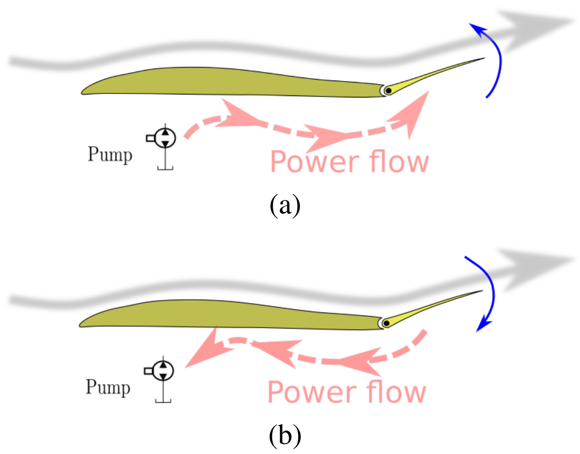


Fig. 1 : Power flow on the actuation of an aircraft control surfaces

and pressure difference ($H_{hydr} = q_v \cdot \Delta p$). In hydraulic valves, there is no power loss when the valve has no resistance to the flow, resulting zero pressure drop ($\Delta p = 0$) or when the valve completely blocks the fluid, resulting in zero flow ($q_v = 0$). As long the valve commutation is not instantaneous, there is a region between completely open and completely close, where the valve itself consumes power. Another source of power waste in this system is the fluid compressibility on the cylinder and pipes. For this reason, among others, one important aspect when controlling this kind of system is reducing the amount of unnecessary switchings.

Sliding mode is a mode of some discontinuous dynamic system, when the its trajectory reaches the set of the discontinuous points, called sliding surface, and gets trapping into this set, sliding along this surface, refer to [10, 11] on sliding mode theory. Due the quantization characteristics of the digital hydraulics, in the general case, the trajectory reaches the equilibrium point only in sliding mode, meaning that the hydraulic valves should be switching between two force levels in order to keep the trajectory around the equilibrium point. Even when the systems trajectory is moving towards the equilibrium point, it might crosses the boundary between two different quantization levels, this boundary can be an attractive surface, producing a sliding mode, resulting in successive unnecessary switching, increasing the energy wasting. More details on this point will be further discussed.

Most of the solutions for controlling quantized systems minimizes the error caused by the quantization [4, 5], regardless to the sliding mode effect. In [14] a previous control gain is defined before designing the sliding mode surface, i.e. the system should be stabilized before designing the sliding surface. When minimizing the whole state variables, the resulting sliding surface may be such that the system trajectory is attacked to the sliding surface far from the equilibrium point, using the switchings to lag the trajectory in reaching the desired equilibrium point. An example of this problem is shown in the Figure 2 with the plot in red, where the equilibrium point is $x_{eq} = [1.0, 0.0]$ and the initiation condition is $x_0 = [0.0, 0.0]$. On the same Figure 2, in blue, a new sliding surface was designed in order to be attractive only near the equilibrium point. In this case, the trajectory crosses the sliding surface at $x = [0.7, 12.5]$, meaning that the sliding surface is not attractive at this point. When the trajectory meets the sliding surface near the equilibrium point, the trajectory actually enters in sliding mode and reaches the desired equilibrium point.

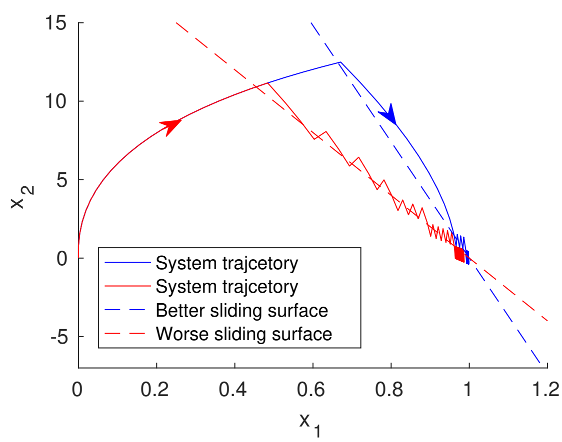


Fig. 2 : Reducing the amount of switches using a different sliding surface

On this paper a new approach to dealing with quantization input is presented in such manner that the goal is to have the system trajectory reaching the equilibrium as fast as possible, instead of reducing the effect of the quantization on the system output. On the proposed designing method, the quantization is seen as both sliding mode, when the desired equilibrium point is on the discontinuity between two quantization levels, and deadband control, when the desired equilibrium point is inside a quantization level. The paper is organized as follows: On the Section 3 is described the link between the quantization and sliding mode and how the sliding mode is inevitable on the equilibrium point. On the Section 4 the system is modeled, considering simplifications from the original non-linear system and convenient definitions for the Section 5, where the main result is given. On the Section 6 it is presented the numeric example and simulations where the results are compared with [7]. On the last Section 7 it is presented the conclusions and final remarks.

2 Nomenclature

Along the paper the time dependence is omitted for simplicity, when it is understood by the context. A' represents the transpose of a matrix A. $Q > 0$ or $Q < 0$ represents that all the eigenvalues are positive or negative respectively. $\|\cdot\|$ is the vector norm. $\mathbf{Tr}(A)$ is the trace of the matrix A. The symbol $\star$ represents the matrix entry that preserves the symmetry. Curly brackets $\{\cdot\}$ are used to define sets, and square brackets for matrices or continuous intervals.

3 Sliding Mode Effect of Quantized Systems

For simplicity, it is considered on this paper systems with scalar inputs, but the presented results are easily extended for vector inputs. Consider the state space system model (1).

$$\dot{x}(t) = Ax(t) + Bq(u(t)), \tag{1}$$

where $x(t)$ and $u(t)$ are the state variables and the input, respectively, and the system matrices are $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{1\times n}$. The function $q(u)$ is the quantization function defined as (2), where $\mathbf{round}(\cdot)$ rounds a number to the nearest integer and Δ is the quantization interval.

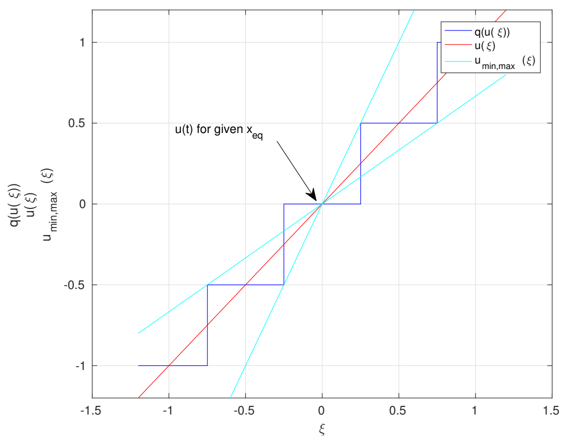


Fig. 3 : Deadband of quantized control

$$q(u) = \Delta \cdot \mathbf{round}(u/\Delta). \tag{2}$$

The system model (1) represents a time-invariant linear system with quantized input. By changing the equilibrium point x_{eq} of (1), the function $q(\cdot)$ may express itself as two two different effects on the control:

i. Deadband control: the equilibrium point x_{eq} is such that the input $u(t)$ lays inside any of the quantization intervals, causing the system (1) to be in open-loop, while the control input in constant, performing no stabilization. This situation is called deadband non-linearity. See Figure 3 where the red line represents $u(t)$ and the blue line its quantization $q(u(t))$.

ii. Sliding mode control: the equilibrium point x_{eq} is such that the input $u(t)$ lays in the border between two quantization intervals. In this case (2) can be written as

$$q(u) = \Delta\left(\frac{1}{2}\mathbf{sign}\left(\frac{u}{\Delta} - k - \frac{1}{2}\right) + k + \frac{1}{2}\right),$$
$$\text{for } u \in \mathcal{B}(k\Delta + \frac{\Delta}{2}),\ k \in \mathbb{Z}, \tag{3}$$

where $\mathcal{B}(p)$ is a (small) open ball centered in p. Note that the therm $\Delta\left(k + \frac{1}{2}\right)$ of $q(u)$ in (3) only shifts the equilibrium point x_{eq} and the control is governed by the $\mathbf{sign}$ function. An example of this is shown in the Figure 4 for $k = 0$. If the

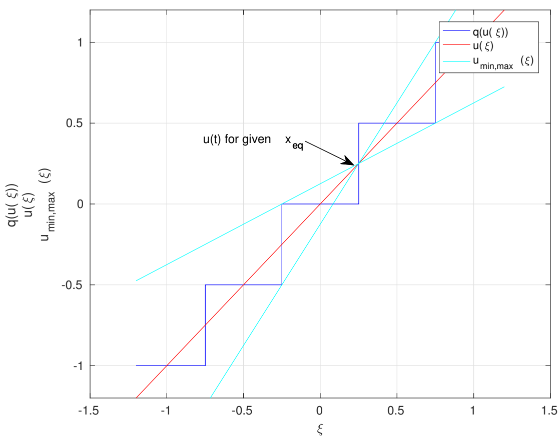


Fig. 4 : Sliding modes in quantized control

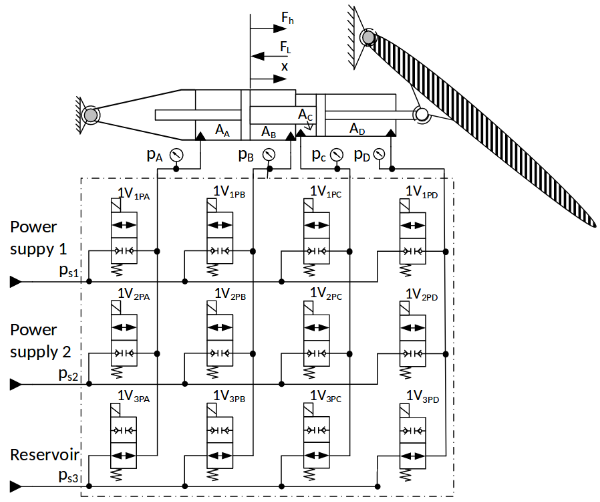


Fig. 5 : Digital hydraulic concept [9]

surface $\frac{u}{\Delta} - k - \frac{1}{2} = 0$ is attractive, the system enters in sliding mode when the trajectory crosses the surface. Please refer to [10] regarding to the Sliding Mode Theory.

4 System Modeling

The basic concept of the of the digital hydraulic connected to a aircraft control surface can be seen in Figure 5. Note the multi-chamber cylinder, with four chambers and the two pressure lines (Power Supply 1 and 2). With properly switching on and off each of the valves $1V_{1PA}$, $1V_{1PB}$, $1V_{1PC}$, $\ldots$, $1V_{3PD}$ it can be obtained the number of possible forces is given by N^M , where N is the number of pressures and M is the number of areas. The circuit proposed on [9] considers gas accumulators, check valves and the use of two reservoirs, which is out of the scope of this paper.

The control strategy involves two approximations of the original hydraulic system, which implies some conservativeness on the solution. The first approximation is neglecting the valve throttling, which is justified by the fact that digital hydraulics uses conventional on/off valves of simple construction, designed to be used completely closed or completely opened having as little throttling as possible, resulting to be highly efficient. A second approximation is time delays, which is part of the valve dynamics and it is necessary in order to avoid hydraulic "short circuit" between lines, as long the opening time and closing time are not the same and changes depending on the working conditions.

The result for four chamber cylinder and neglecting the valve throttling is shown as a linear system in (4), where A_A, A_B, A_C and A_D are the cylinder areas and p_A, p_B, p_C and p_D are the pressure for each cylinder chamber, chosen from available pressures.

$$m\ddot{d}(t) = -kd(t) - b\dot{d}(t) + \\ + A_A p_A(t) - A_B p_B(t) + A_C p_C(t) - A_D p_D(t), \quad (4)$$

where $d(t)$ is the displacement.

Defining the control input by $u(t) = A_A p_A(t) - A_B p_B(t) + A_C p_C(t) - A_D p_D(t)$ with $p_{A,B,C,D} \in \mathcal{P}$, where $\mathcal{P}$ is the set of available pressures, the state variables by $x(t) = \begin{bmatrix} d(t) & \dot{d}(t) & \int d(t)dt \end{bmatrix}^T$ and rewriting (4) in a matrix form, the result is state space model (5), where it was included an extra state variable $\int d(t)dt$ to ensure zero zero steady state error of $d(t)$.

$$\dot{x}(t) = Ax(t) + Bq(u(t)), \quad (5)$$

where the function $q(u)$ is the quantization function defined as (2). The matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-k}{m} & \frac{-b}{m} & 0 \\ 1 & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix}.$$

From (5) considering the two possible behavior of the quantization input as described on the Section 3, the following two models are defined:

i. Deadband control: For deadband control input, once the trajectory enters the region where the control input $u(t)$ constant, the system is on open loop. Without loss of generality, the model considers the equilibrium in zero, so the control law should design a positive invariant set $O \triangleq \{x \in \mathbb{R}^n \,|\, \|u(t)\| < \gamma\}$, i.e. once the system trajectory enters O, implies that it stays inside O. The role of γ will be discussed on the Section 5. In this case the model (5) is described as

$$\dot{x}(t) = Ax(t) + B\phi(u(t)), \tag{6}$$

where $\phi(\cdot)$ is defined in (7).

$$\phi(u) = \begin{cases} 0, & \text{for } \|u\| < {}^{\Delta}\!/_{2} \\ \left[\frac{2}{3}u; 2u\right], & \text{for } \|u\| \geq {}^{\Delta}\!/_{2} \end{cases} \tag{7}$$

The definition (7) is convenient to be used for control design on the Section 5, because it is convex for all $x(t) \notin O$.

ii. Sliding mode control: With the definition (3) and as described on the Section 3, the constant therms in $q(t)$ only shifts the equilibrium point. Modeling the system around the equilibrium point, (5) is rewritten as

$$\dot{x}(t) = Ax(t) + B\frac{\Delta}{2}\mathbf{sign}(u(t)), \quad \text{for } \|u(t)\| \leq \frac{\Delta}{2}, \tag{8}$$

for all $\|u(t)\| \neq 0$ the function **sign** is defined as

$$\mathbf{sign}(u) = \frac{u}{\|u\|}, \quad \forall \|u\| \neq 0. \tag{9}$$

Considering that (8) is valid only for $\|u(t)\| \leq {}^{\Delta}\!/_{2}$, (9) can be written as

$$\mathbf{sign}(u) = \frac{u}{\|u\|} = u \cdot \alpha(u), \quad \text{where } \alpha(u) = \frac{1}{\|u\|} \in \left[{}^{2}\!/_{\Delta}; \infty\right). \tag{10}$$

The definition (10) will be used as an approximation of the function **sign** for the control design on the Section 5, for the cases where the equilibrium point x_{eq} is such that $u(t)$ lays near the boundary between two different quantization levels.

5 Main Result

The goal is to design a linear state feedback control $u(t) = Kx(t)$ to be used in quantized control output, considering the different effects of the control as described on the Section 3 using the models of the Section 4. The design of the control gain K is based on the optimization approach to achieve fast convergence, with the well known $\mathcal{H}_2$ control, subjected to the following two restrictions: *i.* positively invariance in case of $u(t)$ lays inside the quantization interval and *ii.* sliding mode stability in case of $u(t)$ lays between two quantization levels. The optimization is done with the performance signal $z(t) \in \mathbb{R}^{n_z}$ given by

$$z(t) = C_z x(t). \tag{11}$$

Without loss of generality Δ is set to $\Delta = 1$, as long it can be directly multiplied by the input matrix B and posteriorly adjusted on the gain matrix K. With this considerations, we enunciate the following theorem:

Theorem 1 *Given the system (5), suppose that exists symmetric matrices $Q = Q' \in \mathbb{R}^{n\times n}$ and $N = N' \in \mathbb{R}^{n_z \times n_z}$, and a matrix $X = KQ \in \mathbb{R}^{1\times n}$, where $\gamma = 1$ for open-loop stable systems and $\gamma = 2$ for open-loop unstable systems, satisfying the LMI conditions (12, 13, 14) for $\alpha = \{{}^{1}\!/_{2}, \bar{\alpha}\}$ with sufficiently large $\bar{\alpha}$ and $\beta = \{{}^{1}\!/_{2}, {}^{3}\!/_{2}\}$, then the control law $u(t) = Kx(t)$ minimizes the $\mathcal{H}_2$ norm of the system, under the control input (2) with quantization step $\Delta = 1$.*

$$\begin{aligned} &\min_{Q,N,X} \mathbf{Tr}(N) \\ &\begin{bmatrix} N & C_z Q \\ \star & Q \end{bmatrix} > 0, \qquad Q > 0 \\ &AQ + QA' + \beta BX + \beta X'B' + BB' < 0 \end{aligned} \tag{12}$$

$$\begin{bmatrix} AQ+QA'+\beta BX+\beta X'B' & B & \beta X' \\ \star & -\gamma I & \mathbf{0} \\ \star & \star & -\gamma I \end{bmatrix} < 0 \quad (13)$$

$$AQ+QA'+\alpha BX+\alpha X'B' < 0 \quad (14)$$

Proof. The LMI (12) is the usual $\mathcal{H}_2$ norm minimization of a linear time-invariant system [12]

$$\begin{aligned} \|h(t)\|_2^2 &= \int_0^\infty \mathbf{Tr}\left(h'(t)h(t)\right)dt \\ &= \mathbf{Tr}\left(C_z\left(\int_0^\infty e^{At}BB'e^{A't}dt\right)C_z'\right) \\ &= \mathbf{Tr}\left(C_zQ_cC_z'\right), \end{aligned} \quad (15)$$

where $H(s)$ is the Laplace transform of the closed loop of (5) and $h(t) = C_ze^{At}B = \mathcal{L}^{-1}\{H(s)\}$ its inverse. The matrix Q_c is the controllability Gramian. An approximatin of Q_c is obtained by the following LMI (16).

$$\begin{aligned} &\min_{Q,K} \mathbf{Tr}\left(C_zQC_z'\right) \\ &(A+BK)Q+Q(A+BK)'+BB' < 0, \end{aligned} \quad (16)$$

where $\mathbf{Tr}\left(C_zQC_z'\right) > \mathbf{Tr}\left(C_zQ_cC_z'\right)$.

Using Schur Complement[13] and the fact that $N > C_zQC_z'$ implies $\mathbf{Tr}(N) > \mathbf{Tr}(C_zQC_z')$, (16) is easily transformed into (12) with the change of variable $X = KQ$. The inclusion of $\beta = \{1/2, 3/2\}$ covers the variation of $u(t)$ based on the definition (7). In the Figure 3, β can be seen as the change on the slope of u between the two cyan lines.

For the case *i.* deadband control, consider the following inequality:

$$\frac{d}{dt}V(x(t)) + u'(t)u(t) - w'(t)w(t) < 0, \quad (17)$$

where $V(x(t)) = x'(t)Px(t)$, with $P = P' > 0$ and $w(t) = q(u(t)) - u(t)$ is the quantization error.

Integrating (17) leads to

$$\lim_{t\to\infty} V(x(t)) - V(x(0)) + \\ + \int_0^\infty u'(t)u(t)dt < \int_0^\infty w'(t)w(t)dt, \quad (18)$$

for $\Delta = 1$ it is true that $\int_0^\infty w'(t)w(t)dt < \int_0^\infty \frac{1}{4}dt$, because $\|w(t)\| \leq \frac{1}{2}$. If the system is stable and considering zero initial conditions (18) implies

$$\int_0^\infty u'(t)u(t)dt < \int_0^\infty \frac{1}{4}dt, \quad (19)$$

which means that exists a finite T_1, such that $\|u(t)\| < 1/2$ for all $t > T_1$. In other words, the condition (17) implies that the trajectory is such that $u(t)$ enters the ball O, previously defined, and it stays for all $t \to \infty$.

Note that the condition (17) is only valid for open-loop stable system, because $\|u(t)\| < 1/2$ implies $u(t) = 0$ due the quantization. For open-loop unstable system, once the trajectory is such that $\|u(t)\| < 1/2$, $u(t)$ is set to zero, causing the trajectory to leave O. This problem is fixed by considering a scaling factor of γ on O to include the limit cycle of the trajectories entering and leaving the original O. For open-loop unstable systems, by defining $\gamma = 2$, the trajectory is such that the control assumes values of $u(t) \in \{-\Delta, 0, \Delta\}$. From (17), including the scaling factor γ and $u(t) = Kx(t)$ the result is

$$2x'(t)P\left(Ax(t) + B\beta Kx(t)\right) + \\ + \frac{1}{\gamma}x'(t)K'Kx(t) - \gamma w'(t)w(t) < 0. \quad (20)$$

The inequality (20) is easily transformed into (13), using Schur Complement and pre and post multiplying by $Q = P^{-1}$ and with the change of variable $X = KQ$. The scalar β is included to cover the variation of $u(t)$ based on the definition of (7).

The last LMI (14) is the result form the Lyapunov candidate function $V(x(t)) = x'(t)Px(t)$ for the system (8) considering the approximation of the function **sign** as described in (10) with $\alpha = \{1/2, \bar{\alpha}\}$, where α is a sufficiently large scalar, to be interpreted by LMI solvers.

With this, we conclude the proof covering the following consideration: *i.* ensuring that the system trajectory enters in the positively invariant set O, *ii.* sliding mode stability, in case of $u(t)$ be in the border between two quantization interval when the system is in equilibrium; and finely, $\mathcal{H}_2$ norm minimization of the system (1).

6 Numeric Example

Using the model (5), based on the works [9, 7], the numeric values are $k = 150^{\text{kN}}/_{\text{m}}$, $b = 2.5^{\text{kNs}}/_{\text{m}}$ and mass 50kg. The quantization step $\Delta = 1010\text{N}$ is included on the matrix B for designing proposes. The performance signal $z(t)$ in (11) is defined with C_z as in (21), where the values have been chosen to stronger penalize the integral of the position error. As long the system is not open-loop asymptotically stable, $\beta = 2$.

$$C_z = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}. \tag{21}$$

The problem has been solved using the solver SDPT3 [15] in Matlab environment with $\bar{\alpha} = 10^4$. The result is the gain matrix $K = \begin{bmatrix} -61 & -1.6 & -2729 \end{bmatrix}$ without rescaling by Δ.

The simulation has been performed on Hopsan [6], which offers a rich modeling, including for example, the valves and cylinder dynamics of both hydraulic and mechanic characteristics. On this model it has been considered force disturbances and viscous, static and kinematic fictions. The simulation results are compared to the previous work [7].

The simulation result on the cylinder position can be seen in the Figure 6, where the plot a is the result form our previous work [7] and b the result from the proposed methodology. The red line is the position set-point and the blue line is the measured position of the cylinder. The methodology used on the previous work [7] was based on [5]. The objective of [5] is to minimize effect of the quantization error on the output. It can be seen on Figure 6a, that the method considerably reduced the lag between 3s and 5s, but it has induced some unstable sliding mode regimes around 4s. As predicted on the development of the proposed method, as the system is not open-loop stable, there is a limit cycle between 1.5s and 3s in the Figure 6b, where the trajectory crosses the desired position successively.

In the Figure 7 the control output $u(t)$ is presented in blue and the red line its quantization $q(u(t))$, i.e. the force chosen from the available combination of pressures and cylinder diameters. The control output from previous work [7] is shown in the Figure 7a. There is both stable sliding mode at 3.5s and unstable sliding modes at 4s. In our proposed method 7b, does not show any unstable sliding mode, as expected.

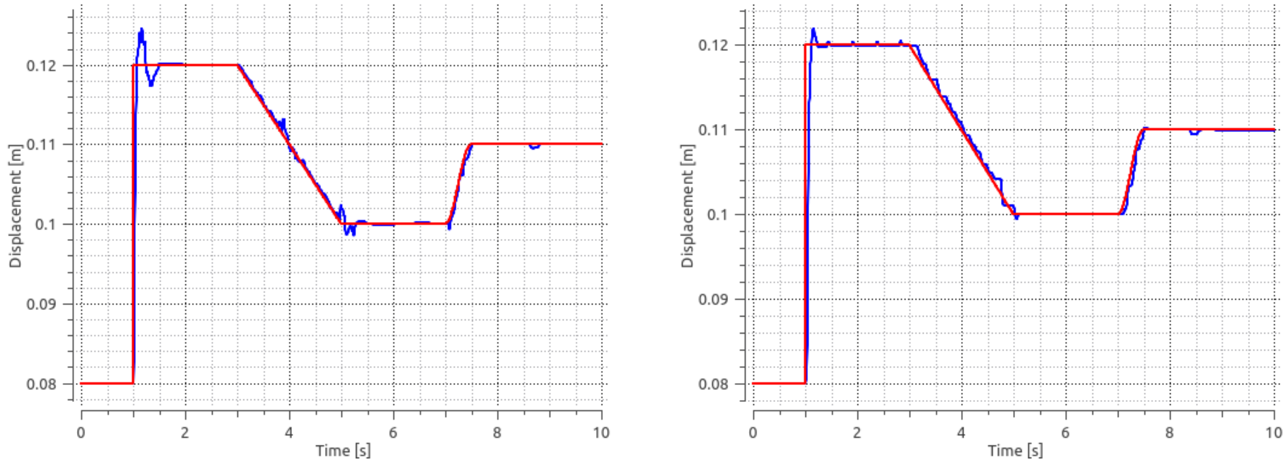
The main advantage of the proposed method is reducing the waste the energy by reducing the amount of switchings. In the Figure 8 it is shown the waste of energy using the both methods, with significant reduction on the energy waste. The energy waste shown in the Figure 8 has been calculated as

$$E_{waste}(t) = \int_0^t H_{hydr}(\tau)d\tau - \int_0^t H_{mech}(\tau)d\tau, \tag{22}$$

where $E_{waste}(t)$ is the energy waste over time. The hydraulic power H_{hydr} is the sum of the power from each individual power supply $H_{hydr} = p_{s1}q_{v1} + p_{s2}q_{v2} + p_{s3}q_{v3}$. The mechanical power H_{mech} is calculated using the velocity v and the force F measured on the cylinder rod $H_{mech} = F \cdot v$. As expected, the energy is mostly wasted when the valve is switched on and off, as seen around 4s or 5s, which are regions of unstable sliding mode. When the cylinder is not moving, the energy waste is very small, as seen by the flats around 6s and after 8s. It is important to mention that for regular proportional valves, the waste of energy occurs mostly when the valve is positioned on the center, as it imposes more restriction to the flow and internal leakage. On this situation the cylinder is stopped, without producing mechanical power.

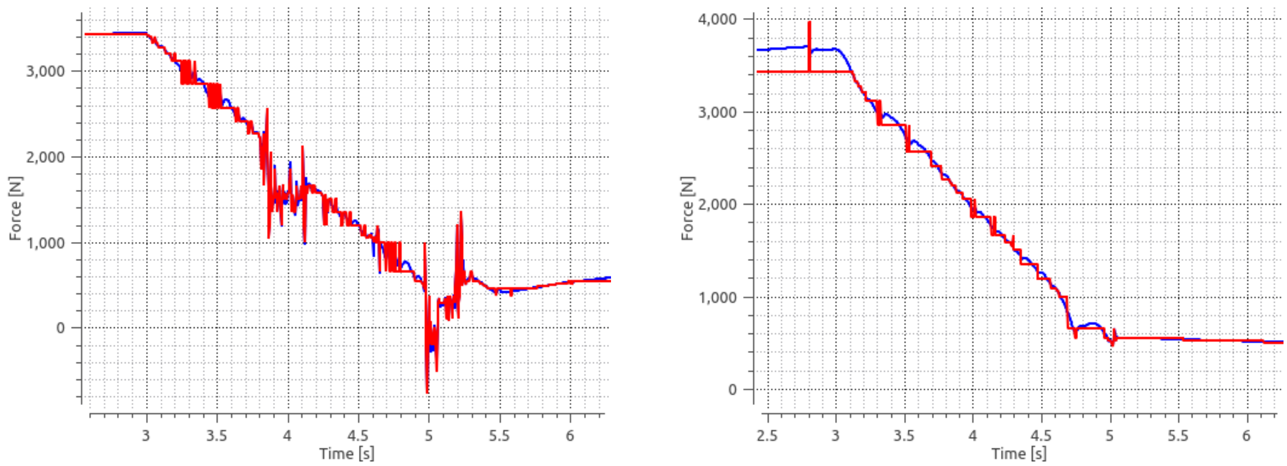
7 Conclusions

It has been presented a new methodology for designing a control gains for digital hydraulic, focusing on aircrafts applications, the proposed method leads to significant reducing of valve switchings, which is important for both energy



(a) Method [7] (b) Proposed Method

Fig. 6 : Position of a digital hydraulic actuator in blue and its set-point in red



(a) Method [7] (b) Proposed Method

Fig. 7 : Control output in blue $u(t)$ and its quantization $q(u(t))$ in red

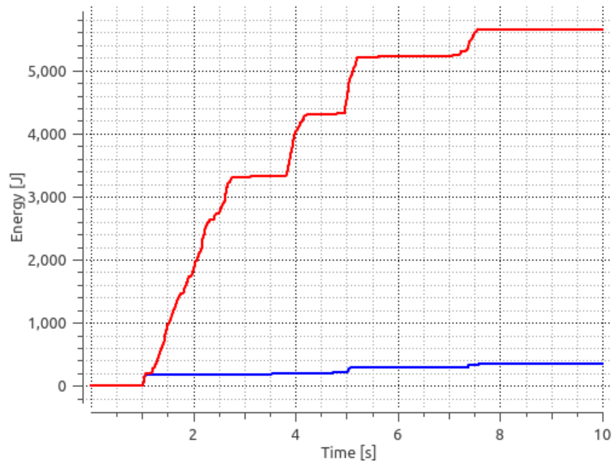


Fig. 8 : Energy waste where red is using the method [7] and blue is the proposed method

efficiency and devices lifespan.

The proposed method considers the possible sliding mode effect that exists between two quantization levels, in order to have stable sliding surfaces, in contrast with the common approach for this kind of problem. Most of the available results found on the literature for quantized control consider that the error between the the actual control output to its quantization is a disturbance to be minimized. This comes out as a hight control gain that forces the system trajectory to the equilibrium regardless how the trajectory moves inside a quantization interval. This kind of approach leads to an excessive switchings, which is not desired in digital hydraulics.

It has been presented a simulation in Hopsan, which offers a rich modeling for all hydraulic and mechanical components, including disturbance and friction.

Remark 1 *Note that the LMIs* (12)*,* (13) *and* (14)*, are linear in A, so the Theorem 1 can be used for designing the control gain for uncertain matrix A, i.e. robust on A. This is not true for the matrix B. For the example on the Section 6, it is possible to design the gain in order to be robust for variations of the spring k and dumping b.*

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