SURROGATE-BASED OPTIMIZATION METHOD APPLIED TO MULTIDISCIPLINARY DESIGN OPTIMZIATION ARCHITECTURES

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Abstract

During the past three decades, various multidisciplinary design optimization (MDO) architectures have been developed, using different ways of dealing with interactions between disciplines, which can be solved by optimization algorithms. In the field of aircraft design optimization, as the complexity of system increases, conventional optimization algorithms applied in MDO architecture exhibit many drawbacks, such as low efficiency, poor robustness or easily getting trapped in a local extra. In this paper, a newly developed surrogate-based optimization (SBO) method is applied to replace traditional optimization algorithms in the MDO architectures so that the efficiency of solving MDO problems can be dramatically improved. To demonstrate its effectiveness, a benchmark MDO test case and Speed Reducer MDO case are employed. It shows that the efficiency of SBO is two-order higher than that of genetic algorithm and the optimization results are better than that obtained by gradient-based algorithm. Besides, SBO is applied to the aerodynamic/structural integrated design of a transport wing in MDF architecture. The weight of the wing is reduced by 30.32% while its aerodynamic performance is retained at a cruise condition.

1 Introduction

Multidisciplinary Design Optimization (MDO) in aircraft engineering is a research field that studies the application of numerical optimization techniques to the design of aircraft systems involving multiple disciplines, while exploring how different disciplines work collaboratively to optimize the comprehensive performance in system level[1][2]. As aircraft design is a complicated multistage progress involving so many disciplines, such as aerodynamics, structural, propulsion and flight control, MDO not only considers the performance of these individual disciplines, but also takes their interactions into account. These interactions can be analyzed and fully utilized through a sound optimization strategy according to the mathematic formulation of a specified MDO problem. Hence, in order to reach the true optimum of the whole coupled complex system, several MDO architectures have been proposed, for instance, multidiscipline feasible (MDF), individual discipline feasible (IDF), all-at-once(AAO), simultaneous analysis and design (SAND), concurrent subspace optimization (CSSO), collaborative optimization (CO), bi-level integrated system synthesis (BLISS), analytical target cascading (ATC), exact and inexact penalty decomposition (EPD and IPD), MDO of independent subspaces (MDOIS), quasi-separable decomposition (QSD) and asymmetric subspace optimization (ASO)[2]. Instead of treating each individual discipline as an isolated island, these architectures reorganize them in a more reasonable coupled way and cut down time and cost of the design cycle, so that the optimization efficiency can be dramatically improved.

During the past three decades, MDO architecture has achieved great development and been successfully applied to engineering design problems[3][4]. Although MDO architecture provides the framework of solving a specific optimization problem, it doesn't participate in searching the design space directly. Optimization algorithms are still used to search for the optimal design. At present, optimization algorithms applied to MDO architecture can be classified into two types[5]. One is gradient-based optimization algorithm, such as sequential quadratic programming (SQP), quasi-Newton method, etc. These algorithms are very efficient with the gradient inexpensively computed by adjoint method[6], but the solution optimality can be sensitive to the start point and the method often becomes trapped into a local minimum. The other one is gradient-free optimization algorithm, also known as heuristic optimization algorithm, including genetic algorithm (GA), simulated annealing (SA), etc. These algorithms are capable of finding a global optimum. However, the computational cost associated with global optimization methods could easily become prohibitive with the increase of the number of design variables, if a high-fidelity, thus expensive computational fluid dynamics (CFD) is employed for functional evaluation. According to these drawbacks of above optimization algorithms, another type of optimization method, surrogate-based optimization (SBO) method has been developed recently[7][8][9].

A surrogate model is essentially an approximation model for the cost function or state function, which is built from the limited information obtained by sampling the design space[8]. In the early development stage of MDO architecture, it was used to replace the expensive disciplinary analysis model such as CFD simulation. Once a sound surrogate model was built, it no longer got updated and the responses of cost functions could be directly predicated by the surrogate model during the optimization process. However, when using this strategy of simply replacing CFD simulations with the surrogate models, the optimization results heavily rely on the global accuracy of the surrogate models. A remedy way is to search for the minimum of current surrogate model and select it as a new sample point for an expensive simulation so that the accuracy of the model can be improved in each iteration. Nevertheless, the practice shows that this method can only find the local optimum in normal conditions. That is to say, it is unable to reach the global optimum of the multidisciplinary system via an optimization mechanism.

In contrast, the core mechanism of SBO is to build surrogate models and solve sub-optimization problems corresponding to the infill-sampling criteria[7][10], whose role is the same as any of conventional gradient-based methods or heuristic optimization algorithms[7], leading to an automatic clustering of sample points near the optimum[8]. Some of the infill-sampling criteria, such as EI (expected improvement), can both improve the current optimal design (exploitation) or the global accuracy of the model (exploration). This paper proposes to use SBO in conjunction with several monolithic MDO architectures by replacing the conventional gradient-based or gradient-free optimizers, so that the global optimum could be found efficiently.

2 MDO architectures

The MDO architecture is regarded as a combination of problem formulation and organization strategy, considering nonlinear couplings between disciplines and ways of transferring information among different modules[2]. For a given optimization problem, the architecture combines its disciplinary knowledge with optimization software and defines how the discipline analysis models or surrogate models are truly organized so that the overall problem can be solved more easily. The architecture can be either monolithic or distributed according to the existence of discipline optimizer[2]. In a monolithic approach, the MDO problem is considered as a single optimization problem with only one system optimizer. At present, the widely used monolithic architectures include MDF, IDF and SAND. In a distributed approach, an optimization problem is partitioned into several sub-optimization problems containing small scales of variables and constraints according to disciplines or demands. The installed discipline level optimizer will optimize variables obtained by discipline analysis first before they are returned to system level optimizer. In current distributed architectures, CO is widely applied in practice because it is quite similar to the structure of engineering-design environment presently. In this paper, SBO method is applied to three monolithic architectures (MDF, IDF, and SAND). More details about MDO architectures can be found in reference [2].

3 Surrogate-based optimization method

The framework of a typical SBO for solving an optimization problem is sketched in Figure 1[7]. It begins by constructing a surrogate with an uncertainty model upon initial sample points, which are determined by design of experiment (DoE) and evaluated by expensive analysis codes like CFD. Once the initial surrogate model is built, we sample the model by solving the sub-optimization problem defined by the infill-sampling criteria, which produce new points and drive them towards a global or local optimum. We proceed this step iteratively until the convergence condition is satisfied.

From the above procedure, one can see that the main ingredients of such a SBO process are: DoE method, surrogate modeling, infill-sampling criteria and sub-optimization[7].

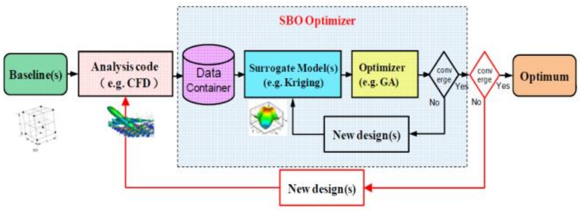


Fig. 1. Framework of SBO-type optimization[7]

3.1 Design of Experiment

Before constructing a surrogate model, SBO selects several initial sample points by design of experiment (DoE) method throughout the design space. DoE is a procedure with the general goal of maximizing the amount of information obtained from a limited number of sample points. Among current DoE methods[7][8], Latin hypercube sampling (LHS) is widely used in the areas of simulation, optimization and reliability computing for it is easy to implement and has high uniformity. Figure 2 shows an example of selecting 20 sample points by LHS for a two-dimensional problem.

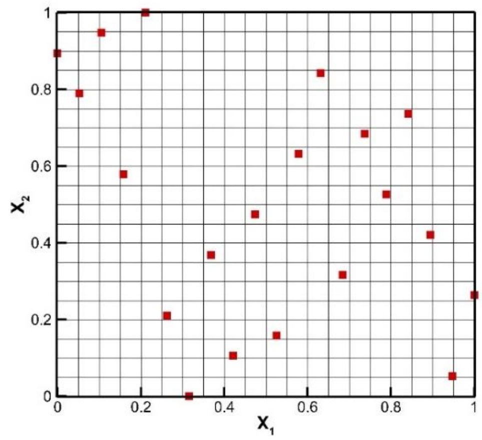


Fig. 2. Schematics of 20 sample points selected by Latin hypercube sampling

3.2 Surrogate modeling

For an m-dimensional problem, suppose we are concerned with the prediction of the output of a high-fidelity and expensive simulation code, which corresponds to an unknown function $y:\mathbb{R}^m \to \mathbb{R}$. By running the code, y is observed at n sites:

$$\mathbf{S} = [\mathbf{x}^{(1)},...,\mathbf{x}^{(n)}]^{\mathrm{T}} \in \mathbb{R}^{n\times m}, \mathbf{x} = \{x_1,...,x_m\} \in \mathbb{R}^n, \quad (1)$$

with the corresponding responses:

$$\mathbf{y}_s = [y^{(1)},...,y^{(n)}]^{\mathrm{T}} = [y(\mathbf{x}^{(1)}),...,y(\mathbf{x}^{(n)})]^{\mathrm{T}} \in \mathbb{R}^n. \quad (2)$$

Surrogate modeling is such a procedure to build an approximation model $\hat{y}(\mathbf{x})$ based on the sampled dataset $(\mathbf{S},\mathbf{y}_s)$. There are a variety of surrogate models at present such as polynomial response surface model, polynomial chaos expansion, kriging and its variants (e.g. gradient-enhanced kriging[11], variable-fidelity kriging[12][13]), radial-basis functions, support-vector regression[8]. Among them, kriging is widely used in the field of aerodynamic design[14][15] because it can effectively represent highly nonlinear and multidimensional functions. In this paper, we focus on the kriging model.

The kriging model is a kind of interpolation model via linear weighted known responses. Assume response as a statistical process as:

$$Y(\mathbf{x}) = \beta_0 + Z(\mathbf{x}), \quad (3)$$

where β_0 is an unknown constant and $Z(\cdot)$ represents the deviation which is a stochastic variable subject to normal distribution $N(0,\sigma^2)$. Then, the covariance matrix of $Z(\mathbf{x})$ is given by

$$\mathrm{Cov}\left[Z(\mathbf{x}),Z(\mathbf{x}')\right]=\sigma^2 R(\mathbf{x},\mathbf{x}'). \tag{4}$$

Here, $R(\mathbf{x},\mathbf{x}')$ is the spatial correlation function, which only depends on the Euclidean distance between two sites, $\mathbf{x}$ and $\mathbf{x}'$. Assuming that the response can be approximated by a linear combination of the observed data $\mathbf{y}_s$, the predictor of $y(\mathbf{x})$ at an untried $\mathbf{x}$ is formally defined as:

$$\hat{y}(\mathbf{x})=\mathbf{w}^{\mathrm{T}}\mathbf{y}_s, \tag{5}$$

where $\mathbf{w}=[w^{(1)},...,w^{(n)}]^{\mathrm{T}}$ is a vector of weight coefficients associated with sampled data. Then, $\mathbf{y}_s=[y^{(1)},...,y^{(n)}]^{\mathrm{T}}$ is replaced by its corresponding random quantities $\mathbf{Y}_s=[Y^{(1)},...,Y^{(n)}]^{\mathrm{T}}$. Once $\mathbf{w}=[w^{(1)},...,w^{(n)}]^{\mathrm{T}}$ are obtained[8][9], the predictor for any untried $\mathbf{x}$ is given by:

$$\hat{y}(\mathbf{x})=\beta_0+\mathbf{r}^{\mathrm{T}}(\mathbf{x})\underbrace{\mathbf{R}^{-1}(\mathbf{y}_S-\beta_0\mathbf{F})}_{=:\mathbf{V}_{krig}}, \tag{6}$$

where $\beta_0=(\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F})^{-1}\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{y}_S$ is a scaling factor. Besides, the kriging model also gives the MSE of the predictor $\hat{y}(\mathbf{x})$:

$$\begin{aligned}&\mathrm{MSE}\{\hat{y}(\mathbf{x})\}=\\&\sigma^2\{1.0-\mathbf{r}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{r}+(\mathbf{r}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F}-1)^2/\mathbf{F}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{F}\},\end{aligned} \tag{7}$$

which can be used to guide us to add new sample points in the optimization.

In this paper, the correlation function $R(\mathbf{x},\mathbf{x}')$ we used is a cubic spline correlation function in the form of:

$$R_k(\theta_k,x_k-x_k')=\begin{cases}1-15\xi_k^2+30\xi_k^3 & \text{for } 0\le\xi_k\le0.2\\ 1.25(1-\xi_k)^3 & \text{for } 0.2<\xi_k<1\\ 0 & \text{for } \xi_k\ge1\end{cases}, \tag{8}$$

where $\xi_k=\theta_k\left|x_k-x_k'\right|$ and the unknown hyper parameters $\theta=(\theta_1,...,\theta_m)$ can be obtained by maximum likelihood estimation.

3.3 Infill-sampling criteria

Once a surrogate model is built, the next step is to select new sample points from the model by solving sub-optimization problems defined by infilling-sampling criteria. The function of infill-sampling criteria is to guide the generation of new sample points which are supposed to be better than any of existing ones. According to infill-sampling criteria, a mathematical sub-optimization problem is defined, which can be solved by conventional numerical optimization methods cheaply, and we finally obtain the new sample point when the sub-optimization loop converges. At present, several matured infill-sampling criteria have been developed[7][8][10][16]. Among them, EI[17] criterion is regarded as an efficient and global method, which takes both the exploration and exploitation into consideration. Its theory and constraint handling are introduced as follows.

Assume that the best observed object function as far is $y_{\min}$ and the prediction of a kriging model obeys a Gaussian normal distribution with mean of $\hat{y}(\mathbf{x})$ and standard deviation of $s(\mathbf{x})$, i.e. $\hat{Y}(\mathbf{x})\in N\left[\hat{y}(\mathbf{x}),s^2\right]$. The probability density function is

$$P(\hat{Y}(\mathbf{x}))=\frac{1}{\sqrt{2\pi}\,s(\mathbf{x})}\exp\left(-\frac{1}{2}\left(\frac{Y(\mathbf{x})-\hat{y}(\mathbf{x})}{s(\mathbf{x})}\right)^2\right). \tag{9}$$

For a minimization problem, the improvement function is

$$I(\mathbf{x})=\max\left(y_{\min}-\hat{Y}(\mathbf{x}),0\right). \tag{10}$$

Hence, the expectation of the improvement function is

$$E[I(\mathbf{x})]=\begin{cases}(y_{\min}-\hat{y})\Phi\left(\dfrac{y_{\min}-\hat{y}}{s}\right)+s\phi\left(\dfrac{y_{\min}-\hat{y}}{s}\right) & \text{if } s>0\\ 0 & \text{if } s=0\end{cases}, \tag{11}$$

where $\Phi(\bullet)$ and $\phi(\bullet)$ are cumulative distribution and probability density functions of a standard normal distribution, respectively. Then we obtain the sub-optimization problem, which is defined as

$$\begin{aligned}&Max. \quad E\left[I(\mathbf{x})\right]\\&s.t. \quad \mathbf{x}_l\le\mathbf{x}\le\mathbf{x}_u\end{aligned} \tag{12}$$

For a constrained optimization problem, one can deal with it by multiplying the possibility of a design being feasible behind $E[I(\mathbf{x})]$ [18]. Considering a situation that we have an inequality constraint $g(\mathbf{x})\le0$, we build a kriging model for it, and assume that the corresponding random variable $G(\mathbf{x})$ is normally distributed, with mean of $\hat{g}(\mathbf{x})$ and standard derivation of $s_g(\mathbf{x})$. Thus the probability of satisfying the constraint at any site is

$$P[G(\mathbf{x}) \le 0] = \Phi\left(-\frac{\hat{g}(\mathbf{x})}{s_g(\mathbf{x})}\right). \tag{13}$$

Then we multiply Eq.(19) by Eq.(17), and obtain the new formulation of our sub-optimization problem, which is defined as

$$\begin{aligned} &Max. \quad E\left[I(\mathbf{x})\right]\cdot P\left[G \le 0\right] \\ &s.t. \quad\ \ \mathbf{x}_l \le \mathbf{x} \le \mathbf{x}_u \end{aligned} \tag{14}$$

If there exist more than one inequality constraint, we just need to multiply their possibilities one by one for they are all independent of each other. For the equality constraint, we transform it into two inequality constraints, and give a quite small tolerance. We build two kriging models for them and multiply them at the same way. We use a hybrid method of combing GA, Hooke&Jeeves pattern search and gradient-based method[7] to solve the above sub-optimization, and then a new optimum $\mathbf{x}^*$ is obtained, which is supposed to be our next sample point and will be evaluated by the expensive analysis code.

4 Example verification

In this section, two examples are employed to exhibit the validity and efficiency of SBO method in MDO architectures, compared with gradient-based algorithm SQP, and heuristic optimization algorithm GA. The SBO method is implemented by an in-house code called "SurroOpt"[7].

4.1 Benchmark MDO test case

An analytical test case by Sellar[19] is employed to verify the application of SBO to different MDO architectures. The optimization problem is defined as

$$\begin{aligned} &Min. && f = x_2^2 + x_3 + y_1 + e^{-y_2} \\ &w.r.t. && x_1, x_2, x_3 \\ &s.t. && g_1 = 8 - y_1 \le 0 \\ & && g_2 = y_2 - 10 \le 0 \\ & && y_1 = x_1^2 + x_2 + x_3 - 0.2y_2 \\ & && y_2 = \sqrt{y_1} + x_1 + x_3 \\ & && x_1 \in [-10,10], \quad x_2, x_3 \in [0,10]. \end{aligned} \tag{15}$$

This problem has a local optimum $f_{local}^* = 8.986721$, and a global optimum $f^* = 8.002860$. There are two coupled disciplines in the above formulation. Their relationship can be illustrated as Figure 3.

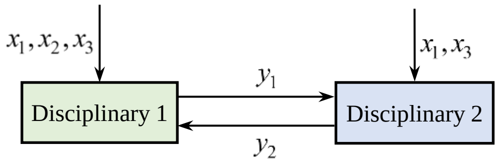


Fig. 3. Schematics of two coupled disciplines for an analytical MDO test case

Firstly, we reconstruct its mathematical formulation via MDF, IDF and SAND architectures respectively. There are two coupled disciplines in the above formulation. Their relationship is illustrated as Figure 3.

MDF:

$$\begin{aligned} &Min. && f = x_2^2 + x_3 + y_1 + e^{-y_2} \\ &w.r.t. && x_1, x_2, x_3 \\ &s.t. && g_1 = 8 - y_1 \le 0 \\ & && g_2 = y_2 - 10 \le 0 \\ & && y_1 = x_1^2 + x_2 + x_3 - 0.2y_2 \\ & && y_2 = \sqrt{y_1} + x_1 + x_3 \end{aligned} \tag{16}$$

IDF:

$$\begin{aligned} &Min. && f = x_2^2 + x_3 + y_1 + e^{-y_2} \\ &w.r.t. && x_1, x_2, x_3, x_4, x_5 \\ &s.t. && g_1 = 8 - y_1 \le 0 \\ & && g_2 = y_2 - 10 \le 0 \\ & && h_1 = x_4 - y_1 = 0 \\ & && h_2 = x_5 - y_1 = 0 \\ & && y_1 = x_1^2 + x_2 + x_3 - 0.2x_5 \\ & && y_2 = \sqrt{x_4} + x_1 + x_3 \end{aligned} \tag{17}$$

SAND:

$$\begin{aligned} &Min. && f = x_2^2 + x_3 + x_4 + e^{-x_5} \\ &w.r.t. && x_1, x_2, x_3, x_4, x_5 \\ &s.t. && g_1 = 8 - y_1 \le 0 \\ & && g_2 = y_2 - 10 \le 0 \\ & && h_1 = x_4 - \left(x_1^2 + x_2 + x_3 - 0.2x_5\right) = 0 \\ & && h_2 = x_5 - \left(\sqrt{x_4} + x_1 + x_3\right) = 0 \end{aligned} \tag{18}$$

Secondly, SBO method and SQP are used to solve the above three problems respectively.

When using SBO, we generate 10 initial sample points randomly by LHS and build a kriging model. The infill-sampling criteria are MSP and EI, which means two sample points are selected at each updating cycle. In order to reduce the randomness, the SBO method is repeated for 30 times and the average value is considered as the final optimum. For SQP, its start points are given by Table 1.

Table 1. Different start points for an analytical MDO test case

Start point	x_1	x_2	x_3
A	1.0	5.0	2.0
B	2.0	2.0	4.5
C	-2.0	1.0	1.5

Table 2 shows results obtained by these two methods. We can see that in the three monolithic architectures, when using SBO, the optimization processes converge to the global optimum, and the number of function evaluations is fewer than that of SQP, especially when using MDF, the discipline analyses of SBO decreases more than 50% and the efficiency of optimization is improved dramatically. Additionally, when we use SQP with start point *C*, the results are all trapped in the local optimum through three MDO architectures, which is avoid in using SBO for all 30 results do converge to the global optimum.

Table 2. Comparison of gradient-based algorithm (SQP) and SBO method for a benchmark MDO test case

Method		MDO Arch.	Objective and constraint Final results			Number of function evaluations		
			Obj.	g_1	g_2	MDA	Dis.1	Dis.2
SBO		MDF	8.00286024	-1.10e-8	-4.1432	22	95	95
		IDF	8.00286033	-9.27e-6	-4.1431	0	33	33
		SAND	8.00286022	-8.8e-10	-4.1431	0	35	35
SQP	*A*	MDF	8.00286376	-3.54e-6	-4.1431	50	574f+38g	574f+38g
	B		8.00307043	-2.10e-4	-4.1432	39	476f+30g	476f+30g
	C		8.98672051	-1.25e-10	-9.8708	47	539f+38g	539f+38g
	A	IDF	8.00286023	-2.0e-15	-4.1431	0	79f+62g	79f+62g
	B		8.00286023	-2.4e-12	-4.1431	0	119f+97g	119f+97g
	C		8.98672051	-2.0e-15	-9.8708	0	69f+58g	69f+58g
	A	SAND	8.00286020	-7.50e-8	-4.1431	0	73f+58g	73f+58g
	B		8.00291526	-5.50e-5	-4.1431	0	58f+48g	58f+48g
	C		8.98672051	-4.52e-9	-9.8709	0	56f+48g	56f+48g

f: function evaluation; g: gradient evaluation

4.2 Speed Reducer MDO case

This case is from reference [20] and the structure of gearbox speed reducer is illustrated as Figure 4. It is one of the NASA's ten benchmark cases for evaluating MDO methods, which represents the design of a simple gearbox and is posed as an artificial multidisciplinary problem comprising the coupling between gear design and shaft design disciplines. The objective is to minimize the weight of gearbox, subject to the bending and contact stresses of gear and torsional deformation and stress of shaft. The theoretical optimum of this problem is $f^* = 2994.341316$, and the optimization model is defined as:

$$
\begin{aligned}
\textit{Min.}\quad & f = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) \\
& \quad -1.508x_1\left(x_6^2 + x_7^2\right) + 7.477\left(x_6^3 + x_7^3\right) \\
& \quad +0.7854\left(x_4x_6^2 + x_5x_7^2\right) \\
\textit{w.r.t.}\quad & x_1, x_2, x_3, x_4, x_5, x_6, x_7 \\
\textit{s.t.}\quad & g_1 = 27x_1^{-1}x_2^{-2}x_3^{-1} - 1 \le 0 \\
& g_2 = 397.5x_1^{-1}x_2^{-2}x_3^{-2} - 1 \le 0 \\
& g_3 = 1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} - 1 \le 0 \\
& g_4 = 1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} - 1 \le 0 \\
& g_5 = 10x_6^{-3}\sqrt{\left(745x_2^{-1}x_3^{-1}x_4\right)^2 + 1.69\times10^7} - 1100 \le 0 \\
& g_6 = 10x_7^{-3}\sqrt{\left(745x_2^{-1}x_3^{-1}x_5\right)^2 + 1.575\times10^8} - 850 \le 0 \\
& g_7 = x_4^{-1}\left(1.5x_6 + 1.9\right) - 1 \le 0 \\
& g_8 = x_5^{-1}\left(1.1x_6 + 1.9\right) - 1 \le 0 \\
& g_9 = x_2x_3 - 40 \le 0 \\
& g_{10} = 5 - x_1x_2^{-1} \le 0 \\
& g_{11} = x_1x_2^{-1} - 12 \le 0 \\
& x_1 \in [2.6, 3.6],\ x_2 \in [0.7, 0.8],\ x_3 \in [17, 28] \\
& x_4, x_5 \in [7.3, 8.3],\ x_6 \in [2.9, 3.9],\ x_7 \in [5.0, 5.5]
\end{aligned}
\qquad (19)
$$

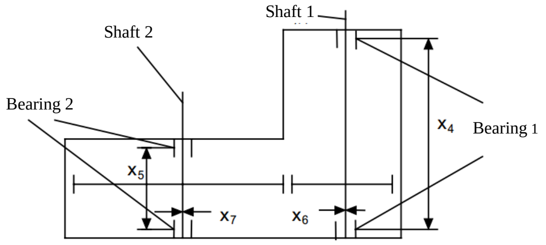


Fig. 4. Model of Speed Reducer[21]

Here, x_1 is the width of gear surface, x_2 is gear modulus, x_3 is the number of pinion teeth, x_4 and x_5 are bearing spacing, x_6 and x_7 are the diameter of the shaft, respectively.

Although this case can be divided into two disciplines: gear and bearing by their physical meaning, there is no coupling between these two disciplines. Thus, we can just recognize it as an individual disciplinary optimization problem and only use MDF architecture. SBO, SQP and GA are implemented to solve this problem. For SBO, initial sample points are chosen by LHS, and a kriging model is built based on them. The infill-sampling criteria are MSP and EI. Also, the optimization process is repeated for 30 times and the results are averaged. For SQP, its three start points are given by NASA in Table 3. For GA, its pop size is 200 and crossover probability factor is 0.8. Table 4 compares results of three optimization methods for speed reducer MDO case. It shows that SBO method uses the least number of analysis model calls, only 28, to converge, and that of SQP is more than 28 from all three start points. For GA, the computation cost is much more than SBO and SQP.

Table 3. Different start points for Speed Reducer MDO case

Start point	x_1 (cm)	x_2 (cm)	x_3 (cm)	x_4 (cm)	x_5 (cm)	x_6 (cm)	x_7 (cm)
A	2.80	0.71	25.0	7.90	7.599	3.00	5.099
B	3.50	0.75	22.0	7.80	8.300	3.35	5.500
C	3.50	0.70	17.0	7.30	7.715	3.35	5.287

Table 4. Comparison of three optimization methods for Speed Reducer MDO case

Method	Start point	Number of function evaluations	Objective finial results
SBO	/	28	2994.341316
	A	85f+76g	2994.341315
SQP	*B*	71f+61g	2994.341284
	C	56f+52g	2994.341315
GA	/	28400	2994.355204

It is important to note that the results obtained by SQP are a little bit lower than the average result by SBO. That is due to the relaxation of SQP algorithm when handling with constraints. Table 5 gives the finial values of 11 constraint functions at the optimum sample point obtained by SBO and SQP (from start point *A*), respectively. We can see that for SQP, the eighth constraint value is slightly violated, $g_8 > 0$, while for SBO, it strictly satisfies all the constraints.

Table 5. Results of constraint functions with SQP(A) and SBO for Speed Reducer MDO case

Constraint	SQP(A)	SBO
g_1	-7.3915e-2	-7.3915e-2
g_2	-0.1979990	-0.1979990
g_3	-0.4991723	-0.4991723
g_4	-0.9046439	-0.9046439
g_5	-2.2700e-13	-5.7515e-10
g_6	-3.7998e-5	-1.6743e-10
g_7	-5.1328e-2	-5.1326e-2
g_8	1.1232e-8	-9.4791e-10
g_9	-28.100000	-28.100000
g_{10}	0.0000000	-1.0840e-10
g_{11}	-7.0000000	-7.0000000

5 Aerodynamic/structural multidisciplinary design of a transport wing

The aerodynamic/structural multidisciplinary design optimization with strong disciplinary coupling is the most typical problem of aircraft MDO problems. In the traditional design process of a transport wing, designers firstly design a wing with the best aerodynamic performance at a cruise condition. Then they carry on the jig-shape design of another wing and try their best to make this wing configuration close to the previous one after static aeroelastic modification. In this way, the designed wing can maintain the cruise performance. However, this two-step design method breaks up the coupling between aerodynamics and structure. In order to avoid this and consider all the interactions of different disciplines, this paper directly regards the transport wing as an elastic body and apply SBO method to the jig-shape design. During the optimization process, we use MDF architecture to perform multidisciplinary analysis (MDA) and take both aerodynamic performance and structural performance of the cruise shape obtained after deformation as design indexes.

5.1 Mathematical optimization model

This paper uses SBO method to conduct aerodynamic/structural integrated design of a transport wing based on MDF architecture. The design condition is cruise condition, with an altitude of 10000m and Mach number of 0.76. The analysis object of aerodynamics discipline is a wing-body configuration (Figure 5). The fuselage data is maintained during the optimization. Taking into account factors such as engine hoisting and landing gear retraction, the length of inner wing is fixed. The structure discipline only carries on the stress-strain analysis of the wing and the thickness of wing ribs and spar webs are fixed to 2mm in the optimization. The detailed parameters can be found in Table 6.

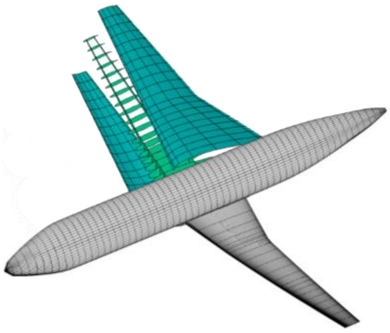


Fig. 5. Configuration of wing-body combination

Table 6. Model parameters for wing aerostructural multidisciplinary design optimization case

Parameter	Value
Cruise altitude	10000m
Cruise Mach number	0.76
Fuselage length	35.0m
Root chord	6.7m
Half span of inner wing	11.8m
Incidence angle	4.0°
Dihedral angle	7.0°
Wing rib thickness	0.002m
Front spar web	0.002m
Back spar web	0.002m

We choose 5 aerodynamic design variables and 20 structural design variables. Here, we divide the wing into 10 segments along the spanwise direction. The ranges of these variables are given in Table 7.

Table 7. The ranges of design variables for wing aerostructural multidisciplinary design optimization case

Structural design variables	Number	Range
Upper skin thickness	10	[2mm, 10mm]
Lower skin thickness	10	[2mm, 10mm]
Aerodynamic design variables	Number	Range
Half span of wing	1	[28m, 34m]
Taper ratio,	1	[0.2, 0.3]
Twist angle	1	[-3.0°, -1.0°]
Leading edge swept angle	1	[26°, 32°]
Angle of attack	1	[0°, 2°]
Total	25	0.002m

The final optimization model is defined as Eq.(28). The objective is to minimize the weight of the wing and maintain its aerodynamic performance after aeroelastic deformation.

$$\begin{aligned} Min.\quad & Weight_{wing} \\ s.t.\quad & \sigma_{\max} \le [\sigma] \\ & \delta_{tip} \le [\delta] \\ & L \ge L_{baseline} \\ & L/D \ge [L/D]_{baseline} \\ & S_{wing} \ge [S_{wing}]_{baseline} \end{aligned} \tag{20}$$

Here, $\sigma_{\max}$ is the maximum equivalent stress, δ_{tip} is tip deformation, L is the lift of the wing, L/D is lift-to-drag ratio, S_{wing} is the wing area. The safety factor of structure is 1.5, allowable stress $[\sigma]=2.7467\times10^{8}\,\text{Pa}$, allowable deformation $[\delta]=1\text{m}$.

5.2 Optimization process based on MDF architecture

Due to the existence of coupled design variables, we use MDF to solve the problem. Its flow chart is illustrated as Figure 6. The aerodynamic analysis is performed by solving full velocity potential equation, and we don't consider viscous effect of fuselage during the optimization so as to guarantee its robustness. When the optimization is finish, we carry on viscous correction of fuselage for both baseline and optimum wing respectively. ANSYS is used in the structural analysis. Loosely coupled method is employed when conducting static aeroelastic analysis.

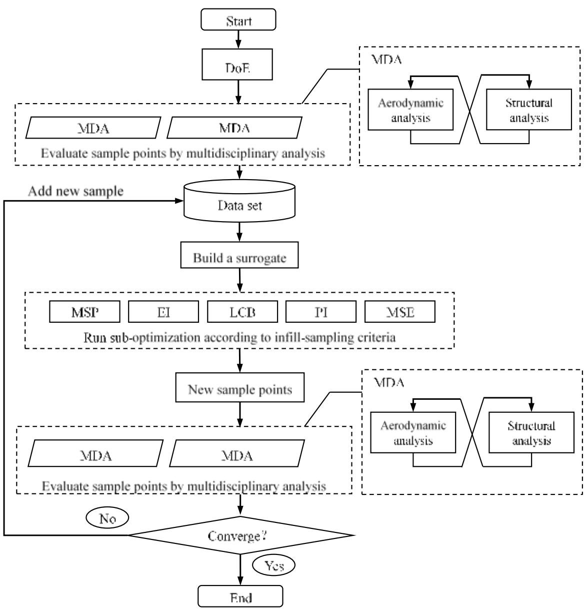


Fig. 6. Flow chart of surrogate-based optimization based on MDF architecture (wing aerostructural multidisciplinary optimization)

5.3 Result and discussion

Figure 7 shows that at the beginning most sample points, including all the initial sample points, are infeasible. With the optimization proceeding, more and more sample points newly added are feasible. After about 400 MDA, the objective function almost converges. Figure 8 gives pressure contours of baseline and optimum wing-body configuration, along with 6 pressure distribution curves of cross sections. Figure 9 shows the jig and cruise shape of baseline and optimum wing respectively. Table 8 gives a comparison of performances of the baseline and optimum configurations. It can be seen that all the constraints are strictly satisfied at the optimum design, and the weight of the wing is reduced by 30.32%, which demonstrates the effectiveness and efficiency of SBO method. Figure 10 shows the comparison of skin thickness of baseline and optimum wings. The optimized skin thickness is cut down which makes the overall weight of the wing decrease.

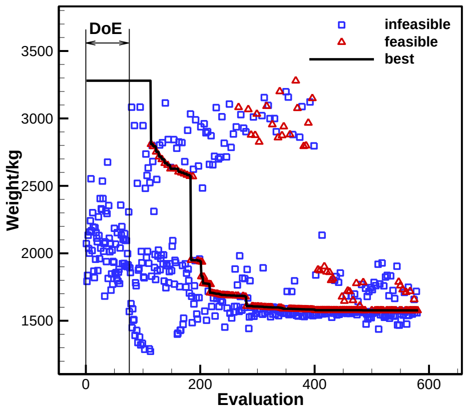
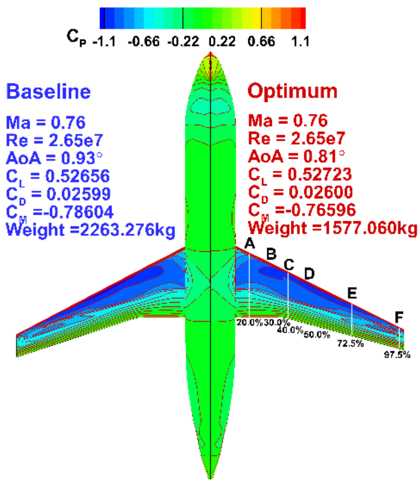
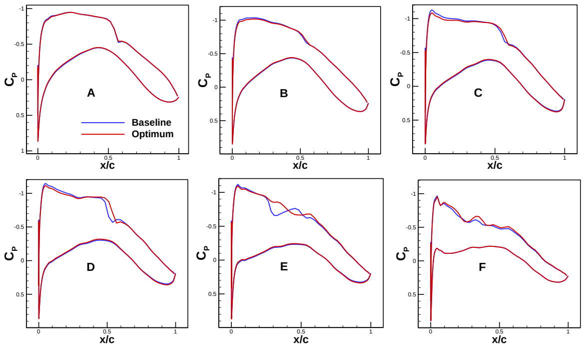


Fig. 7. Convergence history of the wing MDO problem



(a) wing-body combination pressure contour



(b) wing pressure distribution

Fig. 8. Comparison of wing-body combination pressure contours and wing pressure distributions for baseline and optimum configurations

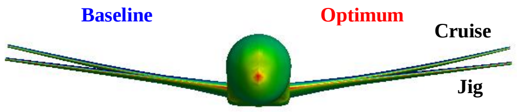


Fig. 9. Comparison of the undeformed and deformed wings (baseline and optimum)

Table 8. Comparison of performances of the baseline and optimum configurations

Wing performance	Baseline	Optimum
Wing weight /kg	**2263.28**	**1577.06(-30.32%)**
Half span of wing /m	31.48	31.45
Taper ratio	0.266	0.236
Twist angle /°	-2.66	-2.50
Leading edge swept angle /°	27.1	26.0
Angle of attack /°	0.93	0.81
Wing area /m^2	115.41	115.58
Lift /N	649582.6	651350.3
Lift-to-drag ratio	20.26	20.28
Maximum equivalent stress /10^8Pa	2.36	2.49(<2.76)
Maximum wingtip deformation /m	0.871	0.999(<1)

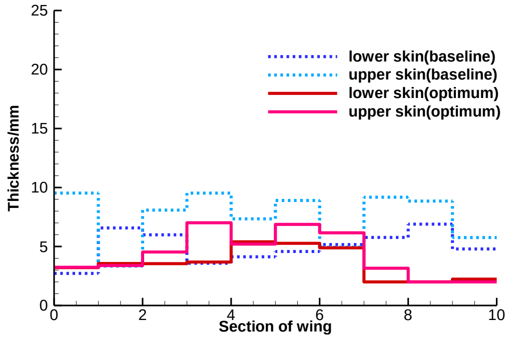


Fig. 10. Comparison of skin thickness of baseline and optimum wings

6 Conclusion

In this paper, the newly developed SBO method is applied to monolithic MDO architectures, which overcomes drawbacks like expensive computation cost and being trapped at local optimums of traditional optimization algorithms. Some conclusions can be drawn as below:

1) SBO method is a global optimization method, and ensures that the optimization converges to the global optimum.
2) Compared with gradient-based or heuristic optimization algorithm, using SBO method decreases the number of both system and discipline analysis calls, and optimization efficiency is dramatically improved.
3) From optimization results, the optimal design obtained by SBO method can satisfy all the constraints strictly.

In the future work, we hope to expand the application of SBO in more MDO architectures such as CO distributed architecture.

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