WING TWIST OPTIMISATION USING AERODYNAMIC SOLVERS OF DIFFERENT FIDELITY

Guangda Yang* , Andrea Da Ronch* , Daniel Kharlamov* , Jernej Drofelnik*
***Faculty of Engineering and the Environment**
University of Southampton, Southampton, SO17 1BJ, U.K.

Keywords: *wing twist optimisation, SU2, hybrid solver, 2.5D+, vortex lattice method*

Abstract

Computational fluid dynamics has become the method of choice for aerodynamic shape optimisation. However, the computational cost poses challenges for the routine use of high-fidelity flow solvers in the early phase of aircraft design. In this paper, a benchmark wing twist optimisation problem is investigated using two aerodynamic solvers of different fidelity. The first solver is the open-source SU2 software, and the second is an efficient hybrid solver which couples a linear vortex lattice method with an infinite-swept wing solver. Numerical details of two optimisation frameworks are presented. It is demonstrated that the hybrid solver can accurately reproduce the aerodynamic loads on the three-dimensional wing. Optimisation results show that both optimisations can effectively minimise the induced drag by recovering a nearly elliptical lift distribution. In comparison to SU2 optimisation, the hybrid approach achieves a speed-up of two orders of magnitude.

1 Introduction

In recent years, the environmental impact of aviation carbon emissions has raised increasing public concern. To address this challenge, aircraft manufacturers have been devoted to reducing fuel consumption when designing new aircraft. Of the many considerations, one important aspect is to improve aerodynamic efficiency through drag reduction.

Induced drag, also known as vortex drag, is produced by lifting wings with finite span and is an inevitable consequence of lift. Induced drag accounts for approximately 40% of the total drag on a conventional aircraft in cruise flight [1]. It is worthwhile developing approaches that can reduce vortex drag. For example, retrofitting an existing wing with winglets is beneficial to reduce the induced drag, but it also raises issues extending on several disciplines. Another way to reduce induced drag in a wing design is to optimise the twist distribution or planform shape, which aims to generate relatively more lift away from the wing tip.

With a rapid development of computer technology, we are able to perform aerodynamic shape optimisation (ASO) with ease using computational fluid dynamics (CFD). In terms of induced drag minimisation, Phillips [2] used a low-fidelity model based on lifting-line theory to find the optimal design for finite twisted wings, and the optimised geometry was subsequently analysed with an Euler flow solver [3]. Hicken and Zingg [4] then explored the use of Euler-based ASO to minimise induced drag. Following their study, a benchmark optimisation problem was formulated and adopted by AIAA aerodynamic design optimisation discussion group (ADODG) [1]. The benchmark case is to minimise the induced drag of an inviscid, rectangular wing through optimising the twist distribution.

[1] https://info.aiaa.org/tac/ASG/APATC/AeroDesignOpt-DG/default.aspx

Several researchers [5, 6, 7] have exercised this optimisation case using their own CFD solvers based on the three-dimensional (3D) Euler equations. From an industrial viewpoint, however, these high-fidelity aerodynamic solvers are prohibitive for routine use as they are computationally expensive, especially in the conceptual and preliminary design phases. Hence, there is the need for an alternative approach, which can provide a trade-off between computational efficiency and accuracy.

To address the above issue, this paper employs an efficient multi-fidelity solver to perform wing twist optimisation of the benchmark problem. The hybrid aerodynamic solver couples a vortex lattice method (VLM) solver, based on Katz and Plotkin formulation [8], with a two-dimensional (2D) infinite-swept wing (ISW) solver [9]. The hybrid solver, which is able to capture nonlinear effects, can rapidly predict the aerodynamic loads. For the purpose of comparison, an open-source high-fidelity solver, SU2 [10], was also used in this study.

This work aims at demonstrating the capability and efficiency of the hybrid solver in solving the wing twist optimisation problem. The technical objectives of this study are to: a) verify the effectiveness of two optimisation frameworks for the same benchmark case; b) investigate the sensitivity of optimal solution to dimensionality of design variables; and c) compare the computational accuracy and efficiency of two aerodynamic solvers of different fidelity and costs.

The remainder of this paper is organised in the following manner. Section 2 describes the benchmark optimisation problem. Section 3 introduces the two aerodynamic solvers and describes corresponding optimisation algorithms. The computational grid is contained in Section 4. The optimisation results are presented and discussed in Section 5. Lastly, conclusions and future work are summarised in Section 6.

2 Case description

The benchmark optimisation problem is the induced drag minimisation of a rectangular wing with NACA 0012 sections in inviscid, subsonic flow. The freestream Mach number is 0.5, and the target lift coefficient is 0.375. The design variables are the twist of sections along the span and about the trailing edge. The purpose of this case is to produce a lift distribution that is close to elliptical and a span efficiency factor approximately equal to one. This optimisation problem is written as

$$
\begin{aligned}
\text{Minimise:} \quad & C_D \\
\text{w.r.t:} \quad & \gamma(y) \\
\text{Subject to:} \quad & C_L = 0.375
\end{aligned}
$$

where C_D and C_L are the drag and lift coefficients of the wing, respectively, and $\gamma(y)$ is the twist distribution along the wing span. The initial wing geometry is illustrated in Fig. 1.

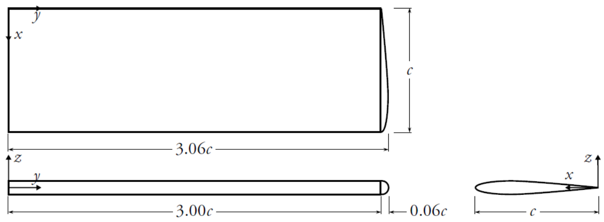


Fig. 1 Initial wing geometry for the benchmark case. (source: ADODG)

3 Methodology

Two flow solvers, SU2 and a hybrid solver, are applied to perform aerodynamic analysis in the benchmark optimisation problem. This section contains a brief summary of these two solvers and the corresponding ASO algorithms.

3.1 SU2

The open-source SU2 code [10] has been recently developed to perform tasks related to partial differential equations (PDE) analysis and PDE-constrained optimisation on unstructured grids. One of the core modules in SU2 is a high-fidelity flow solver, which is capable of solving a wide range of problems. In this work, SU2 is employed to solve Euler equations using finite volume method (FVM).

The SU2 software suite is designed to perform gradient-based shape optimisation using the built-in adjoint approach. Schematics of the optimisation framework can be found in Ref. [11]. In this study, the geometry parameterisation and deformation is manipulated by free-form deformation (FFD) [12] technique. After geometry perturbation, the volume mesh is deformed using an approach based on linear elasticity equations [13]. Sensitivity (or gradient) information is evaluated by continuous adjoint method. The SLSQP optimiser is used to drive the design cycle until the convergence criteria, i.e. Karush–Kuhn–Tucker (KKT) conditions [14, 15], are satisfied.

3.2 Hybrid solver

The hybrid solver [16, 17] is built upon two fidelity levels in aerodynamic analysis to rapidly estimate the aerodynamic loads. A linear VLM solver is used to model 3D effects on finite wings. An ISW solver [9], also referred to as 2.5D+ solver, is employed to capture nonlinear sectional effects. The 2.5D+ solver is specialised to solve flows around ISWs on a 2D stencil, and the flow solution can account for the cross-flow effects. Therefore, the 2.5D+ solver can significantly reduce the computational cost compared with the existing state-of-the-art methods, which rely on a 3D stencil.

The hybrid solver, depicted in Fig. 2, is based on the α-based coupling algorithm [18]. This algorithm corrects the freestream angle of attack, α_∞, at every VLM panel along the wing span using nonlinear sectional data. The procedure of the coupling method is described in Algorithm 1. The reader is referred to Ref. [16] for more details of the coupling approach.

The open-source pyOpt [19] package is used to formulate the optimisation framework by wrapping around the hybrid solver. A variety of optimisers have been integrated into this package. In this study, the gradient-based optimiser SLSQP is employed, and the gradients are computed using finite difference method (FDM). The wing twist optimisation framework is shown in Fig. 3 and further described in Algorithm 2.

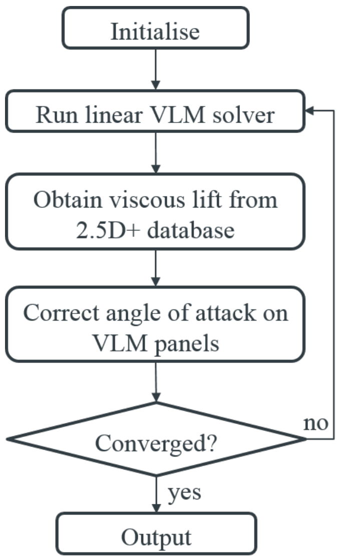


Fig. 2 Flow chart of hybrid solver.

Algorithm 1 α-based coupling method for the hybrid solver.

1: Initialise: $\alpha = \alpha_\infty$ and $\Delta\alpha = 0$
2: Run linear VLM solver to calculate the inviscid lift and induced angle of attack:
$\alpha \Rightarrow C_{L,inv}$ and α_i
3: **for** Every spanwise section j **do**
4: Calculate effective angle of attack:
$\alpha_e(j) = \alpha_\infty - \alpha_i(j)$
5: Obtain the viscous lift from 2.5D+ database at the effective angle of attack:
$\alpha_e(j) \Rightarrow C_{L,visc}(j)$
6: Calculate the angle of attack correction:
$$\Delta\alpha(j) = \Delta\alpha(j) + \nu \frac{C_{L,visc}(j) - C_{L,inv}(j)}{2\pi}$$
7: Update the local angle of attack:
$\alpha(j) = \alpha_\infty + \Delta\alpha(j)$
8: **end for**
9: Repeat Steps 2-8 until $|C_{L,visc} - C_{L,inv}| < \varepsilon$ for all spanwise sections

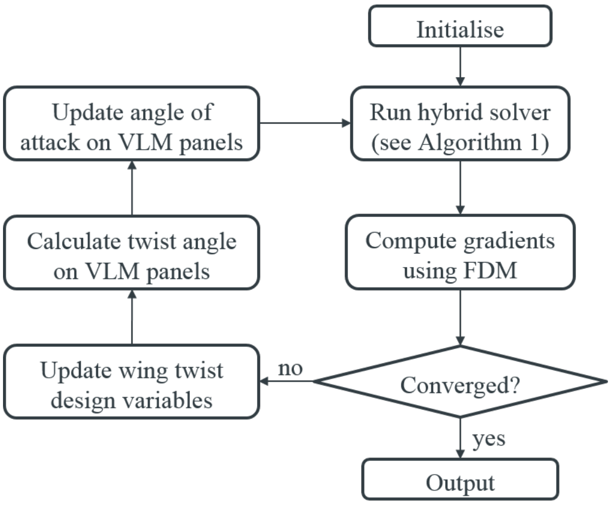


Fig. 3 Wing twist optimisation framework using hybrid solver.

Algorithm 2 Wing twist optimisation using hybrid solver.

1: Initialise: $\alpha(j) = \alpha_\infty$
2: Run hybrid solver for aerodynamic analysis
3: Compute gradients using FDM
4: Update wing twist design variables: $\gamma(k)$
5: Calculate twist angle on VLM panels through interpolation:
$\gamma(k) \Rightarrow \theta(j)$
6: Update angle of attack on VLM panels:
$\alpha(j) = \alpha_\infty + \theta(j)$
7: Repeat Steps 2-6 until convergence (i.e. KKT conditions are satisfied)

At each iteration of the design loop, it is necessary to map the wing twist design variables onto the twist angles of VLM panels, which in general are not coincident. An example of mapping is shown in Fig. 4. In this work, a linear interpolation approach along the spanwise direction is adopted. It is worth noting that the VLM grid remains unchanged during optimisation, whereas the local angle of attack for each VLM panel is updated using Eq. (1) shown below

$$\alpha(j) = \alpha_\infty + \theta(j) + \Delta\alpha(j) \tag{1}$$

where $\theta(j)$ is the geometric twist interpolated from design variables, and $\Delta\alpha(j)$ is angle of attack correction calculated from the hybrid solver.

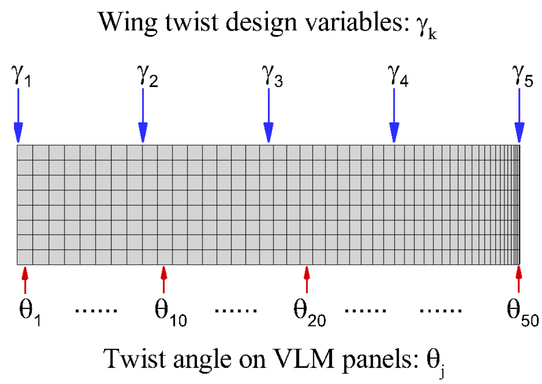
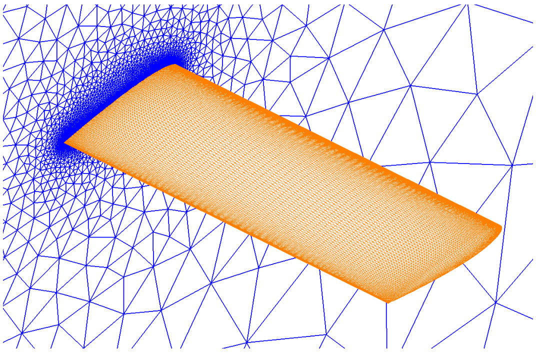


Fig. 4 Schematic of the interpolation on VLM panels.

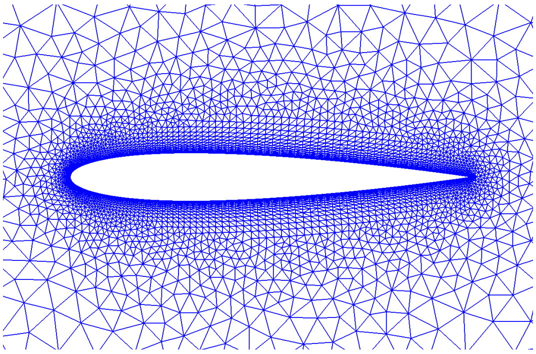
4 Computational grid

For SU2 optimisation, a 3D unstructured grid was generated, shown in Fig. 5. The grid points are clustered at the leading and trailing edge and also at the wing tip to capture the flow physics. A preliminary study was conducted, and the grid was found adequate to guarantee grid independent solutions.

As the sectional aerofoil remains the same along the span, only one section needs to be defined for the hybrid solver. In this work, the 2D grid on symmetry plane was used, which is shown in Fig. 5(b). For the nonlinear predictions of the hybrid aerodynamic code, SU2 was



(a) Aerodynamic surface and symmetry plane



(b) Near view on symmetry plane

Fig. 5 Computational grid for optimisation.

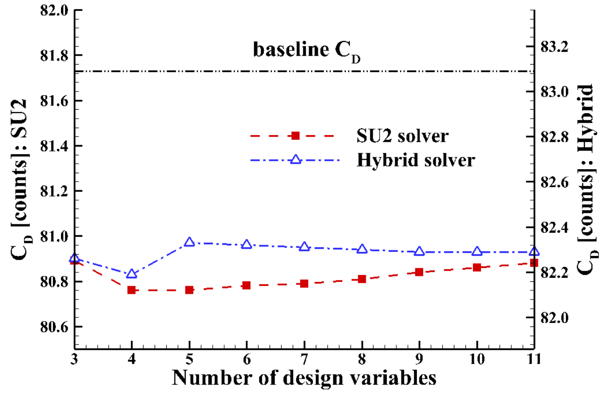


Fig. 6 Drag coefficient in dimensionality study.

used. Because the test case is unswept, the 2.5D+ method corresponds to a pure 2D analysis in this case. The VLM panels, as illustrated in Fig. 4, are uniformly spaced in the chordwise direction and unevenly distributed along the span using "cosine" function, which results in a high resolution at the wing tip.

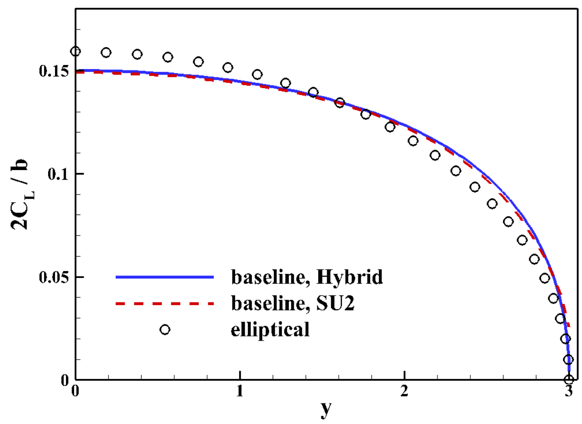
5 Results

For the initial wing geometry, the freestream angle of attack, α_∞, was iteratively updated in the flow simulation to reach the target lift. During the optimisation, the freestream angle of attack is kept fixed, whereas the twist at the wing root is allowed to vary. For both optimisation frameworks, the wing twist design variables are uniformly distributed along the span. A series of optimisations were run to perform dimensionality study, and the number of design variables, n_{dv}, ranges from 3 to 11. Both the initial and final drag results are plotted in Fig. 6.

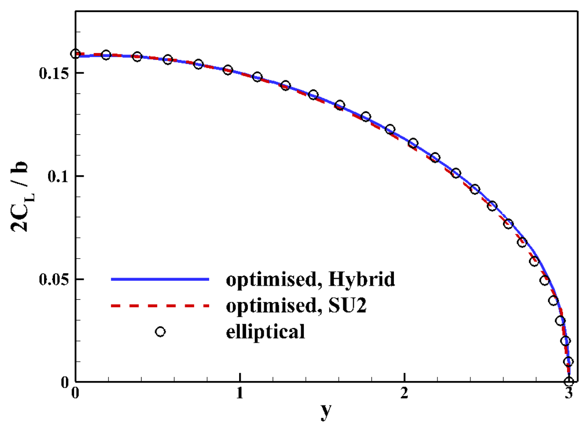
With respect to baseline geometry, the drag results predicted by two flow solvers are very close, with the difference being 1.3 counts. This fact proves that the hybrid solver is capable of estimating drag accurately compared to SU2 solver. It is also observed from Fig. 6 that the variation of drag results on optimised design is very trivial, which indicates that the optimal solution shows insensitivity to the dimensionality of design variables. Moreover, the amount of drag reduction for both optimisation frameworks is approximately one count. Thus, these two optimisation frameworks are equivalently effective in solving this benchmark problem.

The sectional lift distributions of the initial and optimised configuration are plotted in Fig. 7. It is evident that the two flow solvers generated very similar results for the baseline geometry, which further confirms the accuracy of the hybrid solver. In terms of the final design, both optimisation frameworks produced an optimal solution that shows good agreement with the theoretical elliptical distribution. Hence, the benchmark problem can be successfully solved by both optimisation frameworks.

The optimisation case with 5 twist design variables is taken for further analysis. The spanwise twist distribution for optimised design is shown in Fig. 8. It should be noted that the twist values are measured relative to the freestream angle of attack. A similar pattern is observed from these two solutions. The outboard sections produce negative twist to reduce the vortex drag, whereas the inboard sections generate positive twist to increase the sectional lift and thus maintain the desired lift coefficient.



(a) Baseline



(b) Optimised

Fig. 7 Sectional lift distribution for the NACA 0012 wing ($n_{dv} = 11$).

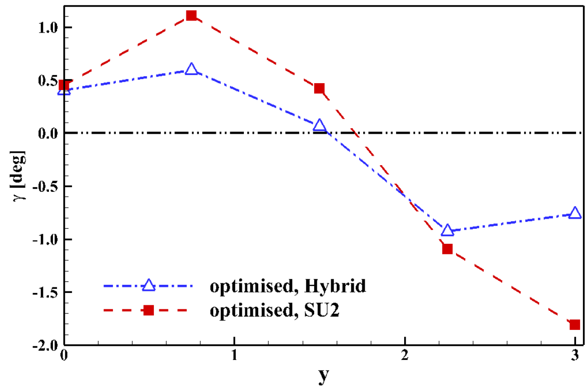


Fig. 8 Twist distribution for the NACA 0012 wing ($n_{dv} = 5$).

In SU2 optimisation, a lattice of control points are uniformly spaced on the surface of FFD box, and multiple control points at the same spanwise position are grouped to perform the twist motion. Consequently, the wing geometry encapsulated in the FFD box undergoes the twist deformation as well. Figure 9 displays the comparison between initial untwisted shape and final deformed one.

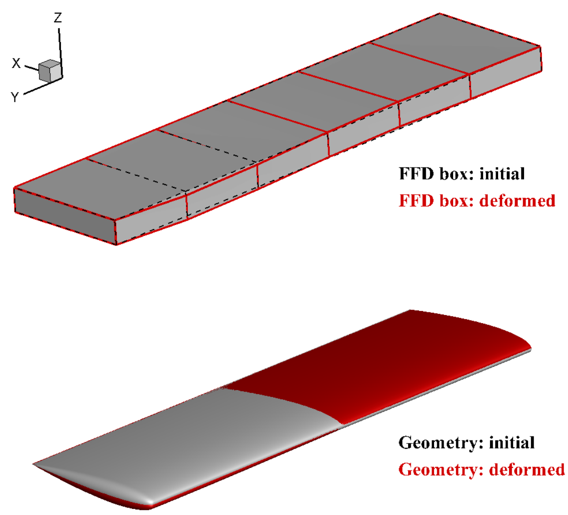


Fig. 9 Comparisons of the FFD box and wing geometry in SU2 optimisation ($n_{dv} = 5$).

The convergence history of the objective function is plotted in Fig. 10. It is apparent that only few major iterations are required for both optimisation frameworks to meet the convergence criteria. This indicates that the optimiser can easily locate the minimum point for this optimisation problem.

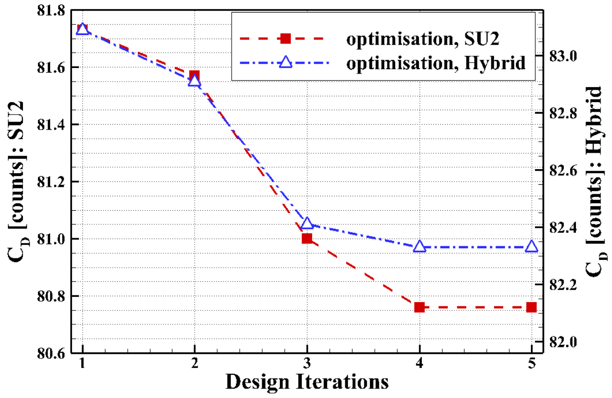


Fig. 10 Convergence history of the objective function ($n_{dv} = 5$).

In terms of computational efficiency, the hybrid solver provides a physically consistent solution in comparison to that obtained from a 3D solution using SU2 solver, but at a fraction of the computational time. Specifically, approximately 190 CPU hours were needed for the optimisation using SU2 solver, whereas the optimisation employing the hybrid approach required around 45 minutes to pre-compute the database and less than 1 minute to perform the optimisation. This corresponds to a speed-up of two orders of magnitude, which demonstrates extremely high computational efficiency of the hybrid solver compared to the 3D CFD simulation. The hybrid solver is thus suitable for aerodynamic analysis and optimisation in the early phase of aircraft design.

6 Conclusions

A benchmark wing twist optimisation problem was investigated using two aerodynamic solvers. The first solver is the high-fidelity three-dimensional (3D) SU2 solver, and the second flow solver employs a hybrid approach that couples two fidelity levels. The multi-fidelity solver combines a linear vortex lattice method solver, capturing 3D effects, with a two-dimensional infinite-swept wing solver, reproducing sectional nonlinear effects. Gradient-based optimisations were performed using SLSQP optimiser.

Three main conclusions may be formulated. Firstly, the hybrid solver can accurately predict the aerodynamic loads compared to the 3D CFD analysis. Secondly, both optimisation frameworks can successfully solve the benchmark problem by producing nearly elliptical lift distribution. Thirdly, the optimisation using the hybrid solver demonstrates high efficiency compared to SU2 optimisation, and a speed-up of two orders of magnitude was achieved.

As the benchmark problem involves an inviscid, unswept wing in subsonic flow conditions, a more realistic test case is thus required. Future work will focus on the twist optimisation of a viscous, swept wing in transonic flow conditions, which can also fully exploit the capability of the hybrid solver.

Acknowledgements

The authors acknowledge the financial support from the Engineering and Physical Sciences Research Council (grant number: EP/P006795/1) and the use of the IRIDIS High Performance Computing Facility, and associated support services at the University of Southampton, in the completion of this work. Yang also acknowledges the financial support from China Scholarship Council (CSC) and the University of Southampton.

References

[1] Kroo I. Drag due to lift: concepts for prediction and reduction. *Annual Review of Fluid Mechanics*, Vol. 33, No. 1, pp 587-617, 2001.

[2] Phillips W F. Lifting-line analysis for twisted wings and washout-optimized wings. *Journal of Aircraft*, Vol. 41, No. 1, pp 128-136, 2004.

[3] Phillips W F, Fugal S R and Spall R E. Minimizing induced drag with wing twist, computational-fluid-dynamics validation. *Journal of Aircraft*, Vol. 43, No. 2, pp 437-444, 2006.

[4] Hicken J E, Zingg D W. Induced-drag minimization of nonplanar geometries based on the Euler equations. *AIAA Journal*, Vol. 48, No. 11, pp 2564-2575, 2010.

[5] Bisson F, Nadarajah S, Shi-Dong D. Adjoint-based aerodynamic optimization of benchmark problems. *52nd AIAA Aerospace Sciences Meeting*, National Harbor, Maryland, 0412, 2014.

[6] Lee C, Koo D, Telidetzki K, et al. Aerodynamic shape optimization of benchmark problems using jetstream. *53rd AIAA Aerospace Sciences Meeting*, Kissimmee, Florida, 0262, 2015.

[7] Poole D J, Allen C B, Rendall T. Control point-based aerodynamic shape optimization applied to AIAA ADODG test cases. *53rd AIAA Aerospace Sciences Meeting*, Kissimmee, Florida, 1947, 2015.

[8] Katz J, Plotkin A. *Low-speed aerodynamics*. Vol. 13, Cambridge university press, 2001.

[9] Franciolini M, Da Ronch A, Drofelnik J, et al. Efficient infinite-swept wing solver for steady and unsteady compressible flows. *Aerospace Science and Technology*, Vol. 72, pp 217-229, 2018.

[10] Economon T D, Palacios F, Copeland S R, et al. SU2: an open-source suite for multiphysics simulation and design. *AIAA Journal*, Vol. 54, No. 3, pp 828-846, 2015.

[11] Yang G, Da Ronch A. Aerodynamic shape optimisation of benchmark problems using SU2. *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Kissimmee, Florida, 0412, 2018.

[12] Samareh J. Aerodynamic shape optimization based on free-form deformation. *10th AIAA/ISSMO multidisciplinary analysis and optimization conference*, Albany, New York, 4630, 2004.

[13] Dwight R P. Robust mesh deformation using the linear elasticity equations. *Proceedings of the Fourth International Conference on Computational Fluid Dynamics*, Berlin, pp 401-406, 2009.

[14] Karush W. Minima of functions of several variables with inequalities as side constraints. *M. Sc. Dissertation. Dept. of Mathematics, Univ. of Chicago*, 1939.

[15] Kuhn H W, Tucker A. Nonlinear programming. *Proceedings of the 2nd Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley, pp 481-492, 1951.

[16] Kharlamov D, Drofelnik J, Da Ronch A, Walker S. Rapid load calculations using an efficient unsteady aerodynamic solver. *AIAA Aviation*, Atlanta, Georgia, 2018.

[17] Drofelnik J, Da Ronch A, Kharlamov D, Sun Y. Rapid calculation of unsteady aircraft loads. *7th European Conference on Computational Fluid Dynamics*, Glasgow, UK, 2018.

[18] Van Dam C P. The aerodynamic design of multi-element high-lift systems for transport airplanes. *Progress in Aerospace Sciences*, Vol. 38, No. 2, pp 101-144, 2002.

[19] Perez R E, Jansen P W, Martins J R R A. pyOpt: a Python-based object-oriented framework for nonlinear constrained optimization. *Structures and Multidisciplinary Optimization*, Vol. 45, No. 1, pp 101-118, 2012.

Contact Author Email Address

Guangda.Yang@soton.ac.uk

Copyright Statement

The authors confirm that they, and/or their company or organization, hold copyright on all of the original material included in this paper. The authors also confirm that they have obtained permission, from the copyright holder of any third party material included in this paper, to publish it as part of their paper. The authors confirm that they give permission, or have obtained permission from the copyright holder of this paper, for the publication and distribution of this paper as part of the ICAS proceedings or as individual off-prints from the proceedings.