# WAVEPACKETS IN TURBULENT FLOW OVER A NACA 4412 AIRFOIL

Leandra I. Abreu* , André V. G. Cavalieri* ,
Philipp Schlatter** , Ricardo Vinuesa** , Dan Henningson**
*Instituto Tecnológico de Aeronáutica, São José dos Campos, SP, 12228-900, Brazil ,
**KTH Royal Institute of Technology, Stockholm, SE-100-44, Sweden

**Keywords:** *Wavepackets, coherent structures, SPOD, reduced order models, resolvent analysis*

## Abstract

*Turbulent flow over a NACA 4412 airfoil with an angle of attack $AoA = 5^{\circ}$ was analysed using an incompressible direct numerical simulation (DNS) at chord Reynolds number of $Re_c = 4 \cdot 10^5$. Snapshots of the flow field were analysed using the method of Spectral Proper Orthogonal Decomposition (SPOD) in frequency domain, in order to extract the dominant coherent structures of the flow. Focus is given to two-dimensional disturbances, known to be most relevant for aeroacoustics. The leading SPOD modes show coherent structures forming a wavepacket, with significant amplitudes in the trailing-edge boundary layer and in the wake. To model coherent structures in the turbulent boundary layer, the optimal harmonic forcing and the associated linear response of the flow were obtained using the singular value decomposition of the linear resolvent operator. The resolvent analysis shows that the leading SPOD modes can be associated to most amplified, linearised flow responses. Furthermore, coherent structures in the wake are modelled as the Kelvin-Helmholtz mode from linear stability theory (LST).*

## 1 General Introduction

Coherent structures play an important role in turbulent flows, for both drag and noise generation. The modelling of the said structures using a linearisation of the Navier-Stokes system, for flows such as jets, boundary layers and wakes, has been attempted recently by McKeon & Sharma [1], Cavalieri *et al.* [2], Abreu *et al.* [3], Towne *et al.* [4], to cite a few examples. Usually such linearised models lead to a definition of a set of modes that together describe coherent turbulent structures. Some of these may play an important role for aeroacoustics [3]. When one considers sound radiation by airfoils, wings and blades at low angles of attack, the dominant aeroacoustic mechanism is referred to as trailing-edge noise. In that case turbulent fluctuations are scattered into acoustic waves by the edge, in a mechanism with high acoustic efficiency at low Mach numbers [5]. It is thus natural to use the cited linearised models to study how coherent turbulent structures may be associated with trailing-edge noise noise.

For that matter, a useful, data-driven approach is Proper Orthogonal Decomposition, most commonly called POD, [6] is a quantitative method, or a signal post-processing approach, often applied to instantaneous fields from experimental, PIV for example [7], or numerical data, such as DNS or LES [3] in order to identify the most relevant coherent structures in turbulent flows. POD generates a set of basis functions for the modal decomposition, called POD modes. The leading POD modes captures the most energetic structures, in terms of energy. Nonetheless if the flow dynamics has predominant structures the data can often be well represented using just a few first modes, which will then reflect the dominant coherent flow structures.

More recently, POD in frequency domain, also labeled as spectral POD, or SPOD [8], has been used to obtain coherent structures in turbulent airfoil flows [3]; its success in the eduction of coherent structures can be attributed to its relationship with the modal decomposition obtained in resolvent analysis, also SPOD is better suited for identifying physically meaningful coherent structures [4]. For this reason, SPOD will be used in this work to study turbulent structures.

In the present study, a DNS of the incompressible flow over a NACA 4412 airfoil with an angle of attack of $AoA = 5°$ with a Reynolds number based on the aerodynamic profile chord $Re_c = 400000$ [9] is studied using a signal post-processing method SPOD. In terms to use a reduced order model to analyse those wavepackets, the turbulent boundary layer mean flow is used as a basis for the computation of linearised flow response to harmonic forcing, obtained by singular modes of the resolvent operator at a specific frequencie and wavenumber. This allows an identification of the optimal harmonic forcing and corresponding flow response, and verify if resolvent modes can be related to observed relevant structures obtained by SPOD. This approach follows closely that of Abreu *et al.* [3]. To analyse the wake region locally parallel resolvent analysis is unsuitable, since an unstable Kelvin-Helmholz mode appears, so the numerical solutions of Orr-Sommerfeld equation are used, called linear stability theory (LST) in this paper. The Kelvin-Helmholtz mode of the wake is obtained as the only unstable mode in the eigenspectrum.

Results of the present study show wavepackets on the airfoil surface that can me modeled using a reduced order model, where the boundary layer is turbulent. This knowledge can be used to better understand mechanisms of trailing-edge noise, which can lead to wing modifications so as to reduce sound radiation. Understanding and modelling such wavepackets can point to novel approaches in order to design more silent aircrafts.

The paper is organized as follows: in Sec. 2 is described some details about the numerical setup used in the present analysis. In Sec. 3 is provided details about the SPOD method, and some results of the SPOD eigenvalues and convergence analysis. The resolvent formulation is described in Sec. 4. In Sec. 5 is presented the mean velocity profiles used to compute the resolvent analysis. In Sec. 6 the results are presented and discussed in the following order: in Sec. 6.1 it is shown the SPOD results, in Sec. 6.2 the comparison between locally parallel resolvent analysis and SPOD for the boundary layer profiles are presented, and finally it is shown in Sec. 6.3 the comparison between Kelvin-Helmholtz mode from linear stability theory (LST) and SPOD for the wake profiles.

## 2 Numerical Setup

The database analised in this work is described by Hosseini *et al.* [9]. In this study, a DNS was performed to analyse the turbulent flow around a NACA 4412 profile at a chord Reynolds number of $Re_c = 400000$, with an angle of attack $AoA = 5°$. An unsteady volume force was used to force transition to turbulence at 10% of the chord on both top and bottom sides of the wing. The mesh was optimized to properly resolve all relevant scales in the flow, and comprises around 3.2 billion grid points. The incompressible spectral-element Navier-Stokes solver Nek5000 [10] was used to carry out the simulation. Fig. 1 shows a visualization of turbulent structures in a flow snapshot.

## 3 Spectral Proper Orthogonal Decomposition

In the present study, POD in the frequency domain, or SPOD [8], was employed to analyze the numerical DNS data, following the procedure outlined by Towne *et al.* [4]. Prior to the SPOD a spanwise average of the fluctuations is performed to focus on the two-dimensional mode, so all the flow quantities now have no longer a $z$-dependence.

SPOD is applied to velocity components $u$ and $v$ to extract the turbulent kinetic energy.

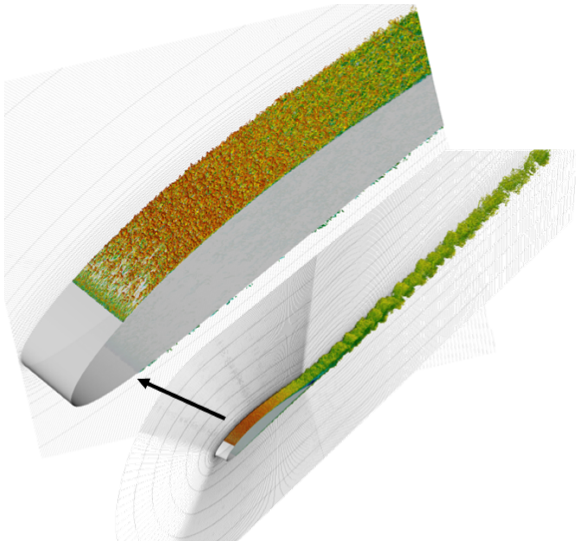


**Fig. 1** Instantaneous visualization of the DNS results showing coherent vortices identified by means of the $\lambda_2$ criterion.

Before to proceed with the SPOD, a Fourier transform is applied to the velocity and pressure fields in time to obtain a specifc frequency of interest $\omega$. So the the transformed quantities becomes $\hat{\mathbf{q}}(x,y,\omega)$, where hats denote Fourier-transformed.

Once the fields are transformed, the SPOD is applied, by solving the following integral equation, also used by Abreu *et al.* [3]:

$$\int R(\mathbf{x},\mathbf{x}',\omega)\Phi(\mathbf{x}',\omega)d\mathbf{x}' = \lambda(\omega)\Phi(\mathbf{x},\omega), \quad (1)$$

where $\Phi$ is the SPOD mode, $\lambda$ is the corresponding eigenvalue and $R$ is the two-point cross-spectral density. The matrix $R$ is Hermitian, and thus eigenvalues are real and eigenfunctions are orthogonal. Since the number of grid points is high, it is more efficient to use the snapshot method [4].

The short-time FFTs required for SPOD to extract the frequency $\omega$ have been taken considering blocks of 64 snapshots with 75% overlap, leading to a total number of blocks $N_b = 66$, so $R$ is a 66 x 66 matrix. SPOD is calculated numerically using the approach described by Meyer *et al.* [11].

In the SPOD method eigenvalues are sorted in decreasing order from highest to lowest energy. However the SPOD ensures that the most important modes in terms of energy are the first modes, which are associated with large scale flow structures. In that case, if the flow has dominant flow structures, these are therefore reflected in the first SPOD modes.

The SPOD was analysed for two different cases: the first case was using the enterely domain with the contribution of each integral weight. The second case focus was given at the boundary layer region by neglecting the integral weights from the wake region ($x/c > 1$). Fig. 2 and 3, for the first and second case, respectively, show the eigenvalues and cumulative contribution to total turbulent kinetic energy by each SPOD mode at $St = 7.93$, which is the same frequency used by Abreu *et al.* [3], defined as $St = fc/U_\infty$, where $f$ is the frequency, $c$ is the aerodynamic chord and $U_\infty$ is the free stream velocity. The convergence of SPOD eigenvalues is relatively fast; we can see for the first case in Fig. 2 that 53% of the total energy can be reconstructed using just the first SPOD mode, and the 6 first SPOD modes represent 82% of the total energy. While for the second case in Fig. 3 the first SPOD mode contributes to 26% of the total energy, and the 6 first SPOD modes represent 62% of the total energy. The energy of the first SPOD modes is lower when the wake contribution is neglected, since the most part of the total turbulent kinetic energy comes from the wake, this is why the wake is neglected in terms to highlight the structures in the boundary layer region.

## 3.1 Convergence Analysis

In terms to verify the convergence of each SPOD mode, or, in other words, verify which modes are reliable to be analized, a normalized scalar quantity $\beta$ is defined. The total dataset is separated into two blocks corresponding to 75% of the original dataset and performing the SPOD on each subset. So $\beta_{i;k}$ is the normalized projection of the $k^{th}$ mode of total dataset $\phi_k$ and the corresponding mode of each subset $\phi_{i;k}$:

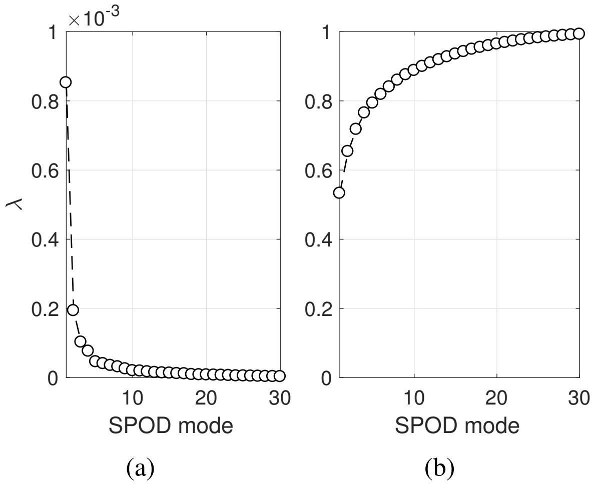



**Fig. 2** (a) Eigenvalues and (b) cumulative contribution to total turbulent kinetic energy by each SPOD mode for $St = 7.93$.

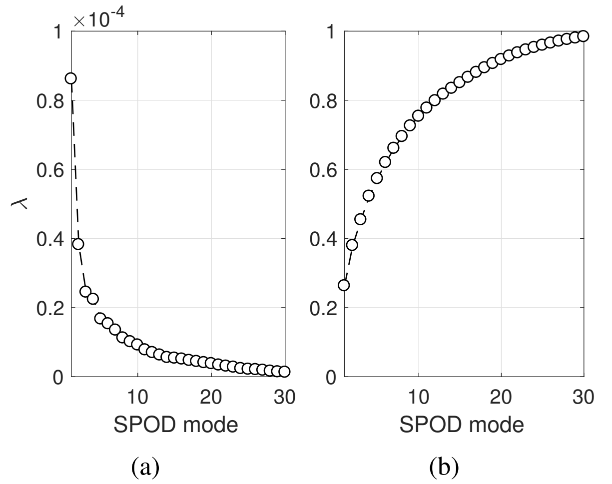



**Fig. 3** (a) Eigenvalues and (b) cumulative contribution to total turbulent kinetic energy by each SPOD mode for $St = 7.93$.

$$\beta_{i,k} = \frac{< \phi_k, \phi_{i,k} >}{\sqrt{|\phi_k|^2 . |\phi_{i,k}|^2}}, \tag{2}$$

where $i = (1;2)$ indicated each subset. The reliable modes are those in which $\beta$ is closest to one, meaning that the same computation with the subset leads to a very similar mode. This kind of analyses was also performed by Abreu *et al.* [3].

Fig. 4 (a) and (b) show the convergence analysis for two different frequencies: $\mathrm{St} = 3.97$ and $\mathrm{St} = 7.93$ respectively. In the present study, we focused on $\mathrm{St} = 7.93$, since it has a better convergence for the first four SPOD modes.

## 4 Resolvent Analysis

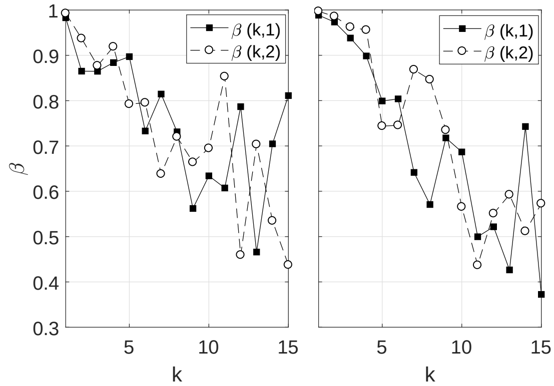
To perform a further analysis of the boundary layer coherent structures found in the SPOD results, a locally parallel resolvent analysis was done for some stations of the airfoil near the trailing-edge, taken at $x/c = 0.7$, $x/c = 0.8$ and $x/c = 0.9$, same stations studied by Abreu *et al.* [3], but for a NACA 0012.

The present resolvent analysis follows the procedure outlined by Tissot *et al.* [12], also used in Abreu *et al.* [3]. Resolvent analysis provides two orthonormal bases for forcings and associated flow responses, and each pair of forcing and response modes is related by a gain. Modes with high gain are expected to be dominant in the flow.

Resolvent analysis requires the linearisation of Navier-stokes equations isolating the nonlinear terms at the right-hand side. We will for simplicity consider the locally-parallel problem, where the mean flow $\bar{\mathbf{q}} = [\bar{\mathbf{u}}\ \bar{\mathbf{v}}\ \bar{\mathbf{p}}]$ has its divergence in $x$ neglected and thus varies only in $y$. The base-flow $U(y)$ becomes thus independent of $x$ and $t$, and the linearised equations are homogeneous along these directions, which allows the parallel-flow *Ansatz*: $\mathbf{q}(x,y,t) = \hat{\mathbf{q}}(y)e^{i(\alpha x - \omega t)}$, where hats now denote quantities that are Fourier transformed in both $x$ and $t$. Substitution of the above equations in the linearised Navier-Stokes and continuity equations, and considering that the Reynolds number, frequency $\omega$ and wavenumber $\alpha$ are given, leads to a forced linear problem. The problem in operator notation becomes

$$L\{\hat{\mathbf{q}}\} = \hat{\mathbf{f}} \tag{3}$$

where the $L = (i\omega - \mathbf{A})$ is the linear operator in $\hat{\mathbf{q}} = [u, v, p]$, and $\hat{\mathbf{f}}$ represents the nonlinear terms. The problem is closed with homogeneous Dirichlet boundary conditions for the velocity fluctuations. On the wall we enforce $u(y=0) = v(y=0) = 0$. In the far field, we have $u(y \to \infty) = v(y \to \infty) = 0$. The resolvent problem becomes




(a) $\mathrm{St} = 3.97$ (b) $\mathrm{St} = 7.93$

**Fig. 4** Convergence of the SPOD modes. Satisfactory convergence is observed for the first four modes.

$$\hat{\mathbf{q}} = (i\omega - \mathbf{A})^{-1}\hat{\mathbf{f}} = R(\omega, \alpha)\hat{\mathbf{f}} \tag{4}$$

where $R$ is the resolvent operator. Now a singular-value decomposition, or SVD, of $R$ allows to identify the optimal forcing $\hat{\mathbf{f}}$ associated with the optimal response $\hat{\mathbf{q}}$ related with a maximum energy gain. The resolvent modes are orthogonal to each other, and they are ranking according to their gain $\sigma$.

The problem was discretised using a Chebyshev pseudo-spectral method [13]. 301 Chebyshev polynomials have been used in the discretisation. We have verified that increasing the number of polynomials does not modify the results.

The frequency $\omega$ and wavenumber $alpha$ are both parameters of the resolvent operator. Frequency was already choosen, since the Strouhal number was defined, $\omega = 2\pi St$. However, the wavenumber $\alpha$ is not known *a priori*, since different wave-lengths may appear in the forcing. To study the effect of $\alpha$, first was varied $\alpha$ and studied the first four gains for three cases: $x/c = 0.7$, $x/c = 0.8$ and $x/c = 0.9$, which are shown in Fig. 5. Similar results were found by Abreu *et al.* [3], but for a NACA 0012. One can notice a peak in all cases at the first gain $\sigma_1$. There is a clear dominance of the optimal forcing, corresponding to an optimal response with a gain that is at more than one order of magnitude higher than the sub-optimals, as shown in Fig. 6. To perform the resolvent analysis was thus used the wavenumber $\alpha$ corresponding to those peaks. These peaks correspond to disturbances inside the boundary layer, where with phase speed lower than the free stream velocity.

Resolvent analysis assumes that the linear locally-parallel operator $L$ in eq. 3 is stable. This was verified to be the case for the turbulent boundary layer profiles studied here. When the analysis is carried out at the wake, an unstable Kelvin-Helmholtz mode appears, and locally-parallel resolvent analysis is no longer appropriate. To analyse the wake, we instead resort to numerical solutions of the Orr-Sommerfeld equation, referred to as linear stability theory (LST) in what follows. The same numerical methods have been used for the discretisation of the Orr-Sommerfeld equation in the spatial stability problem. The Kelvin-Helmholtz mode of the wake is obtained as the only unstable mode in the eigenspectrum.

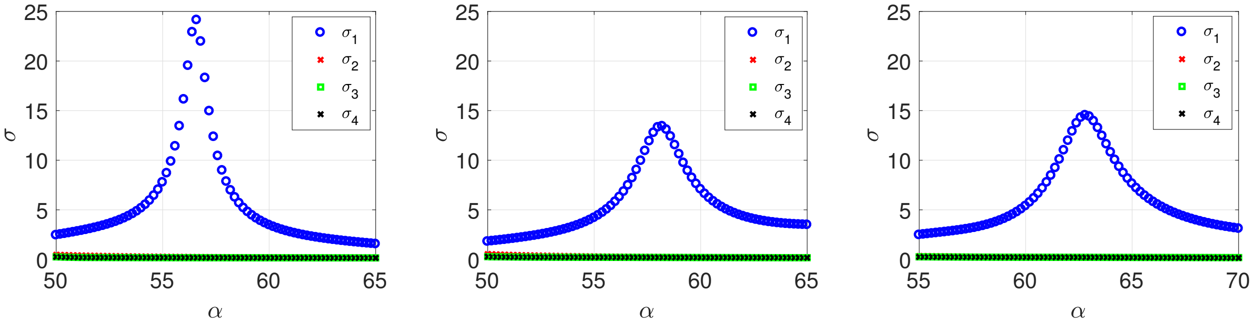
## 5 Mean Velocity Profiles

The analysed positions, where the mean velocity profiles were taken, are show in Fig. 7 (a). The profiles used to solve the resolvent analysis problem for turbulent boundary layer, were taken at three different positions $x/c = 0.7$, $x/c = 0.8$ and $x/c = 0.9$, are show in Fig. 7 (b). The mean velocity profiles used to analyse the wake region are show in Fig. 7 (c) at stations $x/c = 1.05$ and $x/c = 1.1$. The coherent structures at the wake are modelled as the Kelvin-Helmholtz mode from linear stability theory (LST), which is basically a eigenvalue problem from the Navier-Stokes equation.

## 6 Results

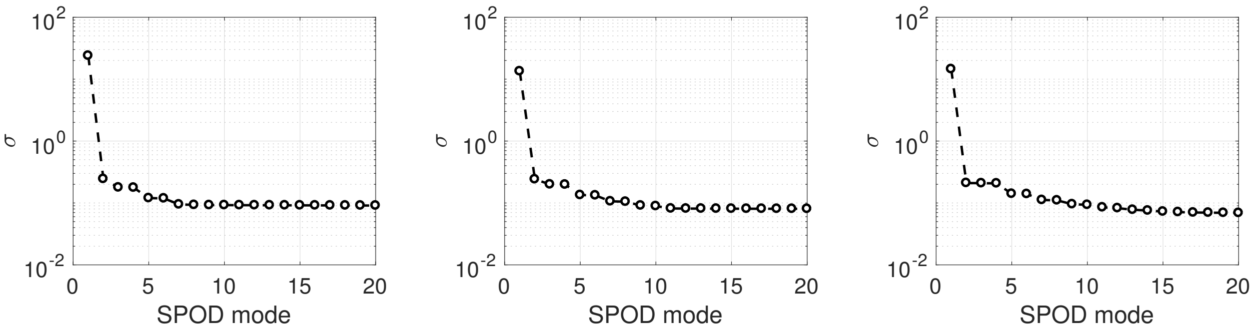
### 6.1 SPOD Results

The SPOD results are show in Fig. 8 for chordwise velocity fluctuations $u$ and in Fig. 9 for chord-normal fluctuations $v$. The results were obtained for $\mathrm{St} = 7.93$, which is the frequency that




(a) $x/c = 0.7$ (b) $x/c = 0.8$ (c) $x/c = 0.9$

**Fig. 5** First four gains as a function of the wavenumber for $St = 7.93$.




(a) $x/c = 0.7$ (b) $x/c = 0.8$ (c) $x/c = 0.9$

**Fig. 6** Resonvent gains for $St = 7.93$.

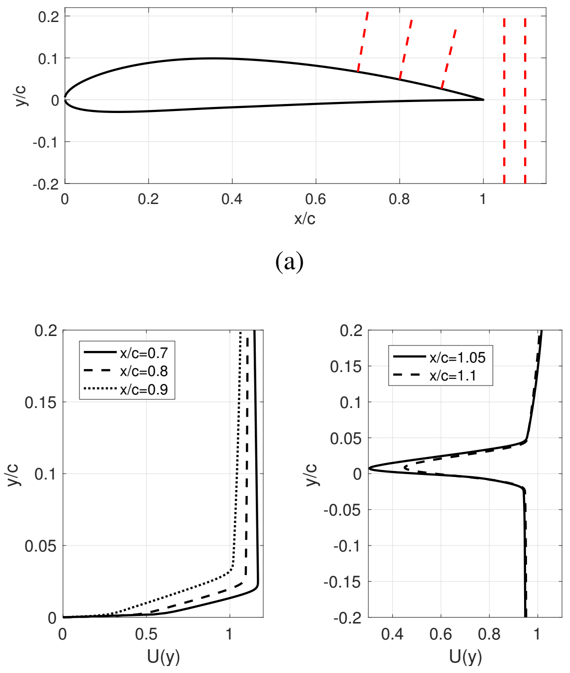
shows better convergence (see Fig. 4).

The first and second SPOD modes (Fig. 8 and Fig. 9) show a weak wavepacket propagating from the trailing-edge toward the wake region, where the amplitudes are highly amplified. In those cases we badly see the boundary layer structures, due to the magnitude of the wake.

In terms to highlight the structures in the boundary layer region, the wake contribution is neglected in SPOD by imposing zero at the integral weights in the wake region ($x/c > 1$). SPOD results in that case are show in Fig. 10 for chordwise velocity fluctuations $u$ and in Fig. 11 for chord-normal fluctuations $v$. The results were also obtained for $\mathrm{St} = 7.93$.

The first and second SPOD modes (Fig. 10 and Fig. 11) show now a wavepacket on the upper suface of the airfoil with more pronounced amplitudes near the trailing-edge region. This remarkably coherent wave shown in all quantities of the first two SPOD modes appears, even though the boundary layer is in a fully-turbulent state. Remind that the two-dimensional mode of the field was taken prior to the SPOD; this is thus a coherent, two-dimensional wave within the turbulent flow.

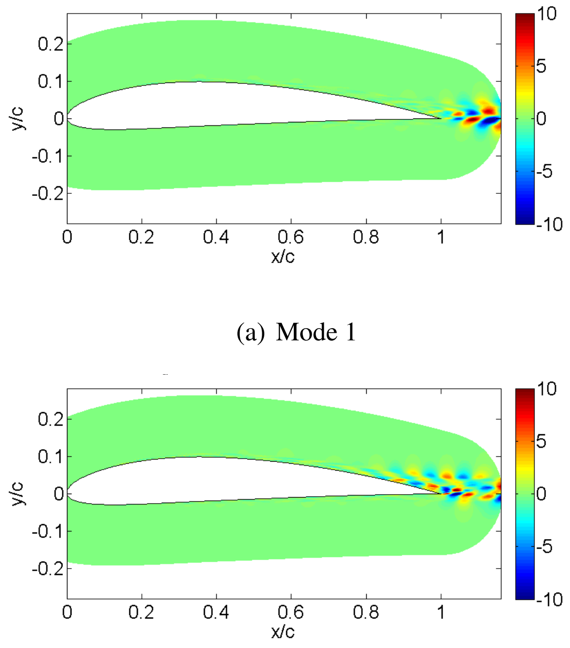
The SPOD results for the $u$ component (Fig. 10) show the fluctuations near the wall with phase opposition to disturbances from the wall to the boundary-layer edge. This suggests a behaviour of Tollmien-Schlichting waves although this is a turbulent boundary layer. The appearance of such waves in turbulent boundary was also found by Hussain & Reynolds [14], where turbulent channel flow was forced using vibrating wires, the turbulent boundary layer appears to exhibit T-S-like waves as well.




(a)

(b) (c)

**Fig. 7** Mean velocity profiles: (a) positions, (b) boundary layer and (c) wake for different x/c positions.




(a) Mode 1

(b) Mode 2

**Fig. 8** SPOD at St = 7.93 for chordwise velocity fluctuations $u$.

## 6.2 Resolvent analysis vs. SPOD Results

To model the waves in the boundary layer obtained using SPOD from Fig. 10 and 11 where the integral weights from the wake region were neglected, resolvent analysis was used as a reduced-order model. The results of the comparison between leading SPOD and resolvent modes are shown for the frequency $\mathrm{St} = 7.93$ for boundary layer stations taken at three different positions $x/c = 0.7$, $x/c = 0.8$ and $x/c = 0.9$ in Fig. 12, 13 and 14 respectively.

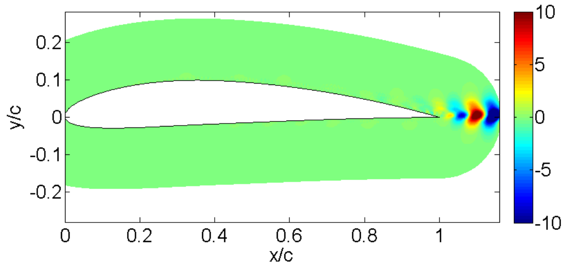
We observe that the first SPOD mode from the DNS in general show a good agreement with the first resolvent mode for $x/c = 0.7$. Since we deal with a turbulent boundary layer, an exact match is not expected, but the comparisons presented here show that resolvent analysis provides a good reduced-order model for the two-dimensional disturbances considered here, which play an important role in the radiation of trailing-edge noise [3]. The relative low amplitude of the two-dimensional disturbances within the boundary layer tends to ease the application of linearised models.

For positions closer to the trailing edge ($x/c = 0.8$ and 0.9) the agreement deteriorates, but the general features of the SPOD mode are captured by the optimal response from resolvent analysis. The worse agreement at downstream positions may be due to the proximity of the trailing edge, which leads to a stronger inhomogeneity in $x$, making the locally-parallel assumption invalid.

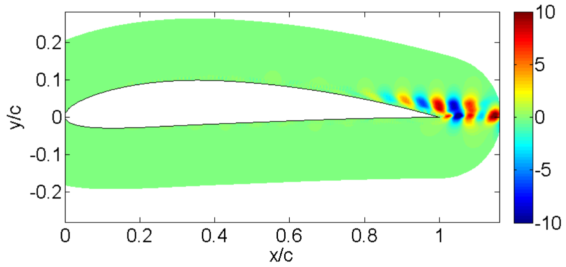
The present results suggest that reduced-order models for trailing-edge noise from airfoils could be built using a source model based on resolvent analysis for two-dimensional disturbances, which, when advected past the trailing edge, are scattered into acoustic waves; the scattering mechanism can be modelled using a tailored Green's function [15].

## 6.3 LST vs. SPOD Results

In terms to model the waves in the wake region, LST, which is also a reduced order model, was

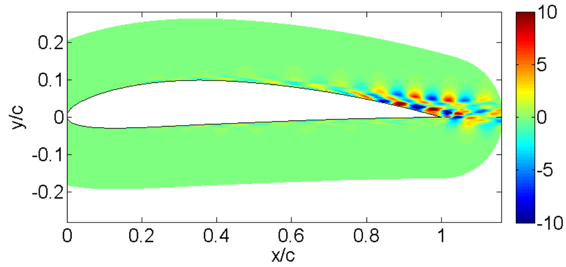



(a) Mode 1

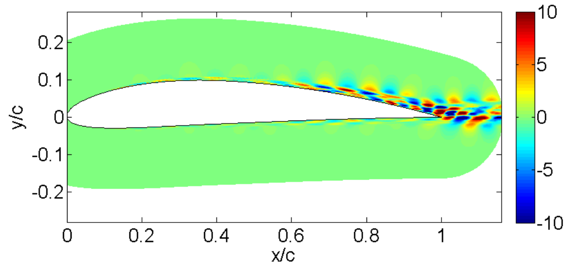



(b) Mode 2

**Fig. 9** SPOD at $\mathrm{St} = 7.93$ for chord-normal fluctuations $v$.




(a) Mode 1




(b) Mode 2

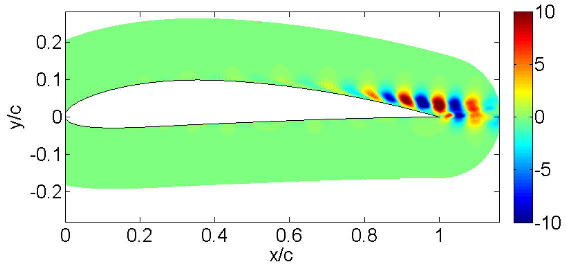
**Fig. 10** SPOD at $\mathrm{St} = 7.93$ for chordwise velocity fluctuations $u$ imposing zero at the integral weights in the wake region ($x/c > 1$).

used. The comparison results between leading SPOD from Fig. 8 and 9 and Kelvin-Helmholtz modes from LST are shown for $\mathrm{St} = 7.93$ at wake stations taken in two different positions $x/c = 1.05$ and $x/c = 1.1$ in Fig. 15 and 16 respectively. Close agreement is found between stability eigenfunctions and SPOD modes at the wake, showing that wake disturbances can also be modelled using the linearised operator.

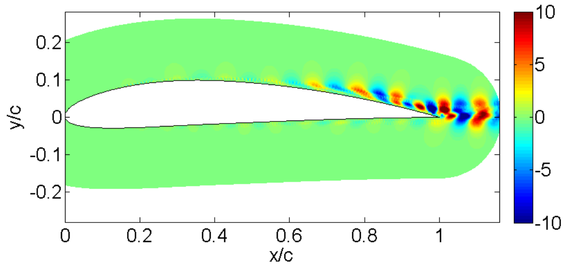
## 7 Conclusion

In the current work we have used SPOD to analyse the flow around a NACA 4412 airfoil with an angle of attack $AoA = 5^\circ$ at chord Reynolds number of $Re_c = 400000$, looking for coherent structures that could be modeled using linear stability theory. By doing so, we achieved a proper separation of spatially and temporally coherent structures, which are either hidden in stochastic turbulent fluctuations or spread over a wide frequency range. We found in the leading SPOD mode, which is statistically converged in our database, a coherent wave that propagates over the upper surface of the airfoil, with higher amplitude at the trailing edge region.

The observed two-dimensional, wavy disturbances in the leading SPOD modes were modelled using locally parallel resolvent analysis for the boundary layers, where turbulent mean profiles were used as base flows, and locally parallel linear stability analysis for the wake. Results for profiles in the trailing-edge region ($x/c = 0.70$, $x/c = 0.80$ and $x/c = 0.90$) show good agreement between the optimal flow response and the leading SPOD mode; moreover, the optimal response has a gain which is at least an order of magnitude higher than the suboptimals, which explains its presence in the turbulent boundary-layer fluctuations. The results highlight resolvent analysis as a pertinent reduced-order model for the relevant fluctuations, leading to predictions of the structure advecting waves reaching the edge. Results for the wake profiles show a good agreement with the linear stability theory as expected, since the Kelvin-Helmholtz mode has significant growth rates for wakes; such strong spatial growth sim-

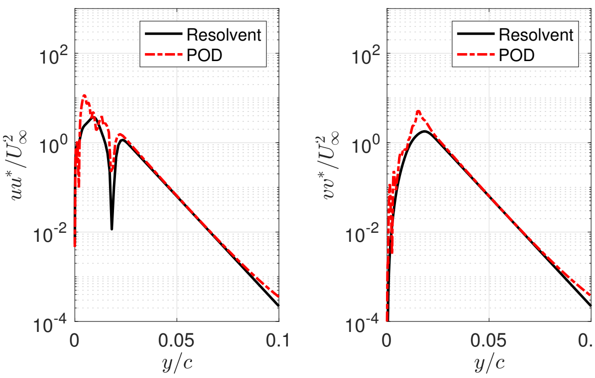



(a) Mode 1




(b) Mode 2

**Fig. 11** SPOD at St = 7.93 for chord-normal fluctuations $v$ imposing zero at the integral weights in the wake region ($x/c > 1$).




(a) u (b) v

**Fig. 12** Resolvent analysis vs. SPOD for $St = 7.93$ at $x/c = 0.7$.

plifies modelling in the near wake, similar to what is seen in turbulent jets [2]. Further numerical and experimental work aiming at the characterisation of such structures is promising.

## References


[1] McKeon B J, Sharma A A. *A critical-layer framework for turbulent pipe flow*. Journal of Fluid Mechanics, Vol. 658, pp. 336-382, 2010.

[2] Cavalieri A. V. G., Rodrigues D., Jordan P., and Colonius T., Gervais I. *Wavepackets in the velocity field of turbulent jets*. Journal of Fluid Mechanics, Vol. 730, pp 559-592, 2013.

[3] Abreu L, Cavalieri A, Wolf W. *Coherent hydrodynamic waves and trailing-edge noise*, 23st AIAA/CEAS Aeroacoustics Conference, Denver-CO, 2017.

[4] Towne A, Shimidt O, Colonius T. *Spectral proper orthogonal decomposition and its relationship to dynamic mode decomposition and resolvent analysis*. ArXiv:1708.04393, 2017.

[5] Ffowcs Williams J E, Hall L H. *Aerodynamic sound generation by turbulent flow in the vicinity of a scattering half plane*, Journal of Fluid Mechanics, Vol. 40, No. 04, pp. 657-670, 1970.

[6] Holmes P, Lumley J L, Berkooz G. *Turbulence, coherent structures, dynamical systems and symmetry*. 2nd edition, Cambridge University Press, 1998.

[7] Leite H F, Avelar A C, Abreu L, Schuch D, Cavalieri A. *Proper Orthogonal Decomposition and Spectral Analysis of a Wall-Mounted Square Cylinder Wake.*, J Aerosp Tecnol Manag, 10: eXX18. doi: 10.5028/jatm.v10.867, 2018.

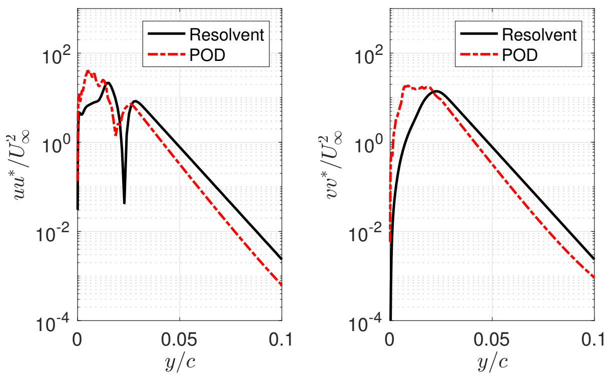
[8] Picard C, Delville J. *Pressure velocity coupling in a subsonic round jet*. International Journal of Heat and Fluid Flow, Vol. 21, pp 359-364, 2000.

[9] Hosseini S M, Vinuesa R, Schlatter P, Hanifi A, Hanningson D S. *Direct numerical simulation of the flow around a wing section at moderate Reynolds number*, International journal of Heat and Fluid Flow, Vol. 61, pp 117-128, 2016.

[10] Fischer P F, Lottes J W, Kerkemeier S G. *Nek5000*, Web page. http://nek5000.mcs.anl.gov, 2008.

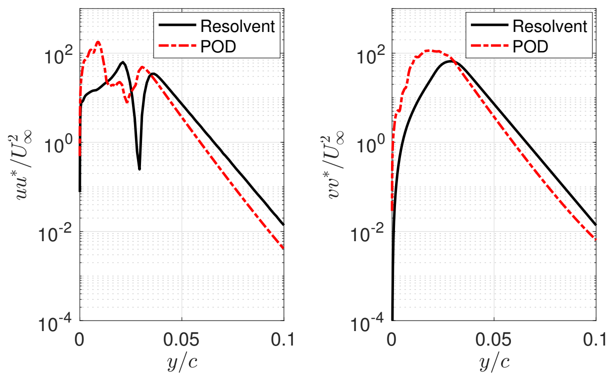
[11] Meyer K E, Pedersen M, Ozcan O. *Turbulent jet in crossflow analysed with Proper Orthogonal Decomposition*. Journal of Fluid Mechanics, Vol. 583, pp 199-227, 2007.

[12] Tissot G, Zhang M, Lajus F, Cavalieri A, Jordan P. *Sensitivity of wavepackets in jets to nonlinear effects: the role of the critical layer*. Journal of Fluid Mechanics, Vol. 811, pp 95-137, 2016.




(a) u (b) v

**Fig. 13** Resolvent analysis vs. SPOD for $St = 7.93$ at $x/c = 0.8$.




(a) u (b) v

**Fig. 14** Resolvent analysis vs. SPOD for $St = 7.93$ at $x/c = 0.9$.

[13] Trefthen L N. *Spectral Methods in MATLAB*. SIAM, Philadelphia, 2000.

[14] Reynolds W C, Hussain A K M F. *The mechanics of an organized wave in turbulent shear flow. Part 3. Theoretical models and comparisons with experiments*. Journal of Fluid Mechanics, Vol. 54(2), pp 263-288, 1972.

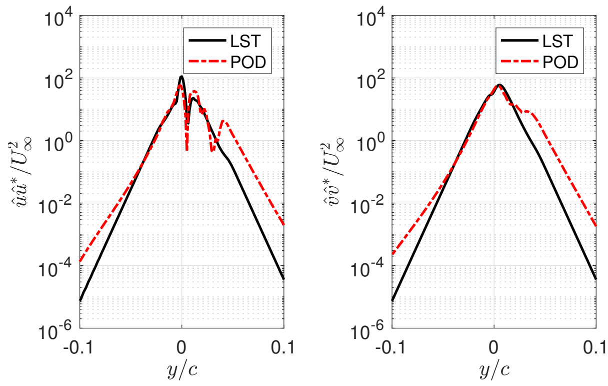
[15] Nogueira P. A. S., Cavalieri A. V. G., Jordan P. *A model problem for sound radiation by an installed jet*. Journal of Sound and Vibration, Vol. 391, 2016.

## 8 Contact Author Email Address

Email: leandraabreu13@gmail.com

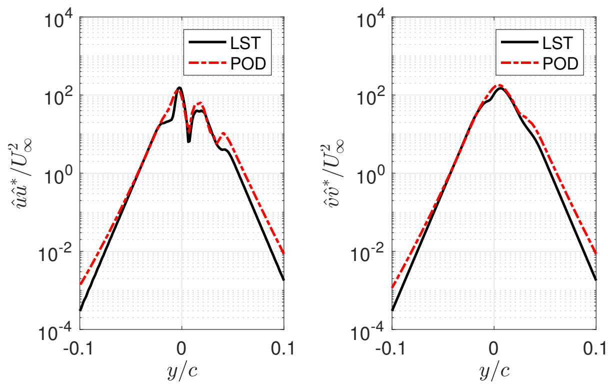
## Copyright Statement






(a) u (b) v

**Fig. 15** LST analysis vs. SPOD for $St = 7.93$ at $x/c = 1.05$.




(a) u (b) v

**Fig. 16** LST analysis vs. SPOD for $St = 7.93$ at $x/c = 1.1$.