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KINEMATICS AND DYNAMICS OF A TENSEGRITY STRUCTURE IN EXPANSION

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Abstract

Tensegrity structures are formed by rigid bodies under compression and cables (or tendons) under traction. This kind of structural arrangement is relevant for aerospace applications not only for its structural efficiency, but also because its stiffness can be controlled by pre-tensioning the tendons. The more those are tensioned, the stiffer the tensegrity is, therefore it is possible to change the total stiffness without changing the shape (if the tension is altered proportionally in all tendons) or change the shape without altering the stiffness (by varying the pre-tension asymmetrically). A tensegrity can also be deployed, saving volume inside a rocket, for instance, and expanded only in space, reaching longer lengths. This paper is focused on the dynamics of this expansion process, through analytical and experimental analyses.

1 Introduction

The term tensegrity is recent and was suggested by Buckminster Fuller, combining the words tension and integrity in 1955. Despite having given it a name, the history of the invention is uncertain and can be shared with Kenneth Snelson and David Georges Emmerich. Furthermore, Karl Ioganson patented in 1920 a three bars and eight cables structure, which could be considered a first step for the configurations of other inventors. [8]

A tensegrity is formed by elements in compression, the rigid bodies, and in tension, usually cables. A rigid bodies configuration that can be stabilized by a determined set of cables without external forces will be called a *tensegrity configuration*, but it will not whether the cables are not necessary or if there is no possible set of cables that stabilizes it. Once connected to the referred set of tendons, it becomes a *tensegrity system*. [18]

These structures can be designed so its elements stress in one direction only. This benefit not only simplifies the equations of motion but leads to more precise models. Also, these unidirectional stresses allow a more efficient material selection, leading to a mass reduction of the structure. [18] For space applications, the mass of the system is a relevant factor due to the limited launching payload, thus tensegrity structures can contribute to reduce the resources needed to put a satellite on orbit by making it lighter.

Another advantage of tensegrities over more traditional structures is the possibility of changing the pre-tension of the cables. If equally altered in all tendons, the stiffness of the structure will change but the shape will keep. However, if the pre-tension is unequally altered, the shape will change. Thus, it is possible to alter the shape without altering the stiffness and vice-versa, enabling numerous applications.

Furthermore, when it comes to control, the traditional structures are “tortured” by the controllers to behave in an unnatural way. The idea behind the controlled tensegrity is transforming the structure so its natural equilibrium change instead of stressing over a fixed equilibrium [18]. The rolling system of the Santos Dumont’s Demoiselle is a good example for this

(Figure 1). One of the wings is pulled by the cables so its shape, and hence its lift, is altered, making one wing's lift greater than the other's and finally leading the aircraft to roll. This system was replaced by the ailerons later.

Fig. 1 Demoiselle 20. Source: http://www.dominiopublico.gov.br.

The civil engineering uses of tensegrity structures are justified simply by their high structural efficiency in terms of mass, as in Rhode-Barbarigos' pedestrians bridge [14]. However, the possibility of having the geometry altered or deployed reinforce the use of tensegrities in space engineering. A growing demand on bigger antennas make this deployability more important for applications on satellites, as there is a limited volume and mass due to the technology available nowadays [23].

In that case, the structure would be sent to space on its deployed shape, saving volume, and expanded only on orbit. Morterolle [11] built and analysed the dynamic behaviour of the "Astromesh" [20], a structure made by struts, cables and a membrane that works as a reflector antenna. Similarly, Abedian [1] studied the dynamic behaviour of the hexagonal prism to perform the same role.

The stiffness control was verified through modal analyses on a tensegrity tower by Furuya [7], who showed that natural frequencies increase with the pre-tension. More recently, Bel Hadj Ali [4] came to the same output, but using not only experimental results but numerical too. Furthermore, Bel Hadj Ali [4] worked with different excitation frequencies and altered the pre-tension gradually so the natural frequency moved away from the excitation.

Motro [12] studied a 3 struts and 9 cables tensegrity through experiments and numerical models and showed that a linear model gives a good approximation of the nonlinear behaviour of simple structures. On the other hand, Yang e Sultan [19] studied a 4 bars and 4 cables membrane tensegrity and concluded this linear elastic model is not proper whether the membrane is under significant displacement amplitudes.

1.1 Retractile Structures

The presentation of retractile structures for space uses is necessary to indicate the existent alternatives and show their limitations [21] and [13]. A retractile structure is formed by connected elements that may change their configuration to reach a certain shape or volume. This characteristic permits the structure to transform from a compact shape into an expanded configuration. For space uses, these structures have the advantages of saving mass and volume, sustaining the launching loads in the compact shape and a good price-performance ratio when considering the compacted form's size and the expanded form's precision.

Tubular structures of thin wall were the first retractile structures to be used in the space industry. These structures take advantage of the elastic behaviour of materials such as steel, copper-beryllium alloys or carbon fibre strengthened polymers to take the tube shape after expanded. There are two main types: STEM (storable tubular extendible member) and CMT (collapsible tubular mast). Telescopic structures, however, consist of concentric cylindrical tubes stored in a compact form. This kind of structure is heavily used as camera stands, for example, in transmission vehicles for TV or mobile watch towers.

Retractile truss structures are designed to solve the issues related to packing and implantation of big space structures during the launch. Zhang [24] used shape memory polymers and glass fibre composites to build retractile trusses of high reliability. Yang and Sultan [22] modeled a control strategy for a deployable tensegrity-membrane system, which had its deployment sequence simulated. The membrane on top of the structure would act as the reflector surface of an antenna, supported by the lightweight and deployable set of struts and cables underneath.

Articulated bar structures are heavily used in the space industry and may appear in numerous configurations. The advantages over the previous structures are the greater stiffness and accuracy. The variable geometry truss and the cable strengthened pantograph are some examples. We may remember the coinable masts, typically used in space. The expanded form is reached by a screw mechanism. The tensegrity structures are interesting for their deployability given by the articulated members.

1.2 Opening Mechanism

There are various possible opening mechanisms. Pellegrino [13] suggests changing the size of a member (cable or strut) and finding the positions of the nodes through geometry. Arsenault [2] started from a prism and replaced six tendons by springs, added actuators that change the length of the bars and found the kinematic relations analytically through the Jacobian matrix.

Russel [16] too endorse the use of tensegrities for the space industry. On his work, the elements under traction are replaced by inflatable films, then the system deploys whenever the air in the films is released. A finite element model was implemented on ANSYS and exported to LS-DYNA to perform the kinematic analysis. Zolesi [25] comments about the form-finding property, suggests a deployable antenna of 12m diameter and simulates its expansion through a numerical model. Rhode-Barbarigos [15] uses a variation of the dynamic relaxation method to perform the same expansion study on a similar structure. This dynamic relaxation method, described and used by Bel Hadj Ali [5], inserts fake masses and damping on the motion equation, transforming a statics problem into a dynamics problem, and solves the equation of motion through finite differences in function of time. The method calculates the positions of each node every instant, and the damping leads these positions to rest in a static equilibrium over time.

The form-finding property allows the designer to find the shape of the tensegrity given the pre-stresses in the cables, therefore, might enhance the expansion analysis of the structure. The force density algorithm suggested by Zhang e Ohsaki [23] starts from an initial pre-stress (usually impossible), finds the closest feasible pre-tension and then the position of the nodes. Finally, the resultant stresses and the properties of the spring (or actuator power) responsible for the expansion can be found by the energy method, described by Moored [10] and also used by Bel Hadj Ali [5].

Schenk [17] built a tensegrity prism with variable cable lengths and studied its many static and symmetric positions. The aim of this work is to develop an analogous system, but with a steady movement. The positions of the nodes can be obtained simply by geometry, but the axial forces involved appear in a complex 3D configuration, therefore the positions will be found analytically and the actuator force through an energy method. Regarding the prototype, the motion data will be collected via image processing by tracking the spots of interest with markers, similar to the methodology used by Lessard [9] and Baltaxe-Admony [3] in biomechanics.

2 Description of the system

The 4 struts tensegrity prism will be studied (Figure 2). The lower base is fixed and the upper base is able to move up and down, but always parallel to the ground. The horizontal cables and the bars have constant lengths. Only the inclined cables, connecting the upper and the lower nodes, can change length, and all of them do it at the same rate, forcing the system remain symmetric, keeping both bases always parallel to each other and requiring only one single engine to vary all the four inclined tendons simultaneously.

The application of this system in reality would be on a reflector antenna. A satellite's signal hits the reflector and gets directed to Earth, therefore the antenna must stand at a minimum distance from the satellite's wall, otherwise part of the signal could hit the wall and come to Earth creating noise. The system presented in this paper represents a deployable stand for the antenna and would replace the cone and the structures below the reflector surface (Figure 3).

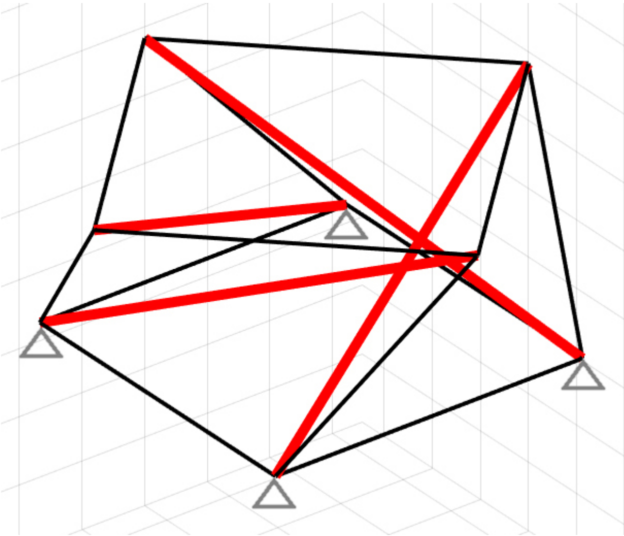

Fig. 2 Four struts tensegrity prism.

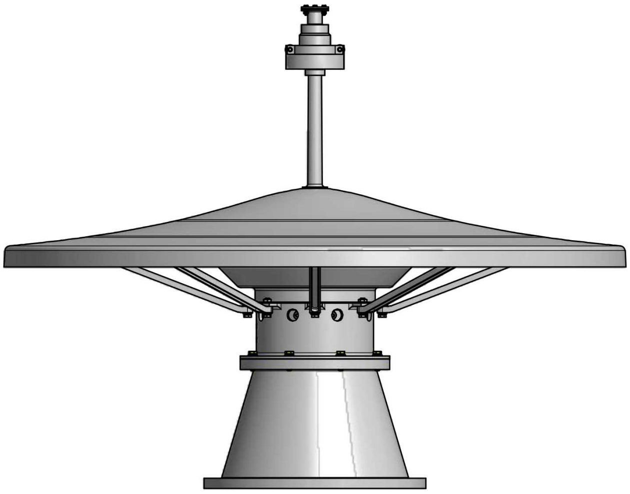

Fig. 3 Antenna.

2.1 Statics

The position of the lower nodes do not change because they are fixed, what will be calculated in this section is the position of the upper nodes, which defines the shape of the tensegrity.

The distance h between the two bases and the twist angle ta change as the length v of the inclined cables vary. The bars' length b and the side l of the square bases are constant. The nodes are numbered as indicated in the top view of the structure (Figure 4).

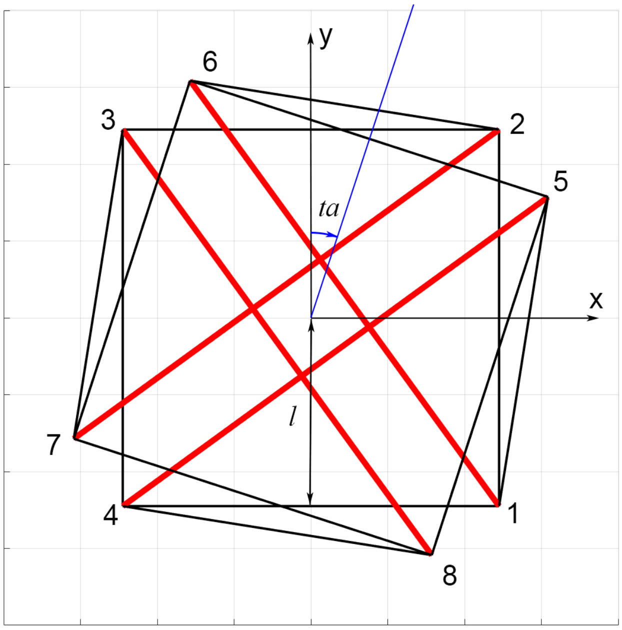

Fig. 4 Top view.

Through loop equations [6], the position of each node can be determined in equation 1, where each column is a node and each to row is assigned one direction (x, y and z). The variables c and s are defined in equations 2 and 3.

$$N = \begin{bmatrix} l & l & -l & -l & l(c-s) & l(c+s) & l(s-c) & l(-c-s) \\ -l & l & l & -l & l(-c-s) & l(c-s) & l(c+s) & l(s-c) \\ 0 & 0 & 0 & 0 & h & h & h & \text{h} \end{bmatrix} \quad (1)$$

$$c = cos(ta) \quad (2)$$

$$s = sin(ta) \quad (3)$$

The distances from nodes 1 to 6, 2 to 7, 3 to 8, and 4 to 5, are known and equal to the strut length b. Then we can isolate the height h in function of b, l and ta (equation 4).

$$h^2 = b^2 - l^2[(1-s+c)^2 + (1+s+c)^2] \quad (4)$$

Finally, the length of the inclined cables v can be calculated (equation 5) from the distance between the nodes 1 and 5, or 2 and 6, or 3 and 7 or 4 and 8.

$$v^2 = l^2[(1-c+s)^2 + (c+s-1)^2] + h^2 \quad (5)$$

2.2 Kinematics

This solution is not so useful yet because its input is the twist angle and the only variable we can control is v. Furthermore, the twist angle cannot be simply isolated because it appears inside the sins and cosines. Applying a numerical method would be a powerful way out, but a simpler one can be used in this case.

The twist angle ta_s that makes a tensegrity prism become stable is defined in equation 6 in function of the n sides of the prism's bases.

$$ta_s = \frac{\pi}{2} - \frac{\pi}{n} \quad (6)$$

Thus, we know the ta's interval in which the mechanism works, and its starting and finishing points will depend on the way the top view was designed and how the nodes were numbered. In this case, starting at 0 and finishing at $\pi/4$ leads to the height varying from zero to maximum and v from maximum to minimum.

Accordingly, a velocity (and/or acceleration) can be assigned to v, and then the time can be calculated. Finally, we have the positions of the nodes in each instant of time for a given velocity/acceleration of v, which is the only variable we can truly control. The distance between nodes 1 and 6, 2 and 7, etc. has to be always constant and equal to the length of the strut, this property can be used to test if the algorithm is working properly.

2.3 Energy

Finding the engine power and the supports reactions is another important step for designing this mechanism. Because the structure is symmetric, we can calculate the engine traction to lift one strut only and then multiply it by 4. Each inclined cable run from the tip of a strut through the lower end of its neighbour and meet the other 3 cables in the centre of the lower base (Figure 5).

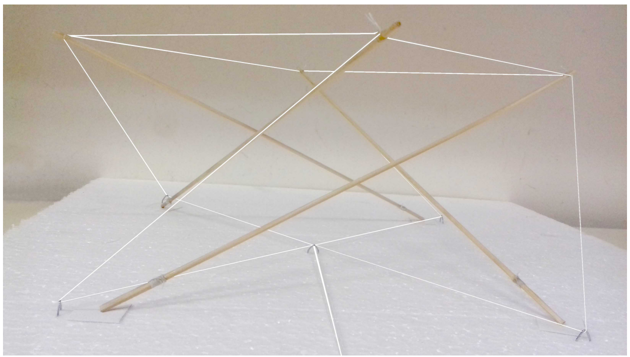

Fig. 5 Prototype.

Neglecting the weight of the wires, all the potential energy gained by the struts, plus the energy lost due to friction with the clips, comes from the engine that pulled the cables (equation 7). Then the force and power of the engine can be found (equations 8 and 9).

$$E_p + E_f = E_e -> 4mg\frac{h}{2} = (F - F_f)\Delta v \quad (7)$$

$$F = \frac{2mgh}{\Delta v} + F_f \quad (8)$$

$$W = \frac{2mgh + F_f}{t} \quad (9)$$

For a higher complexity tensegrity, with more struts, for example, the force and power will be greater because h and m will be greater and Δv

will be lower. However, these parameters depend on the gravity g, and this structure may be designed for space applications, where g can be zero. Therefore, the engine might not be able to lift the structure on Earth but do it easily in space.

2.4 Algorithm

The following algorithm contains the vital steps to find the engine force that is necessary to lift the tensegrity from its deployed position to the expanded shape.

Kinematics:

- Set the twist angle limits and the velocity of the inclined cables.
- Calculate the height h (equation 4).
- Find the nodes' positions N (equation 1).
- Calculate the length of the inclined cable v (equation 5).
- Calculate the time through the given velocity for v.

Energy:

- Calculate the variations of v and h.
- Calculate the force and power of the engine (equations 8 and 9).

The distance between nodes 1 and 2, for example, have to be constant and equal to the square side, this length can be checked every instant to indicate whether the calculations are consistent or not. Plotting the tensegrity would be even better, but these procedures were not described in the algorithm as they are not vital to find the force of the engine or the behaviour of the structure.

3 Results and Discussion

3.1 Simulation

In the prototype, the wires do not connect to the joints (Figure 5) but to the clips, which are $20mm$ farther on the same direction that contain the two joints. Therefore, the numerical model was implemented to copy this characteristic. For $l = 0.1237m$, $b = 0.35m$ and with ta varying from 0 to approximately $\pi/4 rad$, the height h goes from 0 to $0.137m$ (Figure 6). Furthermore, the height varies sharply with a small pull of the inclined cable (Figure 7).

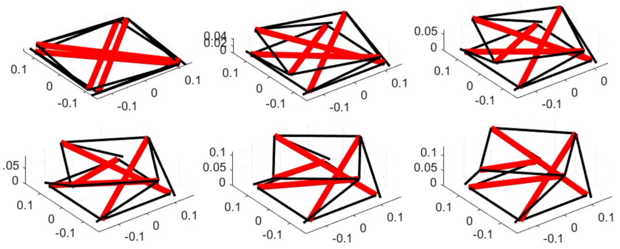

Fig. 6 Expansion.

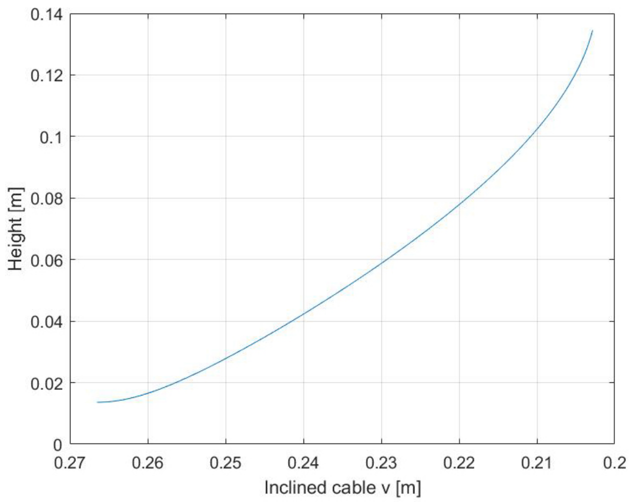

Fig. 7 h and v.

These characteristics so far are only geometrical and therefore not affected by the time or materials properties. The inclined cable was reduced by $64mm$, so for a total time of $6s$, which is reasonable for the expansion process, we may assume a velocity of $-0.01m/s$ for v (negative because the cable must be pulled) and no acceleration. Then it is possible to plot the twist angle and the height versus time (Figure 8).

As the height varies almost linearly with v, and we forced v to vary linearly with time, the height was expected to vary similarly to v with time. Additionally, the twist angle increases with time just as the height does, and these results

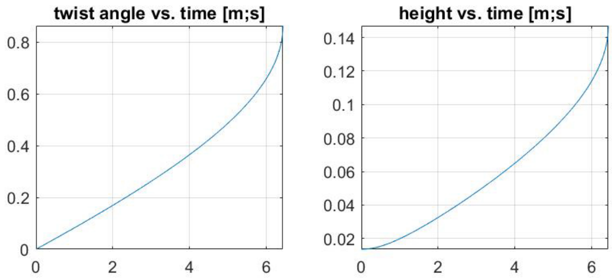

Fig. 8 *h* and *ta*.

combine with the expansion process detailed before (Figure 6).

3.2 Prototype

Recorded by a 30 fps camera, the movement was analyzed through the *Kinovea* software (Figure 9). The height and the pulled cable were tracked, calibrated according to their own minimum and maximum dimensions and plotted against each other over the simulation results (Figure 10).

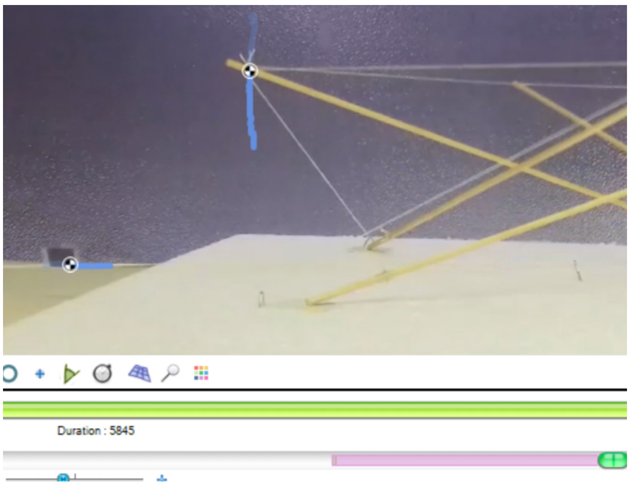

Fig. 9 *Kinovea* 1.

The experimental results do not appear on the left hand side of the graph because the real structure does not deploy shorter than that. In the simulation, the struts are capable of trespassing each other, while in reality the diameter becomes more relevant. However, once the simulation reaches this minimum real height, the experimental results match the simulation outputs

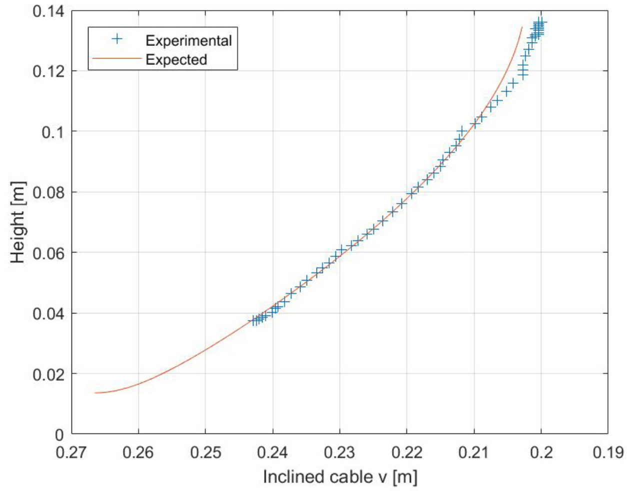

Fig. 10 Experiment 1.

pleasingly with more than 80% of the experimental points precisely on the simulation curve.

When the structure comes close to its maximum height, a difference appears due to the force applied. This force becomes greater in the end because the structure can no longer grow up and all the extra stress turns into buckling of the struts and bending of the clips. The acquisition considered the displacement of a mark fixed on the cable as the amount of cable pulled, when in fact this displacement is the cable amount of cable pulled plus the bending of the clips and buckling of the struts, which explains why the experimental results are slightly shifted to the right when the mechanism comes close to its maximum height.

The experiment was repeated from the top view (Figure 11) to extract the twist angle.

However, from this view, the image was clearer and the software managed to acquire the data more accurately, which lead to an improved match with the expected results.

3.3 Energy Analysis

The cable that lifts the prototype was attached to a scale to measure the necessary force $(0.04kgf \pm 0.01kgf)$ to keep it up. Hence, in this scenario, the friction forces assist the tensegrity to stand, which switches the sign of F_f in the equation 8. As this prototype is so light, four known masses ($15g$ each) were fixed on the struts

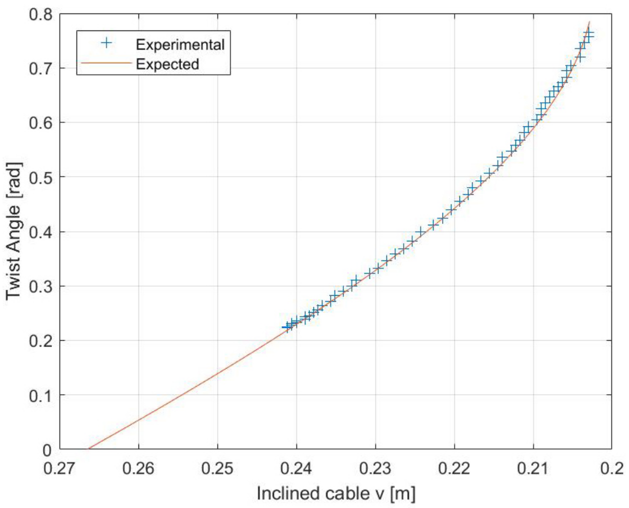

Fig. 11 Experiment 2.

(1.8g each) to provide the structure a considerable weight (Figure 12).

There are 8 contacts between cables and clips, but only 6 of them change the direction of the cables by an acute angle and therefore have significant friction contribution. The remaining 2 were neglected.

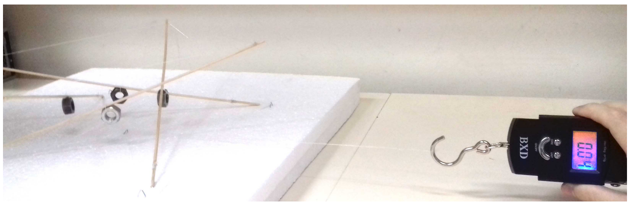

Fig. 12 Force experiment.

Another experiment was conducted to estimate the clip/wire friction. Known masses were hanged by the wire through a clip forming an acute angle, similar to the prototype's condition. The friction forces hold an average of $29.8\% \pm 0.13\%$ of the weight (Table 1).

Table 1. Friction.

$W[kgf]$	0.09	0.17	0.2	0.31	0.36	0.4	0.47	0.51	0.57
$W-F_{fr}[kgf]$	0.06	0.13	0.13	0.21	0.25	0.27	0.34	0.37	0.42
F_{fr}/W	0.33	0.24	0.35	0.32	0.31	0.33	0.28	0.27	0.26

Back to the tensegrity experiment, for the two cables that have only 1 significant clip contact, the force held by the scale is only 70.2% of the force transmitted to the bar, and for the other two wires (with 2 clip contacts each), this relation is $(70.2\%)^2$. The total force measured by the scale is the sum of all 4 wires (equation 10) after passing through the clips.

$$F = 2\left[70.2\% + (70.2\%)^2\right]\left(\frac{mg}{\Delta v}\frac{h}{2}\right) \qquad (10)$$

Solving for the force, we have $F = 0.42N = 0.042kgf$ as read on the scale, which validates the algorithm and the assumptions.

4 Conclusions

The deployability of a 4 struts class 1 tensegrity was studied, tested and verified to be functional. The experiments had some limitations, such as the scale's range, friction on the clip/wire contact and the *Kinovea*'s low accuracy, which forced us to repeat the video recordings on different backgrounds, for example. However, the results of the experiments and the outputs of the algorithm agreed, which leads us to assume that both the kinematic and energy analyses of the algorithm were accurate.

Finally, tensegrities are strong options for space applications in general, for their high structural efficiency and because they can be deployed and then save volume and payload in the rocket. This characteristic makes tensegrity structures unique, and studying their expansion is an important step for defending this example of application. Nowadays, the reflector antennas of satellites have fixed and stiff stands, this work suggested a lighter, deployable and stiffness-controllable alternative.

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