STRESS-BASED TOPOLOGY OPTIMIZATION WITH USING GLOBAL-LOCAL APPROACH

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Abstract

The paper is devoted to development of new approach to topology optimization when stress and stiffness are considered simultaneously. Two topology optimization algorithms are proposed. The first one is for global structural model and the second one is for local zones where stress concentration should be minimized. The proposed method is demonstrated on the examples of L-bracket and aircraft wing.

1 Introduction

Structural stiffness and strength are general aspects at structural design in different technical fields. The most frequent problem is to minimize the objective functions such as compliance and maximum stress. The first of them corresponds to the stiffest structure and the second – to strongest one. The final design should be both stiff and strong, so it is necessary to develop an efficient method for multi-criteria structural optimization.

In such approaches, structural weight is traditionally an objective function, and displacements and stresses are constraints [1–3]. In paper [4] the optimality criteria methods are proposed for simultaneous consideration of stress and stiffness constraints. For stress-constrained problems, the Kreisselmeier–Steinhauser (KS) function [5] was first used in [6]. The authors constructed a multi-objective function of stress and stiffness criteria with respect to material density of finite elements. The KS function was adopted to represent a global stress function. A weighting average scheme was adopted in paper [7] to identify the overall effects on the stiffness and stresses due to varying element thicknesses. A design with maximized static stiffness and minimized peak stress is achieved by gradually shifting material from the under-utilized regions onto the over-utilized ones.

The most general structural optimization method is topology optimization that is used to find in a design domain an optimum material distribution. The geometry and shape of the final design is the result of topology optimization.

The authors' papers [8, 9] show that the significant weight reduction is achieved after sizing optimization with stress/buckling/aeroelasticity constraints of the structural layout after interpretation of the topology optimization result. The sizing optimization must be done because topology optimization was focused primarily on global structural behavior such as structural compliance for the most critical load case. So topology-optimized design did not satisfy local failure criteria under the applied load conditions.

The purpose of this research is to develop an approach to structural optimization in which stresses and stiffness must be simultaneously considered in topology optimization stage. It is known that stress-based topology optimization problems are difficult to solve because a large number of constraints must be considered and stress is highly nonlinear with respect to design variables. Here, global functions are used to approximate local stresses in zones of peak stresses. These zones are found after first stage of topology optimization that is based on the free-size fully-stress design algorithm that leads to large values of densities in the zones. The second stage of topology optimization is to determine material distributions in zones of stress concentration with taking into account gauge constraints on maximum of densities.

An important argument is the decision on the optimality of the designs formed by optimization procedures. While the elements are removed after each finite element analysis throughout the optimization process, the weight of the structure is decreased, maximum-minimum stress values become closer, and the performance of the new design is increased. In a manner of speaking, the efficiency of the material usage for the new design increases iteratively as expected.

This paper presents new approach to structural optimization when stress and stiffness are considered simultaneously. It is based on the two proposed topology optimization algorithms. The approach is demonstrated on the examples of L-bracket and aircraft wing.

2 Problem Statement and Ideas

The most of problem statements for structural optimization and topology optimization can be mathematically formulated as nonlinear programming problem. The differences in the statements are due to what kind of the objective function is considered, what design constraints are imposed and what types of design variables are included in the optimization problem. Usually, in optimization of aircraft structures, the objective function is weight, but the majority of the developed methods of topology optimization deals with compliance. For instance, in the most of commercial programs, the objective function is compliance and the constraint is the weight of saving materials. At the same time, structural optimization of aircraft wing can include many multidisciplinary constraints such as stresses, displacements, eigenvalues, effectiveness of control surfaces, flutter speed, etc. The peculiar difficulty to use topology optimization is related with the type of design variables. Topology optimization has binary design variables while structural optimization has continuous ones.

The proposed approach how to combine these two types of optimizations is based on the three main ideas. The first idea is to make statements of topology and structural optimizations closer. The second idea is to include stress constraints as they play the key role in the problem of minimization of structural weight. Since stress is local response and is singular function the application of nonlinear programming method is complicated. Therefore, the third idea is to use the simplest optimization methods for getting near-optimum solution. In addition, simple optimization methods can treat multiple load cases.

Stress-based topology optimization for 2D problem is formulated as follows:

$$\text{Find} \qquad \min \sum_{i=1}^{N} x_i A_i$$

$$\text{at constraints} \quad \sigma_{il} \le \overline{\sigma}_i,\ i = \overline{1,N},\ \ l = \overline{1,LC},$$

$$\mathbf{Ku} = \mathbf{R},$$

$$x_i \in \{x^l, x^u\}.$$

Here x_i is thickness of i-th element (design variable), x^l and x^u are lower and upper bounds which values are defined from stress constraints, A_i is area of element, σ_{il} is acting stress for l-th load case, $\overline{\sigma}_i$ is admissible stress, N is number of elements, LC is number of load cases, $\mathbf{K}$ is global stiffness matrix, $\mathbf{u}$ are vectors of displacements corresponding to vectors of applied loads $\mathbf{R}$. Note that x^l has near-zero value to avoid singularity of stiffness matrix.

There are some difficulties to solve this problem. It is due to that stress constraint is local state variable, number of such constraints is very large, stress is singular when design variable tends to zero and stress is highly nonlinear with respect to design variable.

In technical literature, there are different proposed solutions to treat these difficulties. Often constraint aggregation is used to reduce the number of constraints and constraint relaxation is applied to access true optima. Then the stated problem can be solved by using non-linear programming methods with some approximations but it can be computationally expensive. There is a drawback in use of constraint aggregations. As the number of local functions increases, it is complicated to reach good approximation of the local stress functions. It is also known that global optimum of the relaxed problem is not necessarily global optimum of initial problem.

Design engineers need no true optimum. Near-optimum solution is sufficient for practical purposes. Therefore, we propose to solve this problem by using simple algorithms like fully stress design (FSD) algorithm. Design engineers often employ such algorithm for structural optimization. Another idea is that the largest stresses in structure are observed in concentration zones. These zones are especially important for topology optimization.

3 General Optimization Procedure

General optimization procedure includes two types of optimization: topology optimization for determination of optimal structural layout and structural optimization to accomplish sizing of structural elements.

In the context of aircraft structural design the procedure begins from specification of geometric outlines of aircraft component that serve as initial data for aerodynamic analysis and topology optimization. After aerodynamic load analysis some parts of the design domain are fixed and another parts are subjected by external loads to perform topology optimization. This is done by using global and local algorithms for topology optimization, which are given below.

Then the engineering interpretation of topology optimization results is accomplished by engineers' intuition to choose several alternative layouts that can be implemented in production. By using conventional design optimization algorithms the sizes of structural elements for alternative layouts are determined to minimize weight with taking into consideration stress/buckling/aeroelasticity constraints. Finally, to find optimal structure the ranking of the considered structures with different layouts is performed on the basis of comparison of obtained optimal weights.

4 Global-Local Algorithms for Stress-based Topology Optimization

The proposed approach includes two topology optimization algorithms. The first one is for global finite element model of structure, and the second one is for local zones with stress concentrations. Sequential application of these algorithms gives possibility to get near-optimal design.

4.1 Algorithm for Global Stress-based Topology Optimization

The algorithm for global stress-based topology optimization consists of four stages. The initial stage includes definition of design domain, generation of refined finite element mesh, application of loads and imposition of boundary conditions. The control parameters of numerical procedure such as threshold values, thickness bounds, allowable stresses, maximum number of iterations should be specified.

The second stage includes internal FSD-like algorithm which operates with availability of minimum and maximum thickness bounds. The main steps of the algorithm are following:

1. Perform structural analysis, calculate stresses in element centers for given load cases.
2. Determine maximum failure indices as maximum ratio of Mises stress to admissible stress in each element for all considered load cases.
3. New value of thickness is calculated by multiplication of old value of thickness by maximum failure index. In elements where thickness becomes less than minimum value, the thickness is considered to have minimum value. In elements where thickness becomes greater than maximum value, the thickness is considered to have maximum value.
4. The steps 1-3 are repeated until maximum failure index for all elements converges to one or the number of iterations exceeds the maximum iteration of FSD algorithm.

The third stage is the outer cycle for material removal in the places where the element thickness less than the threshold value. In the global algorithm at this stage, the threshold value is changing from less value to the larger one. The stage include two steps:

5. Threshold value is determined as minimum threshold value multiplied by a number of cycle of material removal. Thickness values less than threshold

value are taken to be equal to minimum ones.

6. If the number of cycles of removal materials reaches maximum then go to step 7, otherwise go to step 1.

The final stage is also intended for material removal but here material is removed where the maximum failure index is very small. This stage includes smoothing procedure to avoid checker-board effect. Steps in the final stage are

7. Specify minimum values of design variables for the remaining elements where the maximum failure index is very small. Perform the smoothing procedure by adding and/or removing elements with using criterion of element connectivity in obtained structure.
8. Accomplish final structural analysis, calculate maximum failure index for the whole obtained structure, thickness of optimum structure is calculated by multiplication of maximum thickness by the calculated index.

4.2 Algorithm for Local Stress-based Topology Optimization

There are zones (Fig. 1) in structure where stress distribution is peculiar

- places where boundary conditions are imposed;
- places of load applications;
- places of stress concentration.

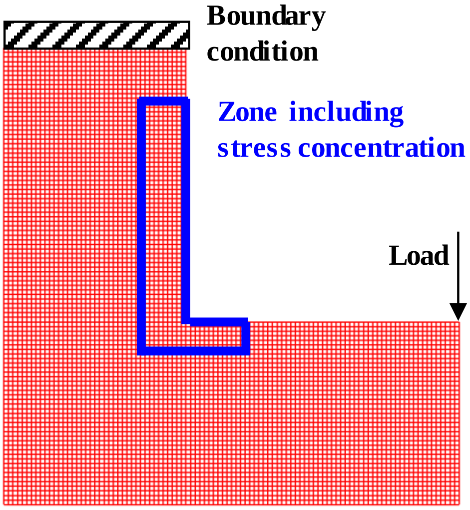

Fig. 1. Example of Peak Stress Zones

The zones where loads applied are usually designed without topology optimization and concentrated loads are replaced by distributed ones. As for zones of boundary conditions, the FSD-like algorithm can treat with them. But it can't treat with zones of stress concentrations as a lot of material is added there. In fact, it is needed to reduce peak stress by optimization of shape of stress concentration zone.

These peculiarities should be taken into account at solution of topology optimization problem. We propose an algorithm which is based on two simple ideas:

- elements with large stresses are removing;
- some elements in neighborhood with removed ones could also become unneeded.

The main steps of the algorithm are following:

1. From engineering viewpoint, the zone including stress concentration is specified and only elements of the zone are considered in optimization. A set of stress threshold values and small value for stress are specified
2. Perform structural analysis
3. The elements where stress is higher than stress threshold value are removed
4. The elements where stress is lower than some given small value are removed
5. Stress threshold value is changed to a new one from the set
6. If all stress threshold values are not looked over then go to step 2
7. From all obtained designs one chooses the best design with taking into account stress, stiffness and weight criteria

5 Numerical Examples

The proposed global-local topology optimization methods and general optimization procedure are demonstrated on the example of L-bracket and aircraft wing, correspondingly.

5.1 L-bracket

The sizes, load application and boundary conditions for numerical model of L-bracket are presented in Fig. 2.

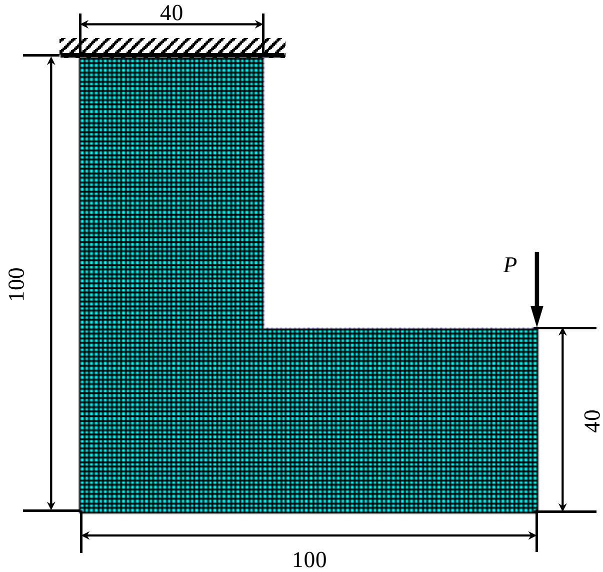

Fig. 2. Sizes, Loads and Boundary Conditions

The structure is made of isotropic material with mechanical characteristics: Young modulus, 1.0, Poisson ratio, 0.3, material density, 1.0. The force P is equal to 1.0, top of the bracket is fixed for all degrees of freedom. Mathematical model contains 6400 elements with size 1×1. Admissible stress for all elements is equal to 1.0. The analysis shows that the maximum stress is observed in the inner corner of L-bracket and it equals to 0.778. The thickness of all elements can be reduced to this value so that stress in this zone becomes equal to the admissible stress. The weight of such structure is equal to 4979.2. The general problem is to minimize this weight at satisfying stress constraints.

5.1.1 Local Topology Optimization

At local level the weight minimization is provided by removal of the most stressed elements. The best design also must have high stiffness, therefore different performance indices including stress, stiffness and weight are calculated. These indices are introduced in Table 1.

In the table, σ, W, U are stress, weight and strain energy, correspondingly, subscript index i corresponds to i-th iteration in local topology optimization, subscript index 0 corresponds to the initial design. The dependence of the performance indices on the stress threshold values is represented in Fig. 3 and 4.

Table 1. Performance Indices

Stress	$PI_S = \dfrac{\sigma_{i,\max}}{\sigma_{0,\max}}$
Weight	$PI_W = \dfrac{W_i}{W_0}$
Stress/Weight	$PI_{SW} = \dfrac{\sigma_{i,\max} W_i}{\sigma_{0,\max} W_0}$
Stress/Stiffness	$PI_{SSf} = \dfrac{\sigma_{i,\max} U_i}{\sigma_{0,\max} U_0}$
Weight/Stiffness	$PI_{WSf} = \dfrac{U_i W_i}{U_0 W_0}$
Stress/Weight/Stiffness	$PI_{SWSf} = \dfrac{\sigma_{i,\max} U_i W_i}{\sigma_{0,\max} U_0 W_0}$

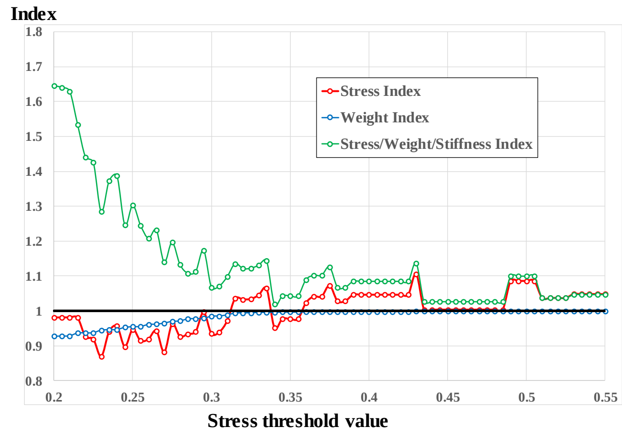

Fig. 3. Performance Indices versus Stress Threshold

In Fig. 4, two local minimums of PI_{SW} are marked by numbers in circles. The design 2 corresponds to the least stress-to-weight ratio, but as it can be seen the stiffness of this design is low if compared with the design 1. Therefore, design 1 can be considered as more preferable.

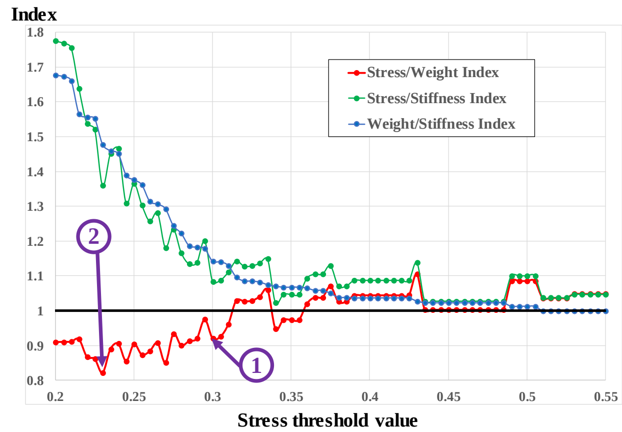

Fig. 4. Performance Indices versus Stress Threshold

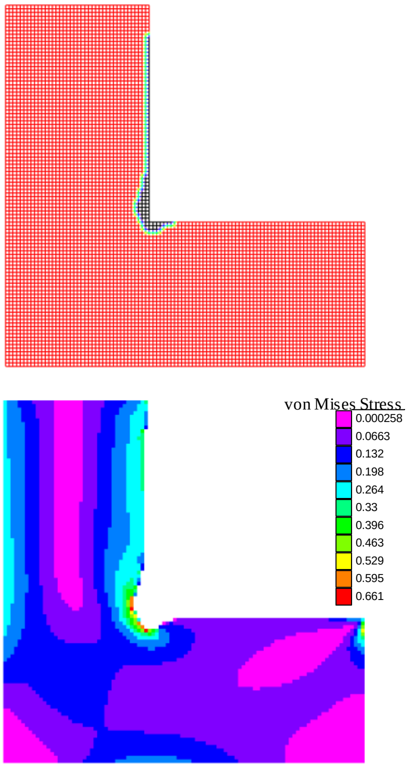

Fig. 5. Local Topology Optimization Result

5.1.2 Global Topology Optimization

The modified FSD algorithm can be written by iterative formulas for resizing thicknesses of elements:

$$x_i^{(k+1)} = \begin{cases} x^l, & \text{if } \tilde{x} \le x^l \\ \tilde{x}, & \text{if } x^l < \tilde{x} < x^u \\ x^u, & \text{if } \tilde{x} \ge x^u \end{cases},$$

where intermediate values are calculated as

$$\tilde{x} = x_i^{(k)} \frac{\max_l \left|\sigma_{il}^{(k)}\right|}{\bar{\sigma}_i}.$$

Using such algorithm, the design with continuous thicknesses is obtained (Fig. 6). This design is infeasible in some zones where thicknesses have upper bound 0.25. These zones can be treated as definitely needed in topology optimization result. They are shown in red color. Some part of structure has near-zero thickness values. The left part is where thicknesses are between minimum and maximum values. In this part, stress is equal about allowable value. After structural optimization and scaling structural thicknesses to satisfy the stress constraints, the obtained weight is reduced by about 58 % from initial design.

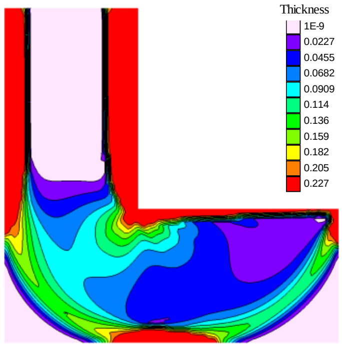

Fig. 6. Optimal Continuous Thickness Distribution after Modified FSD Algorithm

Now, the problem is to transform this continuous distribution of thicknesses by binary one. It can be done by using the third stage of global stress-based topology optimization.

The design with binary distribution of thickness is presented in Fig. 7. The required thickness in this structure to satisfy stress constraints is 0.91 that leads to the reduction of weight by 46 %.

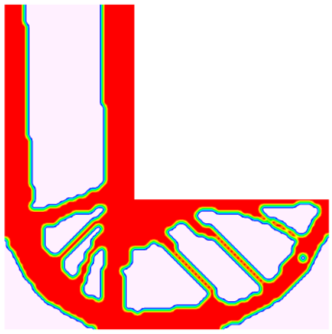

Fig. 7. Optimal Binary Thickness Distribution

To verify the developed algorithm the topology optimization was performed by means of Nastran program. The result of this optimization is presented in Fig. 8.

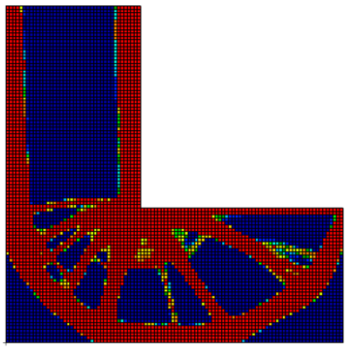

Fig. 8. Topology Optimization Result in Nastran

Notice that the design obtained by using the proposed method is stiffer and stronger. The difference in the maximum displacement is about 1.2 % and in the maximum stress is about 6.7 %.

5.1.3 Global-Local Topology Optimization

The sequential global and local optimizations also give possibility to reduce stress concentration. The final topology optimization result is shown in Fig. 9.

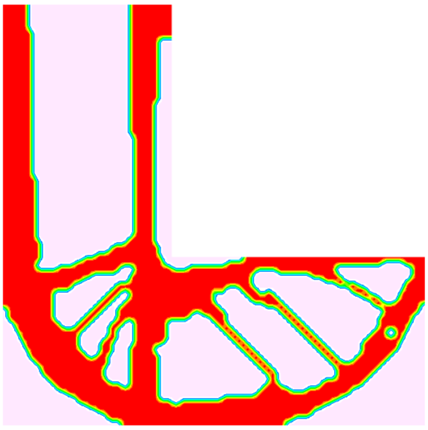

Fig. 9. Final Topology Optimization Result

The design that satisfies to stress constraints has thickness of 0.95 and volume is 2597. The reached additional weight reduction is about 4%. Totally application of global-local optimization leads to the weight decrease of 48 %.

5.2 Aircraft wing

Combining of topology optimization with sizing optimization as described in section 3 allows to get in unified cycle the innovative technical designs for aircraft components [9]. This approach is demonstrated on the example of a small-aspect-ratio wing of aircraft.

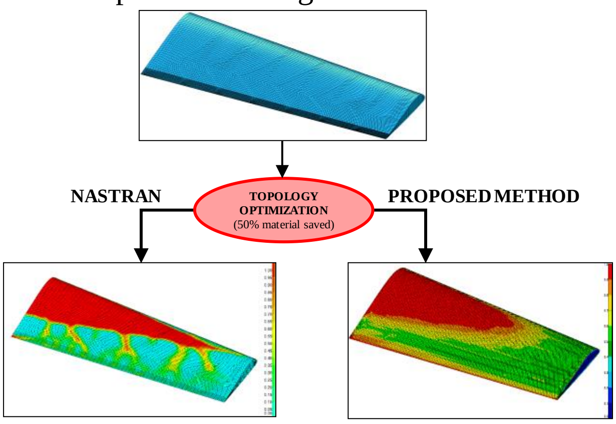

Fig. 10. Optimal Topology of Wing

Fig. 10 shows the initial 3D refined finite element model of wing and two topology optimization results at saving of 50 % structural material obtained by Nastran program and the proposed global FSD-like algorithm. Some difference between the solutions can be highlighted: there are no sloped red zones in weakly loaded wing part in the result of the proposed method. In Fig. 11 are some interpretations of topology optimization results by using these two topology patterns.

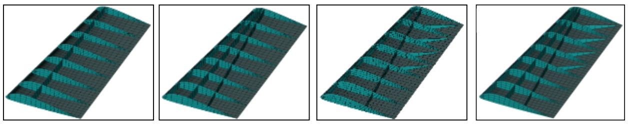

Fig. 11. Engineering Interpretation Structural Layouts

A set of the structural layouts contains one-, two- and three-spar wing-box of different width and some of them have sloped ribs in the wing-tip rear part. Sizing optimization for all these structural layouts has been performed with taking into consideration of stress, buckling and aeroelasticity requirements. Note that buckling constraints were dominant at determination of sizes of panels in root part and end part of the wing for structural layouts without the sloped ribs, aeroelastic constraints were dominant for panels in rear part and local aeroelastic instability was observed there due to small panel thickness.

In summary, the obtained optimal structure of wing turned out by 37.5 % lighter than structure designed by conventional approaches without using optimization methods. The best wing structural layout has three-spar wing-box with additional ribs in the end part, which is shown in Fig. 11 at right.

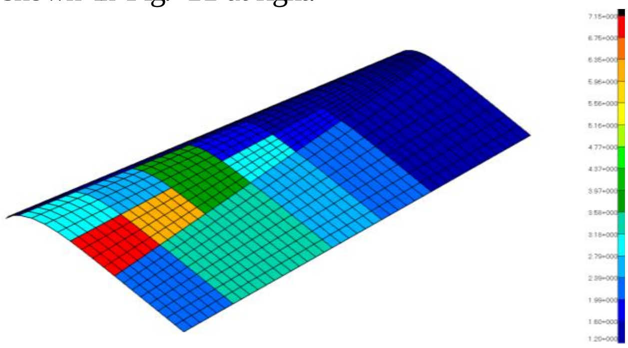

Fig. 12. Optimal Thickness Distribution

Fig. 12 illustrates the optimal structural thicknesses for upper skin of the wing. The maximum thicknesses are observed in the root panels and panels in the end part have gauge sizes.

6 Conclusions

The new approach to topology optimization with taking into consideration of stress constraints have been developed. The proposed efficient method is based on simple fully-stress design algorithm and algorithm for minimization of stress concentration. It was demonstrated on the example of L-bracket structure and numerical results show the opportunity to get near-optimum designs satisfied to stress constraints.

The method is implemented in general structural optimization procedure for searching of reasonable structural layouts of aircrafts structures. The presented example of aircraft wing showed that the mutual use of structural and topology optimization in the procedure allowed to obtain the essential profit in weight if compared with the traditional design of wing.

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