CASCADING FAILURE OF NODES AND THEIR LINKS IN A COMPLEX NETWORK

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Abstract

Cascading Failure has become one of the points of scientific research in complex networks. In this paper, we use the different definition of Load and Capacity in the network, to research the final changing direction of Cascade model with an attack. When the node Capacity is within a specific range, Risk Strength and Elasticity of the link both affected the result of the attack on ER network. Because of the elasticity of link, the degree of Cascading Failure does not increase in pace with attack. The elasticity of the link becomes essential to mitigate the failure under different risks.

1 General Introduction

Since Watts and Strogatz published an article to introduce the small world network model [1], and Barabási also pointed out the characteristics of scale-free (Scale-Free) network [2], scientists studied many topological networks [3-15]. They found that deliberate attacks on critical nodes in networks, especially in scale-free networks, will have a significant impact. In a network, nodes and links always carry a load and need to have an individual ability to handle. If the load exceeds, the failure will occur. One node's failure often causes the fell of the others with the crash of the whole, which called Cascading Failure in Network. Loads of failed nodes or links will reallocate to other connected nodes according to a specific policy. For the next node or link, if the increases load is also beyond the Capacity, they will also fail and cause a new round failure until no new. In the real, the early collapse or impact of individual nodes limits. After spreading with connections, it often leads to broader effects, even the fall of the entire. Therefore, the related research on Cascading Failure has become the leading way to reduce the disastrous crash in a network.

For infrastructures with higher interconnection, Cascading Failure is the leading cause of its disastrous fail. In the power transmission network, the failure of one local node will lead to a large-scale power outage [16]. Because of their importance for modern society, researchers try to build models to capture the typical characteristics of those Cascading Failures. One of the classic Cascade models composed of two ER-dependencies networks with average degree levels. [17] There's a deliberate attack or a random risk happened on node A5. Sergey simulated this Cascading Failure as shown below.

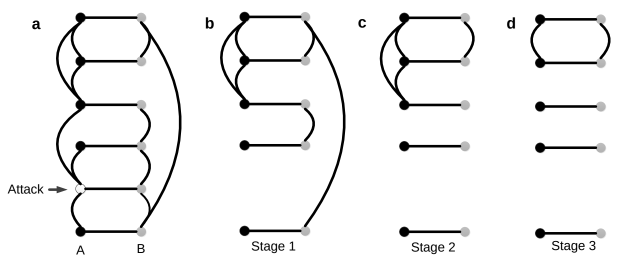

Fig. 1. A Cascading Failure [17]

In Figure 1, we assume there's attack on node A5 which triggered a Cascading Failure. However, the author of literature [17] does not distinguish the variation between links and nodes. All nodes or links have the same features no resilience considering in Fig 1. When risk struck, the node and its ties will fail in all, which is inconsistent with the reality. In real life, we can find the property differences of any of them. The changing in Fig 1 is one example of many cases for ER network. We can recognise the differences of nodes or links in the network which affect the result of the Cascading Failure. In fact, based on the prior research, there are three reasons affect the evolution direction in Cascade models. The original load and Capacity node or link, the removal policy with load's reallocation and the risk's intensity. Thus, we believe the failure of node A5 may not lead to its four links' failure immediately and simultaneously. Besides, the risk diffusion in one-way should exceed the anti-destruction to cause the failure. So, in Cascade model, the changing results are not all shown in Figure 1.

In this paper, we defined the load and Capacity of the network nodes and links, looked for the results of Cascading Failure in Cascade model when node A5 is at risk, and explored the reason. This analysis of the Cascading Failure based on node overload under a risk is critical to lessen the scale of Cascading Failure and improve the survive-Capacity of the network.

2 Defines Cascading Failure of nodes and their links

First, we will explain the idea of node removal. Researchers have taken this for the analysis of network failures [18,19]. It means the failed node will be deleted with all its links for their inability from the net. However, in a real network, the failure of a node does not necessarily cause the problem of all connections. Just like the shown in Fig 2. Decay or failure of any node or edge is not unavoidable which caused by the previous failure. The method of node importance judgement based on the change of network robustness data by this is only an extreme as well.

The congestion occurred on the node, or its link will both cause the node failure, followed by its lessened efficiency and a Cascading Failure. We can define the traditional network Cascading Failure as a node group's [20]. Besides, this Cascading Failure of Nodes and Their Links (CFNL) differs from the former. In our assumptions, any link of the network has the different likelihood of breakage, will produce various Cascading Failures and results. Conceivably, since the overall network failure is cascade [20], the CFNL also. Further, for a directed network, maybe one way of a link has interrupted, but the other still works. For example, on 17 September 2016, the typhoon "Merandi" brought heavy rain, resulted in a severe delay of over 300 departure flights from Shanghai Pudong International Airport. However, the impact on inbound flights was fewer with ten delayed before landing [21].

3 Conditions

3.1 Assumptions

To calculate the final, we use the Cascade model in Fig 1 to simulate the CFNL. Assume there are n nodes in the ER networks. Each node contains a certain number of random primitive links with load L. We numbered the relevant links, $L_n = L_{ij}$ .A further detailed analysis as follows:

- Nodes and links are different because of their load and resistance ability.
- Node failure has an impact on its connections, but not lead to inability.
- When a node fails, its load will reassign along the working path. However, if the load also exceeds the link's anti-destruction, the link will fail;
- One failing node will cause the link becomes unidirectional. Load from the fail will transfer to another side for the last time;
- If the nodes on both sides of a link invalid, the link will be useless;
- The node without a valid connection will isolate in the net.
- If no more faults occur, the CFNL will stop.

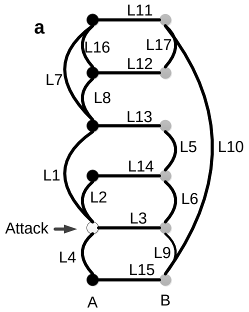

Fig. 2. Cascade model with an attack on node A5. Assuming A5 is under attack. The black nodes are the nodes of network A; the grey belongs to network B; the hollow is the failure.

3.2 Early load and Capacity

Plus, we need to define the first load of the node and link. According to some classic literature, the first load of a node always equals to its betweenness value. That is the total number of the shortest paths between any two points in the network through it. This approach can reflect the node's static Capacity index, but the calculation complicated. Wang proposed another, which is the product of node degree with its next's. [27] This method makes better use of the product which is positive correlate to betweenness of nodes. [28] It can reflect the early load distribution of the node with easy data gaining. Therefore, this paper applies the latter method to define the first load of the node:

$$L_i^0 = \left(k_i \sum_{j\in\Gamma_i} k_j\right)^{\alpha}, i = 1, 2, \cdots, N \qquad (1)$$

Here, L_i^0 is the original load of node v_i. k_i is the degree of node v_i. Γ_i is the set of neighbours of a node v_i. α is an adjustable parameter for controlling the early load strength, and N is the total number of nodes.

The original load for each link is:

$$L_{ij}^0 = L_n^0 = (k_i k_j)^{\beta}, i, j = 1, 2, \cdots, N \qquad (2)$$

β ($\beta > 0$) is a charge parameter. To adjust it, we can control the intensity of the early load, and simulate the different network load distribution in the network. k_i and k_j are the degrees of node i and j as the ends of the link L_{ij}. V is the total number of links.

To be more practicable, we assume the early load and Capacity of the nodes and links in the cascade model conform, $\alpha = 0.5$ * , The preliminary load of each node is,

L_{A1}^0 =5.4772 ; L_{A2}^0 =5.1962 ; L_{A3}^0 =6.9282 ; L_{A4}^0 =3.7417 ; L_{A5}^0 =6.6332 ; L_{A6}^0 =3.7417

L_{B1}^0 =4.8990 ; L_{B2}^0 =3.4641 ; L_{B3}^0 =3.7417 ; L_{B4}^0 =4.5826 ; L_{B5}^0 =5.4772 ; L_{B6}^0 =4.8990

$\beta = 0.5$, The early load of each link is,

L_1^0 =4; L_2^0 =2.8284 ; L_3^0 =3.4641 ; L_4^0 =2.8284 ; L_5^0 =2.4495 ; L_6^0 =3 ; L_7^0 =3.4641 ;

L_8^0 =3.4641 ; L_9^0 =3 ; L_{10}^0 =3 ; L_{11}^0 =3 , L_{12}^0 =2.4495 ; L_{13}^0 =2.8284 ; L_{14}^0 =2.4495 ;

L_{15}^0 =4.2426 ; L_{16}^0 =3 ; L_{17}^0 =2

"Capacity" is a fundamental issue in the research of Cascading Failures. It refers to the maximum load can handle. Higher Capacity, less likely to fail for node or link. The entire network will also have broader ability to withstand Cascading Failures. Therefore, Capacity equals to the survivability of node or link. When the load exceeds, the node or link will fail. If Capacity is enough, there's no Cascading Failure anymore.

About defines "Capacity", here are three styles. Kim and the rest [29] found the Capacity has no linearly relation with load after the study with multiple real networks. They also discovered there are some nodes with a few Capacity but a large remaining [29-31]. The second is the node's Capacity does not concern with the early. These studies define the Capacity satisfy some statistical distribution [32-34]. Among them, the literature [33] defined it as two cases, the ICA model, the Capacity increases by the network size, and ECA model with a constant Capacity. The third defines the Capacity is in proportion

*For the parameters in the CASCADE model, we take a fixed value and does not discuss it in this section. The literature [37] has proved the network has the strongest ability to resist cascade failures in this value.

to the first load of the node. Because of the cost, the Capacity of network nodes often associated with the first load which makes many pupils accepted the third. [35,36]. In this paper, we also set the node's Capacity is proportional to the early load.

$$C_i = (1+\theta)L_i^0, i = 1,2,\cdots,N \qquad (3)$$

Among, C_i is the Capacity of node v_i. $\theta > 0$, it is an adjustable parameter. L_i^0 is the first load of node v_i, and N is the total number of nodes.
Then, the Capacity of each link is

$$C_n = (1+\phi)L_n^0, n = 1,2,\cdots,V \qquad (4)$$

C_n is the Capacity of link n; ϕ is an adjustable parameter and $\phi > 0$; L_n^0 is the first load of the link n and V is the total number of links. In a real network, the value of ϕ is in a specific range, and the Capacity as well.
Let $\theta = 0.5$. Then, the Capacity of each node is,

C_{A1} = 8,2158 ; C_{A2} = 7,7942 ; C_{A3} = 10,3923 ;

C_{A4} = 5,6125 ; C_{A5} =9,9499 ; C_{A6} = 5,6125 ;

C_{B1} = 7,3485 ; C_{B2} = 5,1962 ; C_{B3} = 5,6125 ;

C_{B4} = 6,8739 ; C_{B5} = 8,2158 ; C_{B6} = 7,3485

We assume the link load comes from the node. When the node load exceeds its Capacity, the excess will transfer through the link. When the load increases until it cannot handle, the node will fail and cause the failure of others with interlocking faults in network.

3.3 Failure chance and load redistribution

When the node is fighting with the risk, its load increases but still fewer or equal to Capacity, $L_i(t) \le C_i$, the node will not fail. If $C_i < L_i(t)$, the load is greater than Capacity, the node fails. The likelihood of node failure is,

$$R(L_i(t)) = \begin{cases} 0, L_i(t) \le C_i \\ 1, C_i < L_i(t) \end{cases} \qquad (5)$$

At the early stage of the network, the load of each link and node is less than its Capacity, and the network is running normally. However, any attack or failure will lead to load reallocation happen. Many researchers assume the load will transfer direct to the neighbour in part to a certain percentage. However, in fact, these studies ignored the Capacity of link. The premise of load transfer is that no exceeding the Capacity of the link. Otherwise, the load cannot transfer. If the load increases infinitely, the result can only be failures of the node and all its links. There is no effective transfer, but the total load lessens. For a power network, it will be a break of this node from others.
If $L_n(t)$ represents the load of link n at time t. The likelihood of failure is $Y(L_n(t))$. C_n is the link Capacity. If $L_n(t) \le C_n$, the link does not have a risk of failure; when $C_n < L_n(t)$, the load is higher than Capacity, the link fails. It can express as,

$$Y(L_n(t)) = Y(L_{ji}(t)) = \begin{cases} 0, L_n(t) < C_n \\ 1, C_n \le L_n(t) \end{cases} \qquad (6)$$

$$L_n(t+1) = L_{ni}^0(t) + \Delta L_{ni}(t+1)$$

Assuming the load will transfer as soon as it exceeds. The load reallocation tactics based on the early load of nodes and links is,

Set $\delta_n = \dfrac{L_n^0}{\sum_{n\in\Gamma_i} L_n^0}$, then,

$$\Delta L_{ni} = \begin{cases} \delta_n \cdot (L_i - C_i), L_i(t) \le C_i \\ \delta_n \cdot L_i, \\ [\wedge(L_n(t) > C_n), n \in \Gamma_i, L_i(t) \le C_i] \\ \vee [C_i < L_i(t)] \end{cases} \qquad (7)$$

δ_n is the ratio of load distribution for node i. L_n^0 is the first load of link n; Γ_i is the set of close links of node i; ΔL_{ni} represents the reallocation from the overloaded node i to node n. L_i represents the load need divide. The value of δ_n is not fixable and will change with connections.
According to the first load of the four links, the load transfer ratio now is δ_1=0.3049; δ_2=0.2156; δ_3=0.2640; δ_4=0.2156.

The associated load of node i transfers to another neighbouring node j through each link. A load of node j becomes the early and the increase.

$$L_j(t+1)=L_j^0(t)+\Delta L_{ij}(t+1),\Delta L_{ij}=\Delta L_{ni} \quad (8)$$

4 CFNL in Cascade Model

An attack occurred on node A5 in the model. The load increases and beyond Capacity, cause a failure. The load needs to transfer and begins from:

- If the risk is small, the increased load has not exceeded the Capacity of node A5. No load needs a transfer. Node A5 is still running, and no any failure happens.
- If the load is within a specific range, to ensure node A5 is still reliable, the excess load will transfer. The added on each link should be in the range of the Capacity for successful moving. Then, we can get,

$$\begin{aligned}&(\delta_n\cdot(L_{A5}-C_{A5})+L_n^0\le C_n,n\in\Gamma_i)\\&\wedge(L_{A5}\le C_{A5})\\&(L_{A5}\le\phi\cdot\sum_{n\in\Gamma_i}L_n+C_{A5}\\&=13.1209\phi+9.9499)\\&\wedge(L_{A5}\le 9.9499),\\&\because\phi>0\\&\therefore L_{A5}\le 9.9499\end{aligned} \quad (9)$$

As long as the load of node A5 has not exceeded, they can transfer to other nodes. To keep all connected nodes are valid, it needs,

$$L_1\to A3: L_{A3}^0+\delta_1(L_{A5}-C_{A5})\le C_{A3}\Rightarrow L_{A5}\le 21.3113$$
$$L_2\to A4: L_{A4}^0+\delta_2(L_{A5}-C_{A5})\le C_{A4}\Rightarrow L_{A5}\le 18.6271$$
$$L_3\to B5: L_{B5}^0+\delta_3(L_{A5}-C_{A5})\le C_{B5}\Rightarrow L_{A5}\le 20.3233$$
$$L_4\to A6: L_{A6}^0+\delta_4(L_{A5}-C_{A5})\le C_{A6}\Rightarrow L_{A5}\le 18.6271$$

For $L_{A5}\le 9.9499$, it meets all controls. Nodes can afford the added. We can imply that when the risk is small, node A5 is still working without any failure happen.

- Node A5 will fail if the load exceeds. The transfer begins under one-way without exceeding the links,

$$\begin{aligned}&(\delta_n\cdot L_i+L_n^0\le C_n,n\in\Gamma_i)\wedge(L_i>C_i)\\&\Rightarrow C_i<L_i\le\phi\cdot\sum_{n\in\Gamma_i}L_n^0\end{aligned} \quad (10)$$

$i=A5$,

$$C_{A5}<L_{A5}\le\phi\cdot\sum_{n\in\Gamma_i}L_n^0$$

After the beginning failure, the premise of load transfer is that it is less than or equal to the result of ϕ times early load for all links. ϕ is an adjustable parameter as the Link's Capacity Elasticity (LCE). When the early Capacity of the link is a fixed value, the LCE controls whether the connected load can transfer. Now, we can also infer load transfer has relation with the sum instead of each of the link, assuming LCE of all links are same. Further, there are,

$$9.9499<L_{A5}\le\phi\cdot\sum_{n\in\Gamma_i}L_n^0=13.1209\phi,\ \phi\ge 0.7583$$

We assume that when the attack occurred, $\phi\ge 0.7583$, and all loads can transfer. Because of the failure of node A5, all links also broke. The network will change to graph d1.

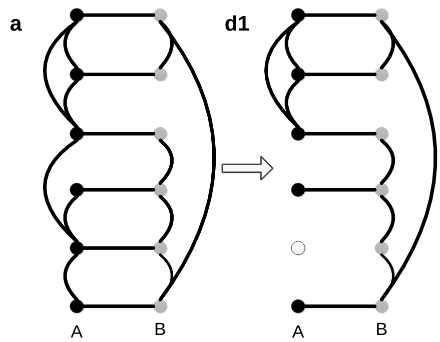

Fig 3 From a to d1

Ensure another node of links will not fail, it needs,

$$\begin{aligned}&(L_n\to j: L_j^0+\delta_n\cdot L_i\le C_j\\&\Rightarrow L_i\le\frac{\theta\cdot L_j^0}{\delta_n},n_{ij}\in\Gamma_i)\wedge(L_i>C_i)\\&(1+\theta)L_i^0<L_i\le\frac{\theta\cdot L_j^0}{\delta_n}\end{aligned} \quad (11)$$

For the fixed value of δ_n and L_j^0, we can get,

$$L_i \le C \cdot \theta \qquad (12)$$

C is a constant. θ is the Node Capacity Elasticity (NCE).Whether the transfer will cause next failure, it depends on the load and NCE. Also,

$$0 < \theta \le \delta_n \cdot \frac{L_i^0}{L_j^0} \Big/ (1 - \delta_n \cdot \frac{L_i^0}{L_j^0})$$
$$\Rightarrow \frac{L_i^0}{L_j^0} < \frac{1}{\delta_n} = \frac{\sum_{n\in\Gamma_i} L_n^0}{L_n^0} \qquad (13)$$

If this meets, the Cascading Failure will stop.

4.1 First way of CFNL

To avoid the next failure because of the load from node A5, it needs,

$$L_1 \to A3: L_{A3}^0 + \delta_1 \cdot L_{A5} \le C_{A3} \Rightarrow L_{A5} \le 11.3614$$
$$L_2 \to A4: L_{A4}^0 + \delta_2 \cdot L_{A5} \le C_{A4} \Rightarrow L_{A5} \le 8.6772$$
$$L_3 \to B5: L_{B5}^0 + \delta_3 \cdot L_{A5} \le C_{B5} \Rightarrow L_{A5} \le 10.3735$$
$$L_4 \to A6: L_{A6}^0 + \delta_4 \cdot L_{A5} \le C_{A6} \Rightarrow L_{A5} \le 8.6772$$

Also, because of the need to meet $L_{A5} > C_{A5} = 9.9499$, when $9.9499 < L_{A5} \le 10.3735$, nodes A4, A6 must be invalid and become isolated nodes; L14, L15 will have the last transfer; the network changes to d2.

The load exceeds the Capacity of nodes A4, A6 and need a transfer. Now, A4, A6 can only transfer the load to B4, B6 no beyond the link's Capacity. There is,

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)$$
$$\Rightarrow C_i < L_i \le \phi \cdot \sum_{n\in\Gamma_i} L_n^0 \qquad (14)$$

$i = A4, A6$, $\delta_{14} = 1$, $\delta_{15} = 1$

$$5.6125 < L_{A4} \le 2.4495\phi;$$
$$5.6125 < L_{A6} \le 4.2426\phi$$
$$\phi \ge 2.2913$$

Assuming the load transferred and node B4, B6 are valid, thus,

$$L_{14} \to B4: L_{B4}^0 + \delta_{14} \cdot L_{A4} \le C_{B4} \Rightarrow L_{A4} \le 2.2913$$
$$L_{15} \to B6: L_{B6}^0 + \delta_{15} \cdot L_{A6} \le C_{B6} \Rightarrow L_{A6} \le 2.4495$$

Nodes B4, B6 will fail; link L14, L16 will erase for the failure; the network changes into d3. Link L5 and L6 will transfer the load of B4. Link L9, L10 will transfer the load of B6.

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)$$
$$\Rightarrow C_i < L_i \le \phi \cdot \sum_{n\in\Gamma_i} L_n^0 \qquad (15)$$

$i = B4, B6$,

$$6.8739 < L_{B4} \le 5.4495\phi;$$
$$7.3485 < L_{B6} < 6\phi,$$
$$\phi \ge 1.2248$$

Since $\phi \ge 2.2913$, the load of nodes B4 and B6 could transfer. $\delta_5 = 0.4495$, $\delta_6 = 0.5505$, $\delta_9 = \delta_{10} = 0.5$, then,

$$L_5 \to B3: L_{B3}^0 + \delta_5 \cdot L_{B4} \le C_{B3} \Rightarrow L_{B4} \le 4.162$$
$$L_{10} \to B1: L_{B1}^0 + \delta_{10} \cdot L_{B6} \le C_{B1} \Rightarrow L_{B6} \le 4.899$$

To node B5, plus the former transfer load from A5 (δ_3 = 0.2640):

$$L_3 \to B5, L_6 \to B5, L_9 \to B5:$$
$$L_{B5}^0 + \delta_3 \cdot L_{A5} + \delta_6 \cdot L_{B4} + \delta_9 \cdot L_{B6} \le C_{B5}$$
$$\Rightarrow 0.2640 L_{A5} + 0.5505 L_{B4} + 0.5 L_{B6} \le 2.737$$

Nodes B3, B5, B1 all fail. L5, L6, L9, L10 need to remove. The network changes into d4. For lack of link, the load of B5 cannot transfer. L13 will transfer the load of the B3 load. L11 and L17 will transfer the load of B1.

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)$$
$$\Rightarrow C_i < L_i \le \phi \cdot \sum_{n\in\Gamma_i} L_n^0 \qquad (16)$$

$i = B3, B1$

$$5.6125 < L_{B3} < 2.8284\phi;$$
$$7.3485 < L_{B1} < 6.4641\phi$$
$$\phi \ge 1.9843$$

Since $\phi \ge 2.2913$, the load of nodes B3, B1 could transfer. $\delta_{13} = 1$, $\delta_{11} = 0.4641$, $\delta_{17} = 0.5359$, then,

$$L_{11} \to A1,: L_{A1}^0 + \delta_{11} \cdot L_{B1} \le C_{A1} \Rightarrow L_{B1} \le 5.9009$$
$$L_{17} \to B2: L_{B2}^0 + \delta_{17} \cdot L_{B1} \le C_{B2} \Rightarrow L_{B1} \le 3.2321$$

To node A3, add the former transfer load from A5 (δ_1=0.3049):

$$L_{13} \to A3, L_1 \to A3: L_{A3}^0 + \delta_1 \cdot L_{A5} + \delta_{13} \cdot L_{B3} \le C_{A3}$$
$$\Rightarrow 0.3049 L_{A5} + L_{B3} \le 3.4641$$

On it, the nodes A3, A1, B2 all fail. L13, L11, L17, L7 will remove. The network changes into d5. The last node is A2. Loads of nodes A3, A1, B2 will transfer to node A2 by links L8, L16, L12. Then,

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i) \Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0 \quad (17)$$

$i = A3, A1, B2$

$$10.3923 < L_{A3} \le 3.4641\phi;$$
$$8.2125 < L_{A1} \le 3\phi;$$
$$5.1962 < L_{B2} \le 2.4495\phi$$
$$\phi \ge 3$$

If we want node A2 still working, it needs,

$$L_{16} \to A2, L_{12} \to A2, L_8 \to A2:$$
$$L_{A2}^0 + \delta_{16} \cdot L_{A1} + \delta_{12} \cdot L_{B2} + \delta_8 \cdot L_{A3} \le C_{A2}$$
$$\delta_{16} = \delta_{12} = \delta_8 = 1$$
$$5.1962 + L_{A1} + L_{B2} + L_{A3} \le 7.7942$$

It shows that node A2 will fail. The network devises into d6 and collapses.

The network fails from node A5, carries on load transfer, causes the failure of A4, A6 and further plan. As long as the value of LCE meets $\phi \ge 3$, the network will collapse no stop.

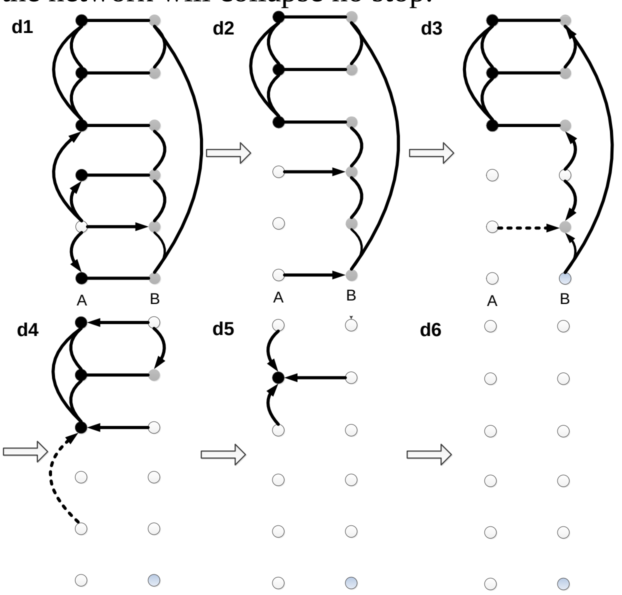

Fig 4 First way from d1 to d6

4.2 Second way

If $10.3735 < L_{A5} \le 11.3614$, nodes A4, A6, B5 will fail and links L13, L15, L6, L9 as well. The network changes into d7. A4, A6 can transfer the load to B4, B6. The load of B5 will also transfer to B4, B6. For success, it needs the Capacity of the link could cover. So,

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i) \Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0 \quad (18)$$

$i = A4, A6, B5$,

$$5.6125 < L_{A4} \le 2.4495\phi \; ;$$
$$5.6125 < L_{A6} \le 4.2426\phi \; ; 8.2158 < L_{B5} \le 6\phi$$
$$\phi \ge 2.2913$$

Assuming the load transferred and the other nodes B4 and B6 are still useful, it needs,

$$L_{14} \to B4, L_6 \to B4: L_{B4}^0 + \delta_{14} \cdot L_{A4} + \delta_6 \cdot L_{B5} \le C_{B4}$$
$$\Rightarrow L_{A4} + 0.5L_{B5} \le 2.2913$$
$$L_{15} \to B6, L_9 \to B6: L_{B6}^0 + \delta_{15} \cdot L_{A6} + \delta_9 \cdot L_{B5} \le C_{B6}$$
$$\Rightarrow L_{A6} + 0.5L_{B5} \le 2.4495$$
$$\delta_{14} = 1 \;,\; \delta_{15} = 1 \;,\; \delta_6 = 0.5 \;,\; \delta_9 = 0.5$$

Nodes B4, B6 will fail and cause removing of link L14, L15, L6, L9. The network changes to d8. A load of B4 will transfer by link L5. A load of B6 will transfer by link L10.

$$(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i) \Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0 \quad (19)$$

$i = B4, B6$,

$$6.8739 < L_{B4} \le 2.4495\phi \; ; 7.3485 < L_{B6} < 3\phi$$
$$\phi \ge 2.8062$$

Also presuming the load of nodes B4, B6 can transfer, $\delta_5 = 1, \delta_{10} = 1$, then,

$$L_5 \to B3: L_{B3}^0 + \delta_5 \cdot L_{B4} \le C_{B3} \Rightarrow L_{B4} \le 1.8708$$
$$L_{10} \to B1: L_{B1}^0 + \delta_{10} \cdot L_{B6} \le C_{B1} \Rightarrow L_{B6} \le 2.4495$$

Nodes B3, B1 are all failed. L5, L10 will remove. The network also changes into d4, d5 and d6.

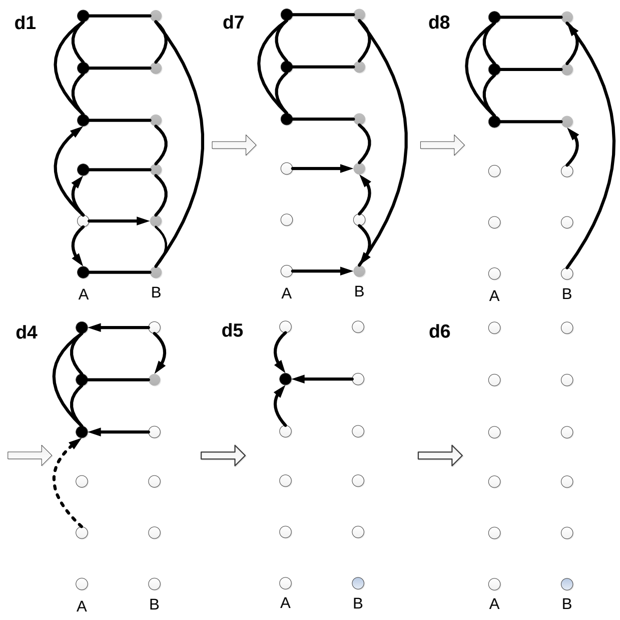

Fig 5 Second way from d1 to d6

4.3 Third one

If $11.3614 < L_{A5}$, node A3, A4, A6, B5 will fail, and the load begin to transfer, there are,

$$\begin{aligned}&(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)\\ &\Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0\end{aligned} \quad (20)$$

$i = A3, A4, A6, B5$,

$$10.3923 < L_{A3} \le 9.7566\phi \;;$$

$$5.6125 < L_{A4} \le 2.4495\phi \;;$$

$$5.6125 < L_{A6} \le 4.2426\phi \;; 8.2158 < L_{B5} \le 6\phi$$

$$\phi \ge 2.2913$$

Assuming the load transferred and the other node still reliable, $\delta_7 = \delta_8 = 0.3551$, $\delta_{13} = 0.2899$, $\delta_{14} = 1$, $\delta_{15} = 1$, $\delta_6 = 0.5$, $\delta_9 = 0.5$, then,

$$L_7 \to A1: L_{A1}^0 + \delta_7 \cdot L_{A3} \le C_{A1} \Rightarrow L_{A3} \le 7.7122$$

$$L_8 \to A2: L_{A2}^0 + \delta_8 \cdot L_{A3} \le C_{A2} \Rightarrow L_{A3} \le 7.3162$$

$$L_{13} \to B3: L_{B3}^0 + \delta_{13} \cdot L_{A3} \le C_{B3} \Rightarrow L_{A3} \le 6.4533$$

$$L_{14} \to B4, L_6 \to B4: L_{B4}^0 + \delta_{14} \cdot L_{A4} + \delta_6 \cdot L_{B5} \le C_{B4}$$

$$\Rightarrow L_{A4} + 0.5L_{B5} \le 2.2913$$

$$L_{15} \to B6, L_9 \to B6: L_{B6}^0 + \delta_{15} \cdot L_{A6} + \delta_9 \cdot L_{B5} \le C_{B6}$$

$$\Rightarrow L_{A6} + 0.5L_{B5} \le 2.4495$$

The nodes A1, A2, B3, B4, B6, must cancel, the links L5, L6, L7, L8, L9, L13, L14, L15, L6 removed, and the network devises to d10. The load of A1 and A2 will transfer through link L11, L12 to B1, B2. Load of B6 will transfer to B1. No active link, the load of B3 and B4 will stop to transfer.

$$\begin{aligned}&(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)\\ &\Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0\end{aligned} \quad (21)$$

$i = A1, A2, B6$

$$8.2158 < L_{A1} \le 3\phi \,; 7.7942 < L_{A2} \le 5.1962\phi \,;$$

$$7.3485 < L_{B6} < 4.899\phi$$

$$\phi \ge 2.7386$$

Assume the load transferred. $\delta_{11}, \delta_{12}, \delta_{10} = 1$, then,

$$L_{12} \to B2: L_{B2}^0 + \delta_{12} \cdot L_{A2} \le C_{B2} \Rightarrow L_{A2} \le 1.7321$$

$$L_{11} \to B1, L_{10} \to B1: L_{B1}^0 + \delta_{11} \cdot L_{A1} + \delta_{10} \cdot L_{B6} \le C_{B1}$$

$$\Rightarrow L_{A1} + L_{B6} \le 2.4495$$

The nodes A1, A2, B3, B4, B6, must cancel, the links L5, L6, L7, L8, L9, L13, L14, L15, L6 removed, and the network devises to d10. The load of A1 and A2 will transfer through link L11, L12 to B1, B2. Load of B6 will transfer to B1. No active link, the load of B3 and B4 will stop to transfer.

$$\begin{aligned}&(\delta_n \cdot L_i + L_n^0 \le C_n, n \in \Gamma_i) \wedge (L_i > C_i)\\ &\Rightarrow C_i < L_i \le \phi \cdot \sum_{n \in \Gamma_i} L_n^0\end{aligned} \quad (22)$$

$i = A1, A2, B6$,

$8.2158 < L_{A1} \le 3\phi$; $7.7942 < L_{A2} \le 5.1962\phi$;

$7.3485 < L_{B6} < 4.899\phi$; $\phi \ge 2.7386$

Assume the load transferred. $\delta_{11}, \delta_{12}, \delta_{10} = 1$, then,

$$L_{12} \to B2: L_{B2}^0 + \delta_{12} \cdot L_{A2} \le C_{B2} \Rightarrow L_{A2} \le 1.7321$$

$$L_{11} \to B1, L_{10} \to B1: L_{B1}^0 + \delta_{11} \cdot L_{A1} + \delta_{10} \cdot L_{B6} \le C_{B1}$$

$$\Rightarrow L_{A1} + L_{B6} \le 2.4495$$

The node B2, B1 and all links failed. The network changes into d6 with the crash.

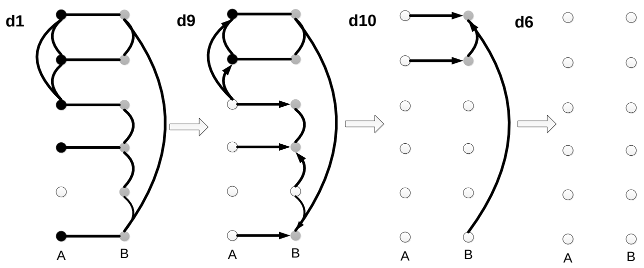

Fig 6 Third way from d1 to d6

5 Conclusion

In this paper, we analyzed the different directions of the evolution of the network. Under the constraints of capacity and resiliency, the attacked network will change in different directions, resulting in different cascading failures. Many research tells us one failure may cause lots unequal finals. In future, we will find more reality to test.

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References

[1] Watts, D.J. and Strogatz, S.H., Collective dynamics of 'small-world' networks. *nature*, Vol.393, No. 6684, pp.440-442. 1998.

[2] Barabási, A.L. and Albert, R., Emergence of scaling in random networks. *science*, Vol.286, No.5439, pp.509-512. 1999.

[3] Newman, M.E., Strogatz, S.H. and Watts, D.J., Random graphs with arbitrary degree distributions and their applications. *Physical review E*, Vol. 64, No. 2, p.026118. 2001.

[4] Barabási, A.L., Jeong, H., Néda, Z., Ravasz, E., Schubert, A. and Vicsek, T., Evolution of the social network of scientific collaborations. *Physica A: Statistical mechanics and its applications*, Vol. 311, No. 3, pp.590-614. 2002.

[5] Girvan, M. and Newman, M.E., Community structure in social and biological networks. *Proceedings of the national academy of sciences*, Vol. 99, No. 12, pp.7821-7826. 2002.

[6] Davis, G.F., Yoo, M. and Baker, W.E., The small world of the American corporate elite, 1982-2001. *Strategic organization*, Vol. 1, No. 3, pp.301-326. 2003.

[7] Ohtsuki, H., Hauert, C., Lieberman, E. and Nowak, M.A., A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, Vol. 441, No. 7092, pp.502-505. 2006.

[8] Albert, R., Jeong, H. and Barabási, A.L., Internet: Diameter of the world-wide web. *nature*, Vol. 401, No. 6749, pp.130-131. 1999.

[9] Barabási, A.L., Albert, R. and Jeong, H., Scale-free characteristics of random networks: the topology of the world-wide web. *Physica A: statistical mechanics and its applications*, Vol. 281, No. 1, pp.69-77. 2000.

[10] Adamic, L.A. and Huberman, B.A., Power-law distribution of the world wide web. *science*, Vol. 287, No. 5461, pp.2115-2115. 2000.

[11] Jaffe, A.B. and Trajtenberg, M., *Patents, citations, and innovations: A window on the knowledge economy*. MIT press. 2002.

[12] Latora, V. and Marchiori, M., Is the Boston subway a small-world network?. *Physica A: Statistical Mechanics and its Applications*, Vol. 314, No. 1, pp.109-113. 2002.

[13] Guimera, R., Mossa, S., Turtschi, A. and Amaral, L.N., The worldwide air transportation network: Anomalous centrality, community structure, and cities' global roles. *Proceedings of the National Academy of Sciences*, Vol. 102, No. 22, pp.7794-7799. 2005.

[14] Jeong, H., Tombor, B., Albert, R., Oltvai, Z.N. and Barabási, A.L., The large-scale organization of metabolic networks. *Nature*, Vol. 407, No. 6804, pp.651-654. 2000.

[15] Jeong, H., Mason, S.P., Barabási, A.L. and Oltvai, Z.N., Lethality and centrality in protein networks. *Nature*, Vol. 411, No. 6833, pp.41-42. 2001.

[16] Rosato, V., Issacharoff, L., Tiriticco, F., Meloni, S., Porcellinis, S. and Setola, R., Modelling interdependent infrastructures using interacting dynamical models. *International Journal of Critical Infrastructures*, Vol. 4, No. 1-2, pp.63-79. 2008.

[17] Buldyrev, S.V., Parshani, R., Paul, G., Stanley, H.E. and Havlin, S., Catastrophic cascade of failures in interdependent networks. *Nature*, Vol. 464, No. 7291, pp.1025-1028. 2010.

[18] Albert, R., Jeong, H. and Barabási, A.L., Error and attack tolerance of complex networks. *nature*, Vol. 406, No. 6794, pp.378-382. 2000.

[19] Bo, C., Centrality of China aviation network and node attack comparison [J]. *Journal of Systems Engineering*, Vol. 1, No. 1, p.003. 2013.

[20] Buldyrev, S.V., Parshani, R., Paul, G., Stanley, H.E. and Havlin, S., Catastrophic cascade of failures in interdependent networks. *Nature*, Vol. 464, No. 7291, pp.1025-1028. 2010.

[21] http://www.sh.chinanews.com/shxw/2016-09-17/11045.shtml

[22] Motter, A.E. and Lai, Y.C., Cascade-based attacks on complex networks. *Physical Review E*, *66*(6), p.065102. 2002.

[23] Sun, H.J., Zhao, H. and Wu, J.J., A robust matching model of capacity to defense cascading failure on complex networks. *Physica A: Statistical Mechanics and its Applications*, Vol. 387, No. 25, pp.6431-6435. 2008.

[24] Li, P., Wang, B.H., Sun, H., Gao, P. and Zhou, T., A limited resource model of fault-tolerant capability against cascading failure of complex network. *The European Physical Journal B-Condensed Matter and Complex Systems*, Vol. 62, No. 1, pp.101-104. 2008.

[25] Ren, J., Shen, M., Mong, R. and Gao, H., Study of reducing size of cascading failures in scale-free network. *Jisuanji Gongcheng yu Yingyong(Computer Engineering and Applications)*, Vol. 47, No. 33, 2011.

[26] DUAN, D.L., WU, J., DENG, H.Z., SHA, F., WU, X.Y. and TAN, Y.J., Cascading failure model of complex networks based on tunable load redistribution. *Systems Engineering-Theory & Practice*, Vol. 1, No. 1, p.026. 2013.

[27] Wang, J.W. and Rong, L.L., A model for cascading failures in scale-free networks with a breakdown probability. *Physica A: Statistical Mechanics and its Applications*, Vol. 388, No. 7, pp.1289-1298. 2009.

[28] Wang, J.W., Rong, L.L. and Wang, D., Model for cascading failures on complex networks based on local characteristics of node. *Journal of Management Sciences in China*, Vol. 13, No. 8, pp.42-80. 2010.

[29] Kim, D.H. and Motter, A.E., Resource allocation pattern in infrastructure networks. *Journal of physics A: mathematical and theoretical*, Vol. 41, No. 22, p.224019. 2008.

[30] WANG, J.S., WU, X.P. and CHEN, Y.Q., Invulnerability of Weighted Scale-Free Networks Against Cascading Failure [J]. *Complex Systems and Complexity Science*, Vol. 1, No. 2, p.001. 2013.

[31] CHEN, S.M., PANG, S.P., ZOU, X.Q., FANG, H.J. and CHEN, Y., Robustness optimization of load-capacity nonlinear model in weighted network for cascading failure [J]. *Control and Decision*, Vol. 1, No. *7*, p.016. 2013.

[32] Moreno, Y., Gómez, J.B. and Pacheco, A.F., Instability of scale-free networks under node-breaking avalanches. *EPL (Europhysics Letters)*, Vol. 58, No. 4, p.630. 2002.

[33] Holme, P., Kim, B.J., Yoon, C.N. and Han, S.K., Attack vulnerability of complex networks. *Physical review E*, Vol. 65, No. 5, p.056109. 2002.

[34] Holme, P., Edge overload breakdown in evolving networks. *Physical Review E*, Vol. 66, No. 3, p.036119. 2002.

[35] Motter, A.E. and Lai, Y.C., Cascade-based attacks on complex networks. *Physical Review E*, Vol. 66, No. 6, p.065102. 2002.

[36] Duan, D.L. and Wu, X.Y., Cascading failure of scale-free networks based on a tunable load redistribution model. 2014.

[37] Wang, J.W. and Rong, L.L., A model for cascading failures in scale-free networks with a breakdown probability. *Physica A: Statistical Mechanics and its Applications*, Vol. 388, No. 7, pp.1289-1298. 2009.

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