

GLOBAL-LOCAL STRUCTURAL ANALYSIS OF COMPOSITE WINGS

E. Carrera* , A. Pagani* , G. HC Silva**

*Politecnico di Torino , **Embraer S.A.

Keywords: *Global/local analysis , Composites , Advanced structural theories , Finite element method
Free-edge effects*

Abstract

The design and certification process of aerospace structures require advanced tools able to conduct detailed stress and strain analysis. Nevertheless, the complexity of modern aircraft structures and the use of composite materials can severely increase the computational costs of finite element models. This work proposes a global/local method that consists on a two-step algorithm (in a "weak" sense) for the description of accurate stress fields in critical (local) regions of complex aerospace composite structures.

1 Introduction

As well known in the engineering practise, the numerical models of aircraft structures are built by combining one-dimensional (1D) and two-dimensional (2D) finite elements, which opportunely discretize mathematical domains of stringers, panels, ribs, and other components. Clearly, this discretization entails a rather simplification of the reality. In fact, structures may contain regions where three-dimensional (3D) stress fields occur. To accurately capture these localized 3D stress fields, solid models or higher-order theories are necessary. However, in order to make the model more efficient, a global/local approach can be eventually employed.

Three main approaches are available in the literature to deal with a global/local analysis: (1) refining the mesh or the shape functions in correspondence with the critical domain [1, 2,

3, 4]; (2) formulating multiple-model methods, in which different subregions of the structure are analysed with different mathematical models [5, 6, 7, 8]; (3) employing methods based on the static condensation also known to as "super-elements methods" [9]. The present paper deals with multiple-model methods. In this case, different subregions of the structure are modelled with kinematically incompatible elements, the compatibility of displacements and equilibrium of stresses at the interface between dissimilar elements have to be achieved. In the *s*-version of the finite element method (FEM) [5, 10], for example, the resolution in a certain subregion of the structure is increased by superimposing additional meshes of higher-order hierarchical elements. In [6], Shim et al. combined 1D and 2D finite elements with 3D solid elements via multipoint constraint equations evaluated by equating the work done on either side of the dimensional interface. In [7], the coupling of structural models with different dimensionality was achieved by exploiting conditions derived from the governing variational principle formulated at the continuous level. Ben Dhia [11] proposed the Arlequin method to couple different numerical models. This method was adopted by Hu et al. [12] for the linear analysis of sandwich beams modelled via 1D and 2D finite elements.

Among the multiple-model methods there are the so called "multi-steps methods", in which the solution within the critical region depends on the analysis of the global structure only for the boundary conditions. For instance, in the

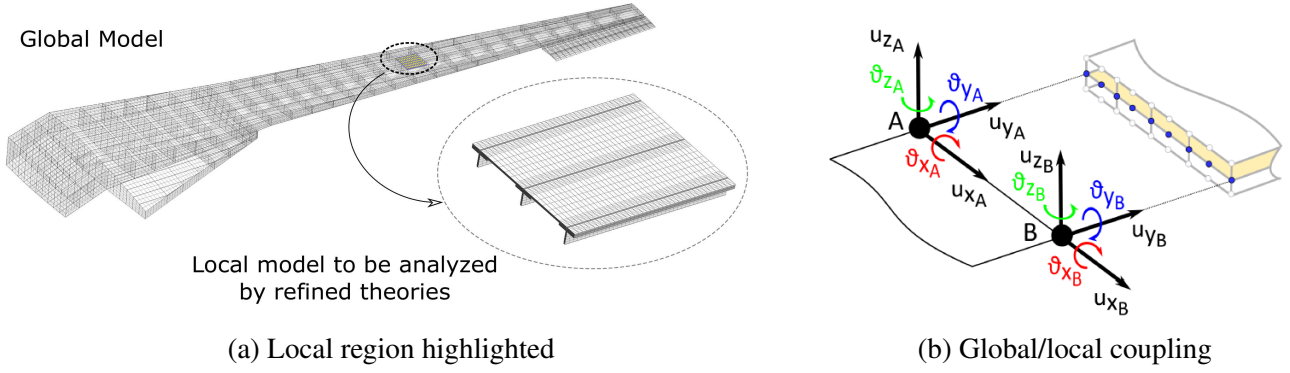


Fig. 1 : Schematic of proposed global/local analysis methodology.

global/local method proposed by Mao et al. [13], a coarse mesh was used to analyse the entire structure to obtain the nodal displacements which were subsequently used as displacement boundary conditions for the refined regions of interest. According to Mao and his co-workers, the application of the boundary conditions in the local analysis introduces errors unavoidably. To minimize the effect of such errors, the local analysis generally requires a region which is larger than the region where stresses are of concern. As a further example, Ransom and Knight [14] presented an innovative method for performing a global/local stress analysis. This method employs spline interpolation functions which satisfy the linear plate bending equation to determine displacements and rotations from a global model, which are used as boundary conditions for the local model. The local analysis is done in a second step and it is completely independent of the global one.

The aforementioned works make use of mathematical artifices to couple global and detailed local models and to force the energy consistency at the interface of kinematically incompatible structural models. On the other hand, the present research proposes a variable kinematic formalism along with a global/local methodology that consists of a two-step algorithm for the evaluation of accurate stress fields of aerospace composite structures. In the proposed method, the first step is devoted to the static analysis of a global model of the structure and it can be done by commercial

software tools (e.g. *MSc-Nastran*, *Abaqus CAE*) using 1D/2D finite elements. After the critical region of interest is identified, the second analysis step is performed by using higher-order models, to obtain accurate stress fields and singularities through the thickness of composite laminates. The utilized refined theories in the detailed analysis are implemented in the domain of the Carrera Unified Formulation (CUF) [15, 16]. In detail, CUF is employed to generate theories of structures which make use of generic expansion of the generalized displacements. In essence, the governing equations are written in terms of *fundamental nuclei*, which are invariant of the approximation order. As a consequence, the solution accuracy can be tuned opportunely to ensure a perfect balance between kinematic consistency with global model and enhanced efficiency at local level. In fact, equivalent-single-layer (ESL) [17] to layer-wise (LW) [18] and component-wise (CW) [19] models can be developed in the domain of CUF. In this work, mainly LW models are employed for capturing 3D stress distributions at the ply level.

2 The proposed global/local approach

Generally, FEM modelling of composite aircraft structures comports a simplification of the reality. Although these models are affected by geometrical inconsistency, because the employed 1D and 2D elements may have incompatible kinematics and the use of fictitious links is usually required, they can provide reliable solutions and

accurate results at global scale. In other words, these simplified models which are the rule in the engineering practise, can give a good estimation of the global structural behaviour in terms of displacement mechanisms. Nevertheless, if accurate stress distributions are needed, for example close to open holes and free edges, detailed analyses which usually employ 3D finite elements are necessary.

In the present work, the detailed (local) analysis is performed by using 1D refined CUF models. According to CUF, a layer-wise (LW) displacement field of the composite beam is written as:

$$\mathbf{u}(x, y, z) = F_\tau(x, z) \mathbf{u}_\tau^k(y) \quad \tau = 1, 2, \dots, M \quad (1)$$

where y is the longitudinal direction of the refined 1D model; (x, z) are the cross-section coordinate; $\mathbf{u}(x, y, z)$ is the three-dimensional displacement field; $\mathbf{u}_\tau^k(y)$ is the vector of generalized displacements for the k -th layer; and $F_\tau(x, z)$ are the expansion functions of the cross-sectional domain. The class and number of expansion functions is arbitrary, being M the maximum number of expansions, which is a user defined parameter. Repeating indexes denote summation. LW models can be implemented by using Lagrange Expansions (LE). This beam theory, introduced by Carrera and Petrolo [18], is based on the use of interpolating Lagrange polynomials as expansion functions F_τ of the cross-sectional coordinates. In this manner, the cross-section of the composite beam can be discretized with an arbitrary number of Lagrangian domains, which are used to represent the surfaces of each layer.

If compared to 3D modelling, CUF has been demonstrated to provide extremely accurate solutions for composite with at least one order of magnitude less of degrees of freedom, see [20]. Moreover, thanks to its intrinsic variable kinematics characteristics, CUF is the ideal tool for global/local analysis as information from global to local model can be transferred with ease and with no accuracy loss. For representative purpose, Fig. 1 shows a typical global/local analysis of a representative aircraft wing. In the present context, given the solution from

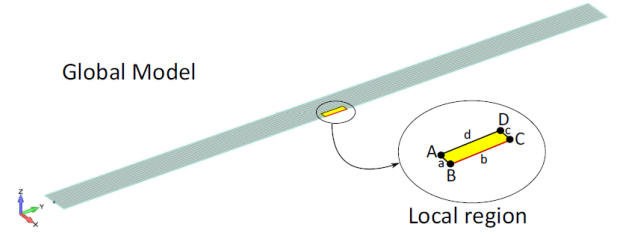


Fig. 2 : Composite coupon test under uni-axial tension.

the global model, the displacement variables at the local interface are transferred to CUF LW model by employing linear shape functions along the mid-plane and Reissner-Mindlin kinematics along the thickness direction. This procedure ensures the minimum information loss and the consistency of the local solution.

3 Numerical results

3.1 Global/local free-edge analysis

The analysis of a composite coupon made of G947/M18 carbon-epoxy as shown in Fig. 2 is carried out. The laminate has four plies with stack sequence $[10^\circ / -10^\circ]_s$ and dimensions as follows: length $L = 200$ mm, width $w = 20$ mm, thickness $t = 0.76$ mm. The material characteristics are given in Lagunegrand et al. [21]; these are:

- $E_{11} = 97.6$ GPa, $E_{22} = 8.0$ GPa, $E_{33} = 8.0$ GPa
- $\nu_{12} = 0.37$, $\nu_{13} = 0.37$, $\nu_{23} = 0.50$
- $G_{12} = 3.1$ GPa, $G_{13} = 3.1$ GPa, $G_{23} = 2.7$ GPa

The coupon is subjected to uniaxial tension (longitudinal strain is $\epsilon_{yy} = 0.001$). The dimensions of the local region are $a = c = 4$ mm, $b = d = 16$ mm. The global model is analyzed with MSc-Nastran using CQUAD4 laminated plate elements. On the other hand, the local region is analyzed using higher-order CUF models with LW capabilities.

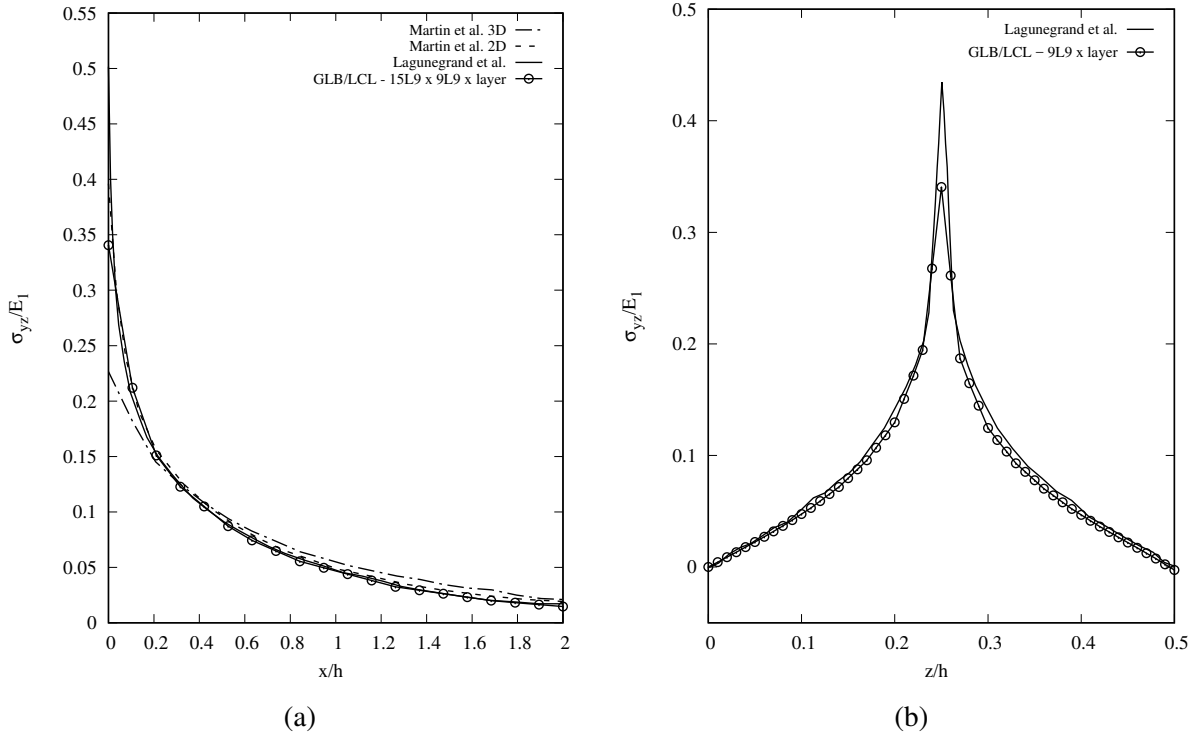


Fig. 3 : Interlaminar stress distribution at free edge.

The proposed analysis aims at demonstrating the capability of the proposed approach to deal with free-edge effect. It is a well-known problem affecting laminated composites, which comports singular stress states that arise at the interfaces between dissimilar layers in the vicinity of geometrical or mechanical discontinuities in the structure. Figure 3 (a) shows the distribution of the transverse shear stress (σ_{yz}) at the interface $-10^\circ/10^\circ$ along axis x in the local region ($x = 0$ is the free edge). Furthermore, Fig. 3 (b) shows the through-the-thickness distribution of the of the same stress component at the free-edge. Whenever possible, the proposed results are compared with those given by Lagunegrand et al. [21] and Martin et al. [22]. It is evident that the provided methodology is able to deal with free-edge effects and stress singularities in accordance with reference from the literature.

3.2 Composite wing

Figure 4 (a) shows the global finite element model of a composite wing of an ultra-light aircraft. The main structure is made of two spars

and 7 ribs. The structure is full composite and is subjected to 4g maximum manoeuvre loading.

The global analysis is performed by using NX-Nastran and the composite laminate is approximated by using PCOMP and, mainly, CQUAD4 plate finite elements. The local analysis, on the other hand, is performed by CUF and aims at describing in detail the interlaminar stress distribution within a small region of the main spar cap, as highlighted in Fig. 4 (b). This local region is a sandwich composite laminate made of 45° - oriented carbon fabric (FAB), carbon unidirectional (UNI), and PVC core. The laminate is made of 25 plies and the material sequence is $[FAB_5/UNI_3/(FAB/UNI)_4/UNI/FAB_2/PVC/FAB]$. The main material characteristics are given in Table 1.

Figure 5 shows the local region analysed. In detail, Fig. 5 (a) shows the material sequence and the relative transverse displacements are depicted in Fig. 5 (b). The through-the-thickness distributions of the in-plane normal stress components as well as important shear stresses along the center of the local region are shown in Figs. 6 (a)

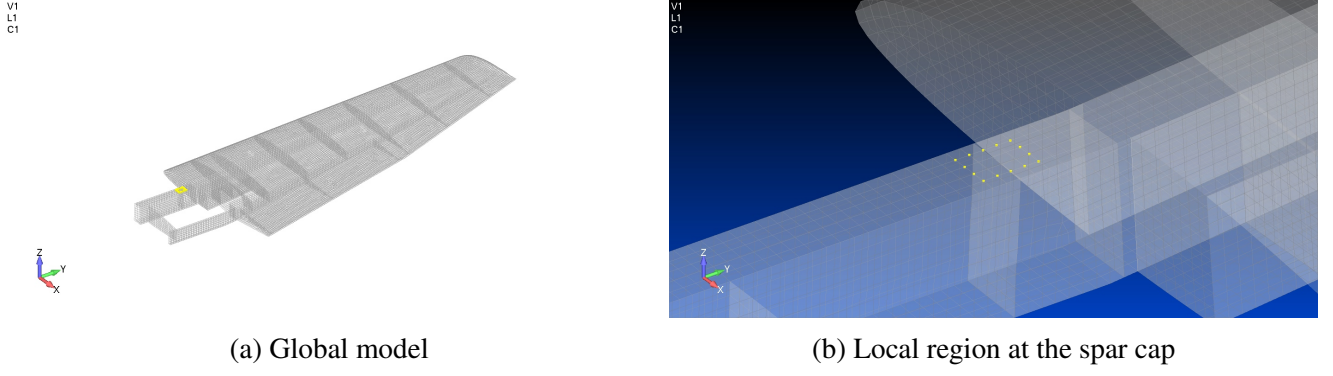


Fig. 4 : Global-local analysis of a composite wing.

Material type	E_1 [GPa]	E_2 [GPa]	$G_{12} = G_{1z} = G_{2z}$ [GPa]	ν_{12}	Thickness [mm]
Carbon fabric	77.2	77.2	2.21	0.25	0.2
Carbon unidir.	109.	8.00	2.21	0.25	0.6
PVC core	1.00	1.00	0.28	0.25	5.0

Table 1: Material properties.

and (b), respectively. As expected, the important contribution is the one related to bending along the y-coordinate. In contrast, the transverse shear stress is negligible.

4 Conclusions

This paper has discussed global/local analyses of composite laminates for aerospace applications. The proposed method couples available commercial finite element tools with advanced theories based on the Carrera Unified Formulation (CUF). Thanks to the variable-kinematics features of CUF and its intrinsic scalable nature, a two-step global/local analysis is straightforward. In fact, the global solution can be used at the local interface to run a local investigation with no information loss and without any kinematics incompatibility between classical 2D finite elements and refined theories. As a matter of fact, the analysis of the free-edge stresses of a composite coupon subjected to uni-axial tension and the global/local analysis of a composite wing have demonstrated that the proposed refined approach allows to describe in a very detailed manner the interlaminar stress distribution in composite laminates.

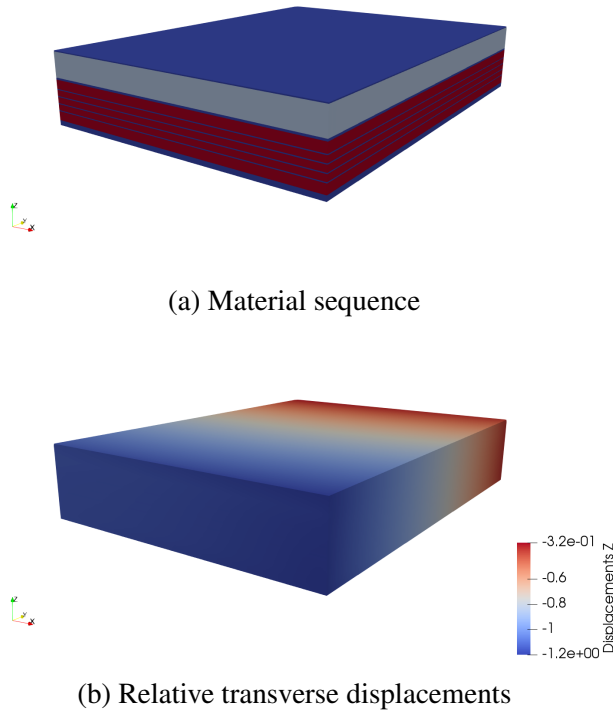
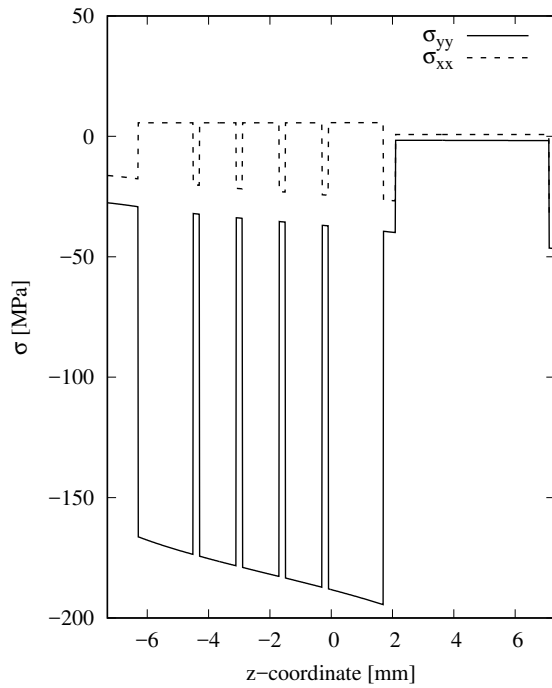
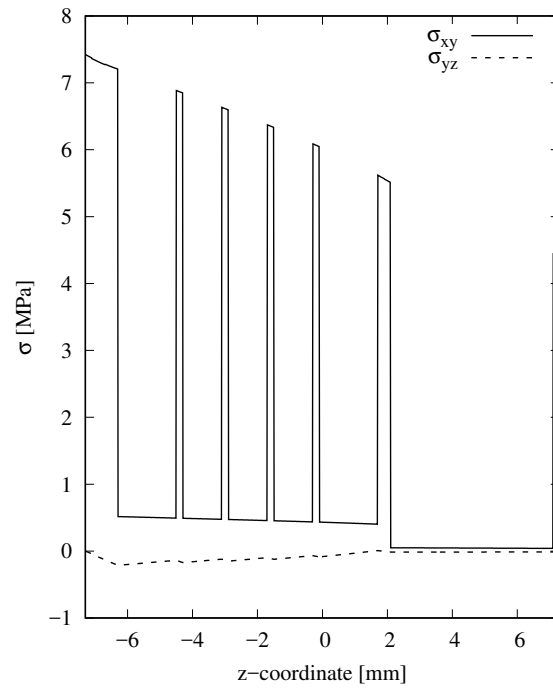


Fig. 5 : Local region of the composite wing.



(a) In-plane normal stress



(b) In-plane and transverse shear

Fig. 6 : Interlaminar stress distribution in the local region of the composite wing.

Acknowledgements

G.A. Fiordilino (Politecnico di Torino), A.G. de Miguel (Politecnico di Torino), P.H. Cabral (Embraer S.A.) and A. Prado (Embraer S.A.) are acknowledged for their essential contribution to this work. This research work has been co-funded by Embraer S.A.

References

- [1] R. E. Bank. *Adaptive computational methods for partial differential equations*. SIAM, 1983.
- [2] B. Szabó and I. Babuška. *Finite Element Analysis*. Wiley Interscience, 1991.
- [3] K. J. Bathe. *Finite element procedure*. Prentice Hall, Upper Saddle River, New Jersey, USA, 1996.
- [4] J. Fish, L. Pan, V. Belsky, and S. Goma. Unstructured multigrid method for shells. *International Journal of Numerical Methods in Engineering*, 39(7):1181–1197, 1996.
- [5] J. Fish. The s-version of the finite element method. *Computers and Structures*, 43(3):539–547, 1992.
- [6] K. W. Shim, D. J. Monaghan, and C. G. Armstrong. Mixed dimensional coupling in finite element stress analysis. *Engineering with Computers*, 18:241–252, 2002.
- [7] P. J. Blanco, R. A. Feijoo, and S. A. Urquiza. A variational approach for coupling kinematically incompatible structural models. *Computer Methods in Applied Mechanics and Engineering*, 197:1577–1620, 2002.
- [8] F. Biscani, G. Giunta, S. Belouettar, E. Carrera, and H. Hu. Variable kinematic beam elements coupled via Arlequin method. *Composite Structures*, 93:697–708, 2011.
- [9] R. J. Guyan. Reduction of stiffness and mass matrices. *AIAA Journal*, 3(2):380–380, 1965.
- [10] J. Fish. Adaptive s-method for linear elastostatics. *Computer Methods in Applied Mechanics and Engineering*, 103:363–396, 1993.
- [11] H. Ben Dhia. Multiscale mechanical problems: the arlequin method. *Comptes Rendus de l'Académie des Sciences - Series IIB - Mechanics-Physics-Astronomy*, 326(12):899–904, 1998.
- [12] H. Hu, S. Belouettar, M. Potier-Ferry, and E. M. Daya. Multi-scale modelling of sandwich struc-

tures using the Arlequin method part I: linear modelling. *Finite Elements in Analysis and Design*, 45(1):37–51, 2008.

- [13] K. M. Mao and C. T. Sun. A refined global-local finite element analysis method. *International Journal of Numerical Methods in Engineering*, 32:29–43, 1991.
- [14] J. B. Ransom and N. F. Knight. Global/local stress analysis of composite panels. *Computers & Structures*, 37(4):375–395, 1990.
- [15] E. Carrera, M. Cinefra, M. Petrolo, and E. Zappino. *Finite Element Analysis of Structures through Unified Formulation*. John Wiley & Sons, Chichester, West Sussex, UK, 2014.
- [16] E. Carrera, A. Pagani, M. Petrolo, and E. Zappino. Recent developments on refined theories for beams with applications. *Mechanical Engineering Reviews*, 2(2):1–30, 2015.
- [17] E. Carrera, M. Filippi, and E. Zappino. Laminated beam analysis by polynomial, trigonometric, exponential and zig-zag theories. *European Journal of Mechanics A/Solids*, 41:58–69, 2013.
- [18] E. Carrera and M. Petrolo. Refined one-dimensional formulations for laminated structure analysis. *AIAA Journal*, 50(1):176–189, 2012.
- [19] E. Carrera, A. Pagani, and M. Petrolo. Classical, refined and component-wise theories for static analysis of reinforced-shell wing structures. *AIAA Journal*, 51(5):1255–1268, 2013.
- [20] A. Pagani, A.G. de Miguel, M. Petrolo, and E. Carrera. Analysis of laminated beams via unified formulation and Legendre polynomial expansions. *Composite Structures*, 156:78–92, nov 2016.
- [21] L. Lagunegrand, T. Lorriot, R. Harry, H. Wagnier, and J. M. Quenisset. Initiation of free-edge delamination in composite laminates. *Composites Science and Technology*, 66(10):1315–1327, 2006.
- [22] E. Martin, D. Leguillon, and N. Carrère. A twofold strength and toughness criterion for the onset of free-edge shear delamination in angle-ply laminates. *International Journal of Solids and Structures*, 47(9):1297–1305, 2010.

Copyright Statement

The authors confirm that they, and/or their company or organization, hold copyright on all of the original material included in this paper. The authors also confirm that they have obtained permission, from the copyright holder of any third party material included in this paper, to publish it as part of their paper. The authors confirm that they give permission, or have obtained permission from the copyright holder of this paper, for the publication and distribution of this paper as part of the ICAS proceedings or as individual off-prints from the proceedings.