MULTILEVEL COLLABORATIVE AERODYNAMIC DESIGN OPTIMIZATION BASED ON SOBOL' GLOBAL SENSITIVITY ANALYSIS

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Abstract

Surrogate model combined with global optimization algorithm is necessary for design space exploration in aerodynamic shape optimization (ASO). However, the "curse of dimensionality" exists to a great extent in those global optimization algorithms. Multilevel collaborative optimization (MCO) method is studies to cope with high-dimensional optimization problems in this paper. The superiority of MCO method over traditional direct full-variables optimization method is confirmed through different test functions. In aerodynamic shape optimization, Sobol' global sensitivity analysis is introduced to quantify the importance degrees of design variables. The design variables are divided in to subcomponents according to their importance degrees and the subcomponents are optimized individually in multiple cycles. The MCO aerodynamic design framework is established by integrating the Sobol' global sensitivity analysis method, efficient shape parameterization method, mesh deformation technique, numerical simulation method and surrogate-based global optimizer. Finally, a commercial airplane in transonic regime is optimized by MCO method and conventional method respectively. Results show that the proposed MCO method is better than conventional method.

1 Introduction

In the past decades, automatic design optimization has been the subject of ever growing interest, thanks to the development of ever more reliable analysis software, efficient optimization methods and powerful computers. In the aerodynamic shape optimization (ASO) community, the use of high-fidelity computational fluid dynamic (CFD) simulation is ubiquitous and searching for an improved design using CFD-based optimization is a common practice[1]-[5]. To achieve a preferable aerodynamic design, various algorithms and approaches have been developed, from conventional gradient-based algorithm including those utilizing adjoint methods[6]-[8], to surrogate-based optimization (SBO) that offer efficient global optimization and substantial reduction of the design cost[9]-[12].

In aerodynamic shape optimization especially in aircraft wing design, a large number of design variables are required to help increase the degrees of freedom and explore more feasible design space. The nature of high dimensional aerodynamic design space, with a large number of constraints that generate multiple infeasible regions and a highly multimodal and fragmented landscape complicates the optimization process considerably. When facing high dimensional aerodynamic design issues, the adjoint method which drastically reducing the cost of computing the gradient is quite popular. Nevertheless, gradient-based optimizers tend to get trapped in local minima and a decent baseline is indispensable. To obtain the global optimum, surrogate-based optimization with global design space exploration should be conducted[13]. Different kinds of global optimization algorithms have been developed: Simulated Annealing (SA), Genetic Algorithms (GA), Differential Evolution (DE), Particle Swarm Optimization (PSO) and so on. Although these global optimization algorithms have shown good optimization performance in solving lower dimensional problems, many of them suffer from the "curse of dimensionality", which implies that their performance deteriorates quickly as the dimension of the problem increases[14].

The "curse of dimensionality" refers to the exponential growth of volume associated with adding extra dimensions to a problem space [15],[16]. For population based optimization algorithms such as PSO, this rapid growth in volume means that as the dimensionality of the problem increases, each of the particles has to potentially search a larger and larger area in the problem space. Furthermore, the number of local optima will grow dramatically. The majority of global optimization algorithms lose the power of searching the optimal solution when the dimension increases. Therefore, more efficient search strategies are required to explore all the promising regions in a given time budget[17],[18].

In recent years, large-scale global optimization has attracted more and more interest. The most usual approach for solving large scale optimization problems is collaborative coevolution (CC) frame proposed by Potter and De Jong [19]. CC adopts a divide-and-conquer strategy, which decomposes a high dimensional problem into several subcomponents and evolves the subcomponents individually in multiple cycles. Via this divide and conquer method, CC is able to solve many separable or weak nonseparable problems effectively[20],[21].

There are many decomposition strategies dealing with high-dimensional problem in the literature. In our work, we present a decomposition strategy based on global sensitivity analysis[22],[23]: the design variables are decomposed in to different low dimensions according to their global sensitivity indices. Sobol' global sensitivity analysis method is introduced which studies how the variation in the output of a model can be apportioned quantitatively to different design variables. Combined with Sobol' global sensitivity analysis, the multilevel collaborative optimization (MCO) framework is established.

The remainder of the paper is organized as follows: Section 2 reveals the "curse of dimensionnality" in population-based global optimization algorithms. The advantage of multilevel collaborative optimization method over conventional optimization method is confirmed in section 3. The theory of Sobol' global sensitivity analysis is presented in section 4. In section 5, the superiority of proposed method is further studied through design optimization of a transonic wing. Section VI outlines conclusions and future work..

2 The "curse of dimensionality"

The "curse of dimensionality'', which was first coined by Bellman[15],[16], is the term used to describe the problem caused by the exponential increase in volume associated with adding extra dimensions to a mathematical space.

Many optimization methods suffer from the "curse of dimensionality", which implies that their performance deteriorates quickly as the dimensionality of the search space increases. A variation of the Rastrigin function is employyed to uncover the "curse of dimensionality" of global optimization algorithm. The function is a widely used multimodal test function. It has the following definition:

$$y = \frac{1}{\mathrm{D}} \cdot \left[10 \cdot D + \sum_{i=1}^{D} \left(x_i^2 - 10 \cdot cos\left(2\pi x_i\right) \right) \right] \qquad (1)$$

$$(-5.12 < x_i < 5.12)$$

In the function, D represents the dimensions of input variables. Rastrigin function is slighted modified and the division of D is to eliminate the impact of dimension on the function value. An overview of the function in 2D situation is shown below.

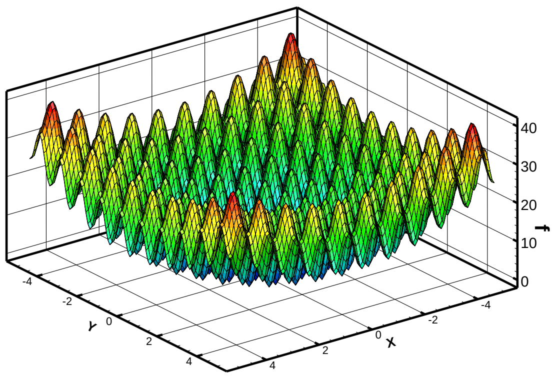

Fig. 1. Visualization of 2D Rastrigin function

The PSO algorithm is used to optimize the function. In the first case, the same number of function evaluations is conducted for different dimensional functions. There are 100 particles and 400 optimization steps for each dimensional function. In the second case, the number of particle is 10 times the dimensionality, i.e. the function evaluations is increasing with the dimensionality.

In both case, the optimization is run 10 times and the averaged optimization steps and optimum are shown in Fig. 2-Fig. 5. From the result, it is clear that the final optimum becomes worse as the dimensionality increases: when the dimensionality is high, the increased problem space, along with the sparse population of particles causes the algorithm quickly converging on relatively poor solution. Consequently, PSO's predisposition towards premature convergence worsened and stuck in local optimum as the dimensionality of the problem grew.

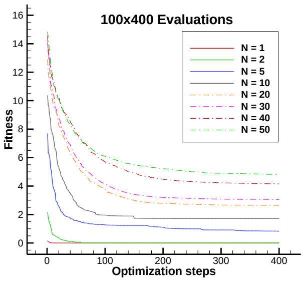

Fig. 2. Optimization histories of the first case

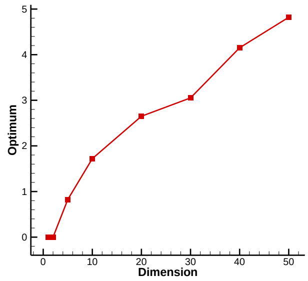

Fig. 3 .Different dimensional results of the first case

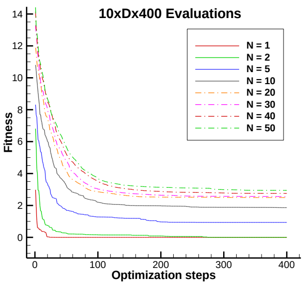

Fig. 4. Optimization histories of the second case

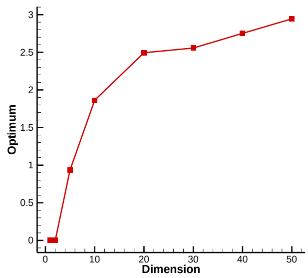

Fig. 5. Different dimensional results of the second case

3 Multilevel Collaborative Optimization

The increasing dimensionality causes the deterioration of the performance of most global optimization algorithms. Therefore, it is very essential for modern optimization strategy to be able to be scalable for high-dimensional problems. The idea of multilevel collaborative optimization (MCO) is introduced in this work. In multilevel collaborative optimization, high dimensional design variables are decomposed into several subcomponents and these low dimensional design variables are optimized collaboratively in multiple cycles.

To show the superiority of multilevel collaborative optimization over conventional direct full-variables optimization, the Rastrigin function is also employed to be optimized. In this case, the dimensionality of the problem is set to 40. For multilevel collaborative optimization, the input variables are decomposed into four sub-dimensions, i.e. there are 10 variables in each sub optimization loop. Each sub optimization is run with 100 steps and the total number of function evaluations is the same with that of direct optimization in section 2. The result is shown in figure 6, which shows that the multilevel optimization is better than that of direct optimization.

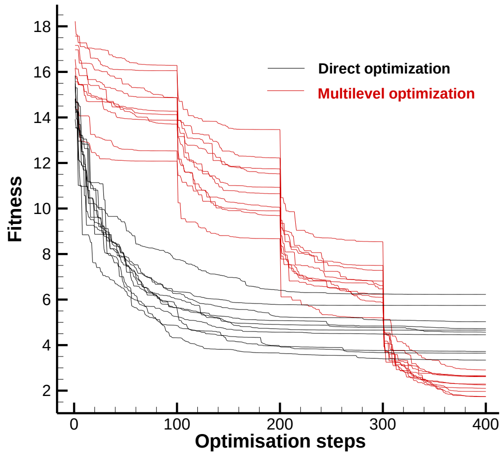

Fig. 6. Optimization histories of Rastrigin function

The result may be not surprising for Rastrigin function which is a separable function. To further study the performance of multilevel optimization strategy, two non-separable functions are chosen to be optimized. They have the following definition:

Griewangk function:

$$y = \frac{1}{4000}\sum_{i=1}^{D} x_i^2 - \prod_{i=1}^{D} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \qquad (2)$$

$$(-600 < x_i < 600)$$

Ackley function:

$$y = -20 \cdot exp\left(-\frac{1}{5}\sqrt{\frac{1}{D}\sum_{i=1}^{D} coef_i \cdot x_i^2}\right) - exp\left(\frac{1}{D}\sum_{i=1}^{D} coef_i \cdot cos\left(2\pi x_i\right)\right) + 20 + exp\left(1\right) \quad \left(-32 < x_i < 32\right) \tag{3}$$

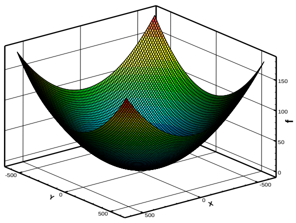

Fig. 7. Visualization of 2D Griewangk function

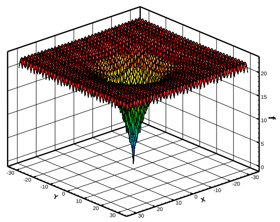

Fig. 8. Visualization of 2D Ackley function

The optimization histories of these two non-separable functions are presented in Fig. 9-Fig. 10. From the results, the multilevel optimization also shows good performance for non-separable functions. Thus, it can be inferred that, the multilevel collaborative optimization strategy would hold true for aerodynamic shape design problems.

In aerodynamic shape design, we have no prior knowledge of the design space property and the separability of design variables. To author's knowledge, there is also no guideline for how to conduct multilevel optimization in aerodynamic design. In a very natural sense, the design variables can be divided in to sub-components according to their importance degrees, i.e. the sensitivities and optimized separately. The reason lies in that for global optimization algorithm used in ASO, the design variables in each dimension are randomly distributed and the contribution of less important variables would be concealed by the important ones. This idea is somewhat like the design variable screening method. However, MCO do not ignore any design variables, and the design space is not shrinked. In this paper, Sobol' global sensitivity analysis is introduced to quantify the importance degrees of design variables and details of the sensitivity analysis method is presented in the next section.

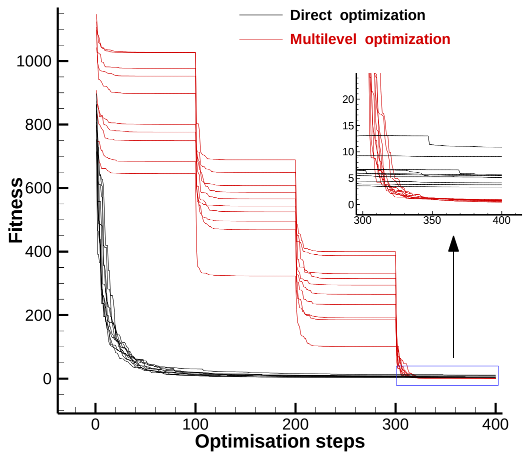

Fig. 9. Optimization histories of Griewangk function

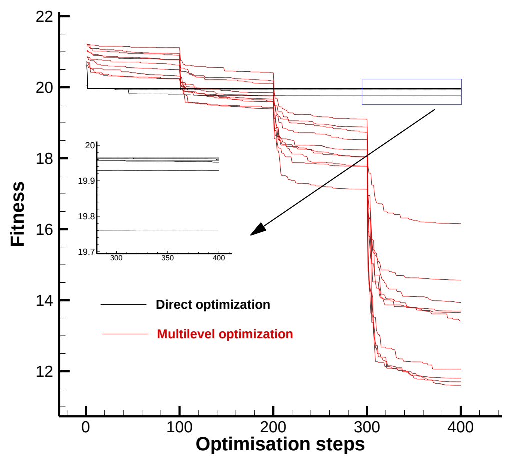

Fig. 10. Optimization histories of Ackley function

4 Sobol' Global Sensitivity Analysis

Sensitivity analysis is the study of how the variation in the output of a model can be apportioned, qualitatively or quantitatively, to different inputs. Sensitivity analysis may help understanding the contribution of the model inputs to the model output and system performance in general.

Methods for sensitivity analysis are typically classified as local perturbation or global methods. Local methods perturb the inputs along coordinate directions around a nominal value and measure the effects on the outputs. Though relatively inexpensive, local methods are dependent on the choice of the perturbation step and the local sensitivity measured at the nominal condition may be very different elsewhere in the parameter space. Global methods address these issues by providing integrated measures of the output's variability over the full range of parameters. The global sensitivity approach does not specify the input. Therefore, global sensitivity indices should be regarded as a tool for studying the mathematical model rather than its specified solution[24].

Sobol' global sensitivity analysis is a variance-based method[25]. The variance-based analysis is sampling-based and therefore applies Monte Carlo simulation. Moreover, it relies on the computation of conditional variances. The main advantage of the methods is that the analytic structure of the model to be analyzed has not to be known.

As the effect of inputs upon the output can be independent and cooperative, it is natural to express the model output as a finite hierarchical correlated function expansion in terms of the input variables:

$$\begin{aligned} f(x) &= f_0 + \sum_{i=1}^{n} f_i(x_i) \\ &+ \sum_{1\le i<j\le n} f_{ij}(x_i, x_j) \\ &+ \sum_{1\le i<j<k\le n} f_{ijk}(x_i, x_j, x_k) + \cdots + \\ &\sum_{1\le i_1<\cdots<i_l\le n} f_{i_1 i_2 \ldots i_l}(x_{i_1}, x_{i_2}, \ldots, x_k) + \\ &\cdots + f_{12\ldots n}(x_1, x_2, \ldots, x_n) \end{aligned} \tag{4}$$

The total number of summands in (4) is 2^n. We assume that the members in (4) are orthogonal and can be expressed as integrals of $f(x)$:

$$\begin{aligned} & f_{i_1 i_2 \ldots i_l}(x_{i_1}, x_{i_2}, \cdots, x_{i_l})|_{x_{i_s}=\bar{x}_{i_s}} = 0, \\ & i_s \in \{i_1, i_2, \ldots, i_l\} \end{aligned} \tag{5}$$

We have:

$$\begin{aligned} \int f(x)\,dx &= f_0 \\ \int f(x) \prod_{k\ne i} dx_k &= f_0 + f_i(x_i) \\ \int f(x) \prod_{k\ne i,j} dx_k &= f_0 + f_i(x_i) + f_j(x_j) \\ & \quad + f_{ij}(x_i, x_j) \end{aligned} \tag{6}$$

Assume that $f(x)$ is square integrable. Then, all the $f_{i_1\cdots i_s}$ are square integrable. We get

$$\int f^2(x)\,dx - f_0^2 = \sum_{s=1}^{n} \sum_{i_1<\cdots i_s}^{n} \int f_{i_1\cdots i_s}^2 dx_{i_1} \cdots dx_{i_s} \tag{7}$$

The constants

$$\begin{aligned} D &= \int f^2(x)\,dx - f_0^2 \\ D_{i_1\cdots i_s} &= \int f_{i_1\cdots i_s}^2 dx_{i_1} \cdots dx_{i_s} \end{aligned} \tag{8}$$

are called variances and

$$D = \sum_{s=1}^{n} \sum_{i_1<\cdots i_s}^{n} D_{i_1\cdots i_s} \tag{9}$$

The ratios $S_{i_1\cdots i_s} = \frac{D_{i_1\cdots i_s}}{D}$ are called global sensitivities. All the $S_{i_1\cdots i_s}$ are nonnegative and their sum is

$$\sum_{s=1}^{n} \sum_{i_1<\cdots i_s}^{n} S_{i_1\cdots i_s} = 1 \tag{10}$$

The main breakthrough in Sobol' method is the computation algorithm that allows a direct estimation of global sensitivity induces using values of $f(x)$ only. Monte Carlo integration is utilized to obtain those sensitivities[26].

Sobel's g function is utilized to assess the accuracy of this sensitivity analysis method:

$$\begin{aligned} Y &= \prod_{i=1}^{k} g_i(x_i), \\ g_i(X_i) &= \frac{|4x_i - 2| + a_i}{1 + a_i} \end{aligned} \tag{11}$$

The constant a_i determine sensitivities of different variables. First order variance of g function is:

$$V_i = \frac{1}{3(1+a_i)^2} \tag{12}$$

The total variance of g function is:

$$V = -1 + \prod_{i=1}^{k} (1 + V_i) \tag{13}$$

In this test case, the constant a_i are presented in table 1. 10000 Monte Carlo simulations are performed to obtain Sobel's sensitivities. Sobel's sensitivities and the theoretical sensitivities are shown in Fig. 11. It can be seen that Sobel's sensitivities are quite close to the theoretical sensitivities.

Table 1 Values of constant a_i

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
90	1	0.1	2	80	4	0.5	3	70	1

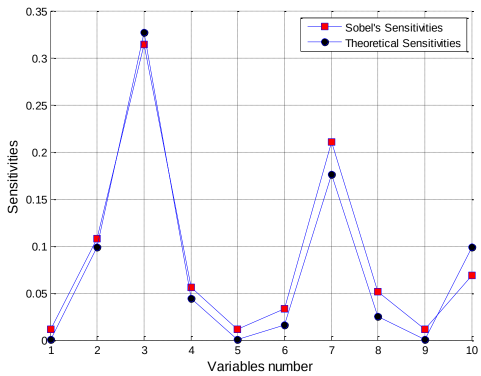

Fig. 11. Sensitivity comparison

5 Design Optimization of a Transonic Wing

5.1 Framework of multilevel collaborative optimization

The Framework involves integrating an efficient shape parameterization and mesh deformation technique with a numerical simulation method and surrogate-based optimizer. The flowchart of multilevel collaborative optimization is presented in Fig.12 and the computation methods are discussed below.

1. CFD Solver

The RANS solver is used for aerodynamic analysis and SST turbulence model is applied; Roe scheme is used for inviscid terms and central difference for viscous terms; Implicit time marching method LU-SGS is utilized.

2. Geometric Parameterization

The parameterization scheme is the critical factor for an efficient exploration of the design space. We use free-form deformation (FFD) approach to parameterize the geometry[27],[28]. The FFD approach can be visualized as embedding the spatial coordinates defining a geometry inside a flexible volume. The parametric locations (u, v, w) corresponding to the initial geometry are found using a Newton search algorithm. Once the initial geometry has been embedded, perturbations made to the FFD volume propagate within the embedded geometry via the evaluation of the nodes at their parametric locations. NURBS volumes are used for the FFD implementation, and the displacements of the control point locations are design variables.

3. Mesh Movement

Mesh movement operation is required to propagate surface perturbations to the remainder of the volume mesh. A robust mesh deformation technique utilizing quaternion spherical interpolation[29] and inverse distance weighted interpolation developed in our previous work is utilized[30]. In this method, the movement is divided in to a rotation part and a translation part, and distributed parallel-computing is utilized to accelerate efficiency. Therefore, high quality CFD Grid can be generated automatically in a very short time.

4. Surrogate-based optimization

Surrogate-based optimization is a very efficient method for global optimization of computationally expensive engineering problems. In this work, Kriging model is used as the surrogate of CFD simulations for its outstanding performance in data fitting problems[31],[32] and PSO algorithm is chosen as the optimizer due to its algorithmic simplicity and effectiveness [33],[34].

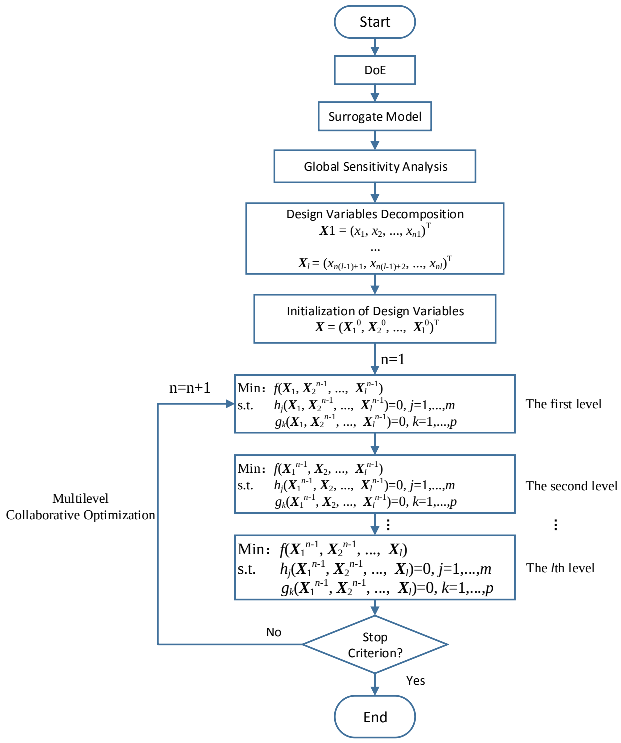

Fig.12. Flowchart of the multilevel collaborative optimization framework

5.2 Design optimization of a transonic wing

A commercial airplane in transonic regime is chosen as the design case. The drag coefficient is minimized at a prescribed lift coefficient. The single design point is:

$$Cl = 0.54, \quad Ma = 0.78, \quad Re = 2e7$$

FFD technique is utilized to parameterize the wing as shown in Fig. 13. The leading edge and trailing edge of the wing remain unchanged. Vertical coordinates of the control points are the design variables and the wing shape is characterized with 48 variables.

The Latin Hypercube Sampling (LHS) scheme is utilized as the DoE method. 3000 samples are performed using high-fidelity CFD simulations. After the Kriging surrogate model is constructed, the surrogate model is used to calculate global sensitivities and assess candidate designs.

By Performing Sobol' sensitivity analysis approach, global sensitivities of different variables are presented in Fig. 14 and then they are sorted as shown in Fig. 15. The 48 design variables are decomposed into four subcomponents according to their global sensitivities indices. These subcomponents are optimized individually in several cycles.

Conventional direct optimization and proposed multilevel optimization are both conducted in this case. Optimization settings are as follows:

- Direct optimization: 100 particles, 400 steps (100x400).
- Multilevel optimization: 100 particles, 20 steps, 5 cycles (100x20x4x5).

The number of function evaluations is the same in these two cases and the optimization histories are shown in Fig. 16. As expected, multilevel optimization shows better results than direct optimization. Fig. 17-Fig. 18 illustrates pressure contours of the baseline and optimized shapes. Fig. 19 gives airfoil and pressure coefficient comparisons on different wing span sections. It can be seen that, the shock wave is weaken on both optimized shapes. By comparison, the shock wave on direct optimized shape is stronger than that of multilevel optimized shape. The aerodynamic shape optimization case shows the effectiveness of proposed method.

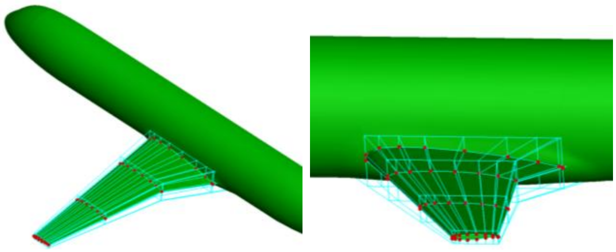

Fig. 13. The airplane configuration and FFD box

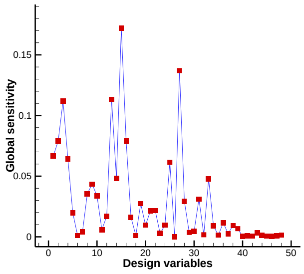

Fig. 14. Sensitivities of different design variables

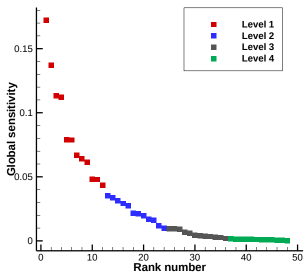

Fig. 15. Sensitivity decay of design variables

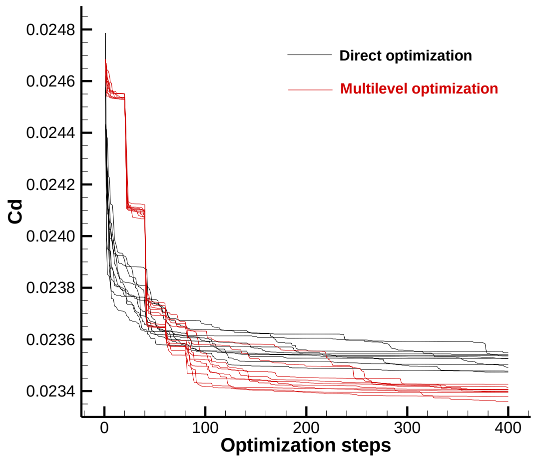

Fig. 16. Optimization histories of different methods

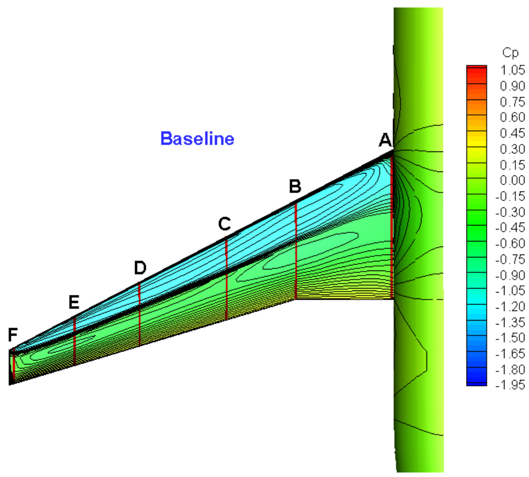

Fig. 17. Pressure contour of the baseline

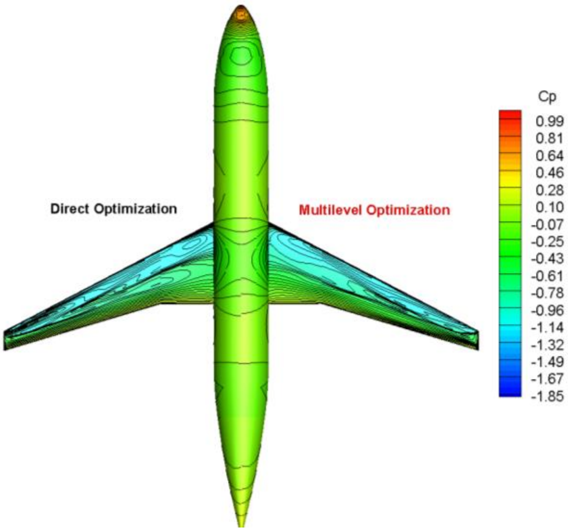

Fig. 18. Pressure contour comparison of different optimization results

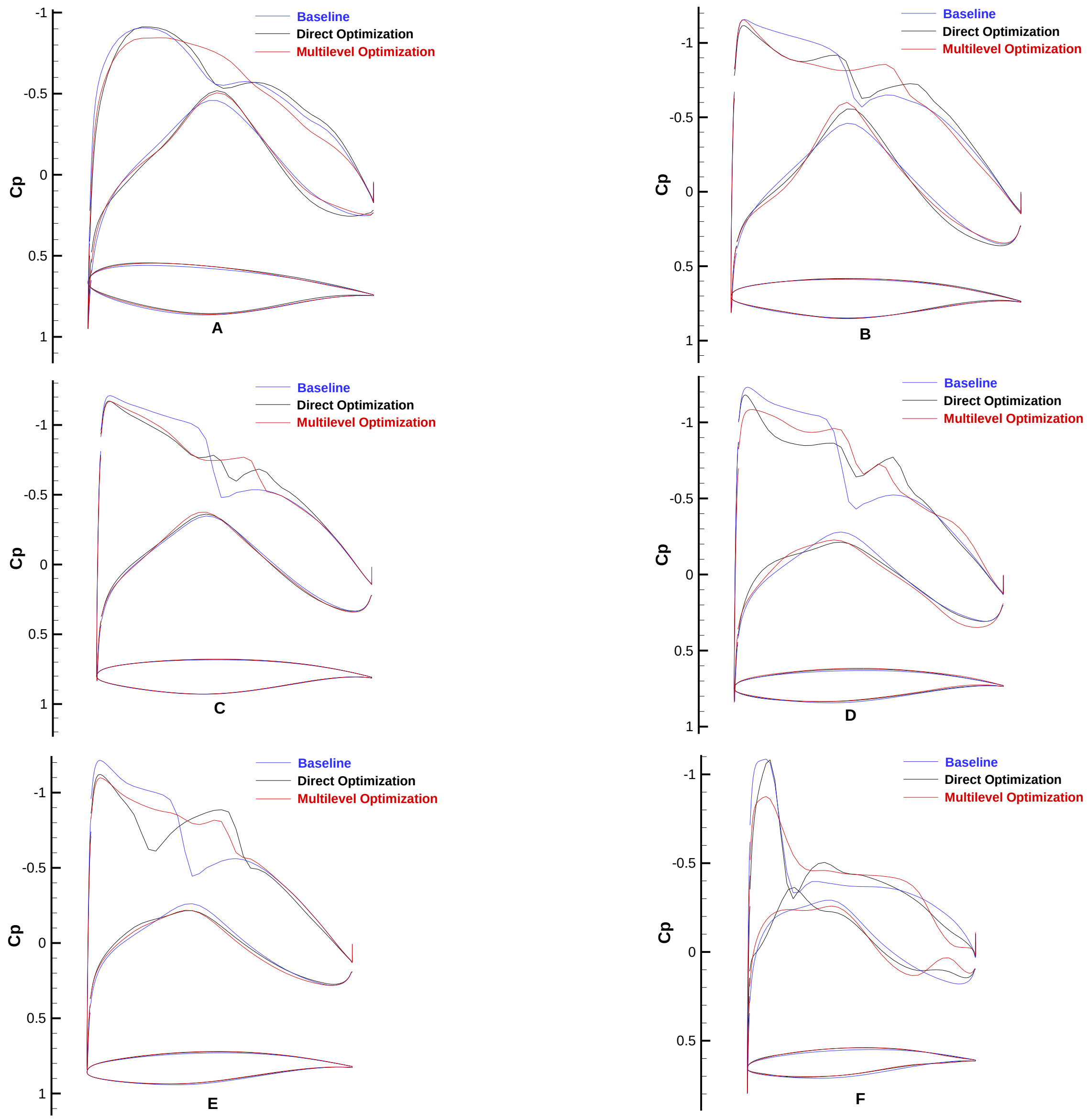

Fig. 19. Comparisons of section shapes and pressure distributions

6 Concluding Remarks

In the aerodynamic shape optimization community, searching for an improved design is still a common practice. The huge search space and multiple local minima restrict the ability of an optimization algorithm to achieve a globally optimal design given a limited budget. This work proposed a multilevel collaborative optimization method based on Sobol's global sensitivity analysis. The design variables are decomposed into subcomponents according to their importance degrees and then the subcomponents are optimized individually in multi cycles. The aerodynamic design case of a transonic wing is conducted to confirm the effectiveness of the proposed method.

Different from design variables screening method, the multilevel collaborative optimization approach does not eliminate any design variables and the design space does not shrink. Although there is no theoretical proof that the multilevel collaborative optimization can find the true global optimum, the blessing of MCO is that it can accelerate the design optimization. There remain several open questions in the reparability of design variables and how to separate them in aerodynamic shape design optimization. Future work will focus on deep study of theoretical background and improving the knowledge of multilevel collaborative optimization.

7 Acknowledgments

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