QUASI-LPV MODELING AND PARAMETER-DEPENDENT OPTIMAL CONTROL OF MORPHING WING AIRCRAFT

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Abstract

A near-optimal gain-scheduling strategy for the morphing aircraft flight control design is proposed. Nonlinear parameter-dependent dynamics of the morphing aircraft is transformed into a quasi-linear parameter-varying form. Assuming that the morphing parameters are determined in an outer-loop to meet system requirements, an inner-loop controller is designed by optimal control scheme. An algebraic Riccati equation using the parameter-dependent coefficient matrices is solved to design a near-optimal controller. Numerical simulation is performed to demonstrate the effectiveness of the proposed scheme.

1 Introduction

Aircraft capable of in-flight, controlled, large-scale shape changes with the object of improving efficiency, versatility, and performance over a wide range of flight conditions and variant missions are called morphing aircraft [1]. The concept of variable-geometry aircraft has a long history starting from the beginning of manned flight, and there exist many operational aircraft which can perform shape changes [2]. However, utilization of traditional mechanical and hydraulic systems has been restricted mainly due to their low energy density. Recently, the efforts to develop morphing aircraft has been re-ignited according to smart technologies for materials, sensors, and actuators [3], which offer researchers new possibilities to design efficient morphing aircraft [4]. The design process often involves complex trade-off studies as well as multidisciplinary optimization. In [5], impact of wing morphing on aircraft performance was demonstrated. The morphing strategies were also considered to improve the performance of a cruise missile [6–8].

Morphing configuration may be determined online to maximize the system-level benefits, e.g., lift-to-drag ratio. Reinforcement learning-based approaches were used to learn the optimal policy for shape change [9–11]. Since the determined configuration is realized by manipulating a series of morphing actuators, the problem of designing morphing aircraft control system can be divided into three sub-problems: shape determination, shape control, and flight control. This separation can be partly justified by the fact that response of morphing actuators is significantly slower than that of conventional flapped control surfaces. In this study, only flight control design is mainly focused. Shape changes are often parametrized by a set of measurable time-varying signals, which continuously affect the dynamics in a given manner. In addition to aerodynamic characteristics, mass properties are also significantly changed. Therefore, morphing aircraft is often modeled as a multi-body dynamic system with varying degrees of approximating assumption [12, 13]. The obtained dynamic model of morphing aircraft is, in general, a parameter-dependent nonlinear system.

Flight control system of morphing aircraft should ensure consistent closed-loop behavior for the considered flight regimes and shapes. Accordingly, several control design approaches to address the parameter-varying nature of the system have been studied. They often schedule or adapt the controller parameters by measuring or estimating the external parameters. One standard approach is a classical gain scheduling method [14]. This approach provides a simple yet powerful way to design a gain-scheduled controller despite the lack of theoretical guarantee of stability. Another approach is the linear parameter-varying (LPV) control method [15–19], which can be viewed as a mathematically rigorous version of the gain-scheduled design. Adaptive dynamic inversion control based on feedback linearization was also applied to account for the uncertainties due to the parameter variation [9–11, 20, 21]. Other popular nonlinear control methods have also been studied, which include sliding mode control [22,23] and backstepping control [13,24].

In this study, a near-optimal gain-scheduling strategy for the morphing aircraft flight control design is proposed. A variable-span morphing aircraft model is obtained by modifying a conventional high-performance aircraft model. The obtained nonlinear parameter-dependent dynamics of the morphing aircraft are then transformed into a quasi-linear parameter-varying (qLPV) form. In earlier related work, a parameter-dependent Riccati equation was utilized in a pitch-yaw autopilot design where the parameter was exogenously supplied [25]. In fact, there is no fundamental difference between qLPV model and state-dependent coefficient (SDC) form used in the state-dependent Riccati equation (SDRE) method. Both modeling strategies can exploit endogenous as well as exogenous parameters in the coefficient matrices. Quasi-LPV modeling approach, however, tries to separate the non-scheduling state variables from the system matrices and often involves some approximations if necessary. Therefore, unnecessary dependency of the controller gain on the non-scheduling variables can be removed. An algebraic Riccati equation (ARE) using the qLPV model is then solved at every time step to give the near-optimal control law. The resulting controller is self-scheduled on the altitude and morphing parameter (exogenous parameters) as well as airspeed and angle of attack (endogenous parameters). The morphing parameters are assumed to be determined and controlled externally to meet the system requirements, and therefore their determination is not addressed explicitly in this study. Numerical simulation is performed to demonstrate the effectiveness of the proposed scheme.

2 Morphing Aircraft

In this study, longitudinal dynamics of a morphing aircraft model is considered. Variable-span morphing aircraft model is obtained by modifying a conventional fixed-wing aircraft model. The target aircraft is assumed to be capable of symmetric variable-span morphing, which changes the aerodynamic characteristics of the aircraft. The longitudinal equations of motion can be expressed as

$$\dot{V}_T = \frac{T\cos\alpha - D}{\mathrm{m}} - g\sin\gamma \tag{1}$$

$$\dot{\alpha} = -\frac{T\sin\alpha + L}{\mathrm{m}V_T} + \frac{g\cos\gamma}{V_T} + q \tag{2}$$

$$\dot{\gamma} = \frac{T\sin\alpha + L}{\mathrm{m}V_T} - \frac{g\cos\gamma}{V_T} \tag{3}$$

$$\dot{q} = \frac{m}{J_y} \tag{4}$$

where V_T, γ, α, q, and h are the true airspeed, flight path angle, angle of attack, pitch rate, and altitude, respectively, m, J_y, and g are the mass, y-axis moment of inertia, and gravitational acceleration, respectively, and L, D, m, and T are the lift, drag, pitching moment, and thrust, respectively. The thrust T is expressed as

$$T = T_{\max}\delta_t \tag{5}$$

where $T_{\max}$ is the maximum thrust, and δ_t is the throttle. Aerodynamic forces and moment are defined as

$$L = QSC_L \tag{6}$$
$$D = QSC_D \tag{7}$$
$$\begin{aligned} m = QS\bar{c}\bar{C}_m - x_{\text{ref}}(D\sin\alpha + L\cos\alpha) \\ + z_{\text{ref}}(D\cos\alpha - L\sin\alpha) \end{aligned} \tag{8}$$

with

$$Q = \tfrac{1}{2}\rho_a V_T^2 \tag{9}$$

where Q is the dynamic pressure, ρ_a is the atmospheric density determined by the altitude in the international standard atmosphere (ISA), S is the reference area, $\bar{c}$ is the reference length, and x_{ref} and z_{ref} are the displacements of the center of pressure from the center of gravity along the body x- and z-axes, respectively.

Note that S and $\bar{c}$ are the planform area and mean aerodynamic chord, respectively, which are parameters chosen to derive dimensionless aerodynamic coefficients. Therefore, S and $\bar{c}$ are constant regardless of morphing. Instead, aerodynamic coefficients bear whole morphing-induced aerodynamic variations. In the considered aircraft model, dimensionless aerodynamic coefficients are constructed as follows,

$$C_L = C_{L_0} + \frac{\bar{c}}{2V_T}C_{L_q}q + C_{L_{\delta_e}}\delta_e \tag{10}$$
$$C_D = C_{D_0} + \frac{\bar{c}}{2V_T}C_{D_q}q + C_{D_{\delta_e}}\delta_e \tag{11}$$
$$\bar{C}_m = \bar{C}_{m_0} + \frac{\bar{c}}{2V_T}\bar{C}_{m_q}q + \bar{C}_{m_{\delta_e}}\delta_e \tag{12}$$

where δ_e is the elevator deflection. Control derivatives are functions of α, and the other six coefficients are functions of both α and morphing parameter η. To simplify the expressions for the pitching moment coefficients, new coefficients including the c.g. effect are defined as

$$\begin{aligned} C_{m_0} = \bar{C}_{m_0} - \tfrac{x_{\text{ref}}}{\bar{c}}(C_{D_0}\sin\alpha + C_{L_0}\cos\alpha) \\ + \tfrac{z_{\text{ref}}}{\bar{c}}(C_{D_0}\cos\alpha - C_{L_0}\sin\alpha) \end{aligned} \tag{13}$$
$$\begin{aligned} C_{m_q} = \bar{C}_{m_q} - \tfrac{x_{\text{ref}}}{\bar{c}}(C_{D_q}\sin\alpha + C_{L_q}\cos\alpha) \\ + \tfrac{z_{\text{ref}}}{\bar{c}}(C_{D_q}\cos\alpha - C_{L_q}\sin\alpha) \end{aligned} \tag{14}$$
$$\begin{aligned} C_{m_{\delta_e}} = \bar{C}_{m_{\delta_e}} - \tfrac{x_{\text{ref}}}{\bar{c}}(C_{D_{\delta_e}}\sin\alpha + C_{L_{\delta_e}}\cos\alpha) \\ + \tfrac{z_{\text{ref}}}{\bar{c}}(C_{D_{\delta_e}}\cos\alpha - C_{L_{\delta_e}}\sin\alpha) \end{aligned} \tag{15}$$

Now, let us define

$$C_m = C_{m_0} + \frac{\bar{c}}{2V_T}C_{m_q}q + C_{m_{\delta_e}}\delta_e \tag{16}$$

Then, the pitching moment m can be expressed as

$$m = QS\bar{c}C_m \tag{17}$$

The original model can be found in [26], and further details for morphing aircraft model modification can be found in [27].

3 Quasi-LPV Model

In this section, basic concept of an LPV system is introduced. A standard procedure to transform a specific class of nonlinear systems into a qLPV model is explained. Equations of motion of the morphing aircraft are embedded in a qLPV model, which is considered for the control design.

3.1 LPV Model

LPV system is defined as a linear system whose system matrices in state-space representation depend on a set of exogenous parameters. Note that the exogenous parameters are measurable online but not known a priori. LPV system can be represented as

$$\dot{x} = A(\rho)x + B(\rho)u \tag{18}$$
$$y = C(\rho)x \tag{19}$$

where $x \in \mathcal{R}^{n_x}$ is a state, $u \in \mathcal{R}^{n_u}$ is a control input, $y \in \mathcal{R}^{n_y}$ is a measurement output, and $A(\rho)$, $B(\rho)$, and $C(\rho)$ are parameter-dependent matrices with proper dimensions. For a given compact set $\mathcal{P} \subset \mathcal{R}^{n_\rho}$, a scheduling parameter $\rho \in \mathcal{P}$ is assumed to be measurable in real-time, where $\rho(\cdot)$ is a piecewise continuous trajectory. Some of the most common techniques to transform a nonlinear system into an LPV model are Jacobian linearization, velocity-based linearization, and qLPV linearization.

The qLPV technique is an approach to overcome the local validity of the resulting model in classical Jacobian linearization schemes. In qLPV linearization, nonlinear system is transformed into an LPV form by hiding the nonlinear terms in the scheduling variable. Since this process is a transformation rather than a linearization in a classical sense, the resulting LPV model is exactly equal to the original nonlinear system [28].

Consider a class of input-affine nonlinear parameter-dependent system of the form

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}A_{11}(x_1,p) & A_{12}(x_1,p)\\ A_{21}(x_1,p) & A_{22}(x_1,p)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}B_1(x_1,p)\\ B_2(x_1,p)\end{bmatrix}u + \begin{bmatrix}f_1(x_1,p)\\ f_2(x_1,p)\end{bmatrix} \tag{20}$$

Assuming that there exist differentiable functions $\bar{x}_2(x_1,p)$ and $\bar{u}(x_1,p)$ which satisfy following equation for every x_1 and p

$$\begin{bmatrix}0\\ 0\end{bmatrix} = \begin{bmatrix}A_{11}(x_1,p) & A_{12}(x_1,p)\\ A_{21}(x_1,p) & A_{22}(x_1,p)\end{bmatrix}\begin{bmatrix}x_1\\ \bar{x}_2\end{bmatrix} + \begin{bmatrix}B_1(x_1,p)\\ B_2(x_1,p)\end{bmatrix}\bar{u} + \begin{bmatrix}f_1(x_1,p)\\ f_2(x_1,p)\end{bmatrix} \tag{21}$$

Let us consider the following transformation.

$$\xi_1 = x_1 \tag{22}$$

$$\xi_2 = x_2 - \bar{x}_2 \tag{23}$$

$$v = u - \bar{u} \tag{24}$$

$$\tilde{A}_{22} = A_{22} - \frac{\partial \bar{x}_2}{\partial x_1}A_{12} \tag{25}$$

$$\tilde{B}_2 = B_2 - \frac{\partial \bar{x}_2}{\partial x_1}B_1 \tag{26}$$

Now, qLPV system can be obtained as follows,

$$\begin{bmatrix}\dot{\xi}_1\\ \dot{\xi}_2\end{bmatrix} = \begin{bmatrix}0 & A_{12}\\ 0 & \tilde{A}_{22}\end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} + \begin{bmatrix}B_1\\ \tilde{B}_2\end{bmatrix}v + \begin{bmatrix}0\\ E_2\end{bmatrix}\dot{p} \tag{27}$$

where $E_2 = -\frac{\partial \bar{x}_2}{\partial p}$. Finally, Eq. (20) is transformed to a qLPV model in a more concise form as

$$\dot{\xi} = \tilde{A}\xi + \tilde{B}v + E\dot{p} \tag{28}$$

If there are nonlinearities in the non-scheduling variables, the qLPV system becomes a first-order approximation along the family of equilibrium points [29]. If reliable measurement is not available for $\dot{p}$, it can be treated as a disturbance to be rejected.

3.2 Embedding

Dynamics of morphing aircraft given in Eq. (1-4) falls in a class of nonlinear systems given in Eq. (20) by defining the state, input, and parameter as

$$x_1 = \begin{bmatrix}V_T\\ \alpha\end{bmatrix}, x_2 = \begin{bmatrix}\gamma\\ q\end{bmatrix}, u = \begin{bmatrix}\delta_t\\ \delta_e\end{bmatrix}, p = \begin{bmatrix}h\\ \eta\end{bmatrix} \tag{29}$$

To eliminate the nonlinear entries in terms of the non-scheduling variable x_2, a first-order approximation with respect to a trim value is performed for the flight path angle γ. Assuming that flight path angle is small, we have

$$\cos\gamma \approx 1, \quad \sin\gamma \approx \gamma \tag{30}$$

Given altitude, airspeed, and morphing configuration, a typical trimming subroutine for straight and level flight with constant airspeed has three independent variables: angle of attack, throttle, and elevator deflection. Unfortunately, there is no guarantee that one can always find the equilibrium given arbitrary x_1 and p. Thus, in this study, nearest trim point is found in terms of the angle of attack in every time step, which is used to form a qLPV model.

In the trim condition, following result is obtained because $\bar{\gamma}$ and $\bar{q}$ are zero.

$$\frac{\partial \bar{x}_2}{\partial x_1} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \tag{31}$$

$$\frac{\partial \bar{x}_2}{\partial p} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \tag{32}$$

Let us define the system matrices as follows,

$$A_{12} = \begin{bmatrix}-g\cos\bar{\gamma} & -\frac{QS\frac{\bar{c}}{2V_T}C_{D_q}}{\mathrm{m}}\\ -\frac{g\sin\bar{\gamma}}{V_T} & -\frac{QS\frac{\bar{c}}{2V_T}C_{L_q}}{\mathrm{m}V_T}+1\end{bmatrix} \tag{33}$$

$$A_{22} = \begin{bmatrix}\frac{g\sin\bar{\gamma}}{V_T} & \frac{QS\frac{\bar{c}}{2V_T}C_{L_q}}{\mathrm{m}V_T}\\ 0 & \frac{QS\bar{c}\frac{\bar{c}}{2V_T}C_{m_q}}{J_y}\end{bmatrix} \tag{34}$$

$$B_1 = \begin{bmatrix} \frac{T_{\max}\cos\alpha}{\mathrm{m}} & -\frac{QSC_{D_{\delta_e}}}{\mathrm{m}} \\ -\frac{T_{\max}\sin\alpha}{\mathrm{m}V_T} & -\frac{QSC_{L_{\delta_e}}}{\mathrm{m}V_T} \end{bmatrix} \tag{35}$$

$$B_2 = \begin{bmatrix} \frac{T_{\max}\sin\alpha}{\mathrm{m}V_T} & \frac{QSC_{L_{\delta_e}}}{\mathrm{m}V_T} \\ 0 & \frac{QS\bar{c}C_{m_{\delta_e}}}{J_y} \end{bmatrix} \tag{36}$$

Now, qLPV model of the morphing aircraft can be obtained as in the form of Eq. (27). Note that computation of the partial derivatives $\frac{\partial \bar{x}_2}{\partial x_1}$ and $\frac{\partial \bar{x}_2}{\partial p}$ is not required in this case. In general, partial derivatives are computed by performing analytical (desirable if possible) or numerical differentiation. The time rate of exogenous parameter variation $\dot{p}$ is obtained by the following kinematic relation.

$$\dot{h} = V_T \sin\gamma \tag{37}$$

It is assumed that the morphing parameter rate $\dot{\eta}$ is available. Thus, the complete information for the disturbance signal $E\dot{p}$ is available. However, in this case, the term $\dot{p}$ do not explicitly appear in the control design model because E is a zero matrix, although the parameter rate affect the dynamics unless the airframe is completely confined within a trim condition. The possible influence of the parameter rates and exogenous disturbance signal can be treated as unknown disturbances to be rejected, which are implicitly addressed by designing a robust controller. Finally, the LPV system of the following form is considered for the control design.

$$\dot{\xi} = A(\rho)\xi + B(\rho)v \tag{38}$$

4 Control Design

Consider infinite-horizon quadratic cost function of the form.

$$J = \frac{1}{2}\int_0^\infty (\xi^T Q\xi + v^T Rv)dt \tag{39}$$

where $Q \geq 0$ and $R > 0$ are positive semi-definite and positive definite symmetric weighting matrices with proper dimensions. Note that the resulting controller with constant Q and R is automatically scheduled by the parameter ρ. However, Q and R can also be designed as functions of ρ to accommodate the parameter variations. These design parameters add further flexibility to the controller design. The first step is to solve the parameter-dependent ARE of the form.

$$A^T P + PA + Q - PBR^{-1}B^T P = 0 \tag{40}$$

where all the matrices in the ARE are functions of ρ. Therefore, ARE solution P is also a function of ρ. Finally, the near-optimal gain-scheduled feedback solution is obtained as follows,

$$v = -K\xi \tag{41}$$

where

$$K = R^{-1}B^T P \tag{42}$$

The actual control input can be recovered as

$$u = -K\begin{bmatrix} x_1 \\ x_2 - \bar{x}_2 \end{bmatrix} + \bar{u} \tag{43}$$

5 Numerical Simulation

Numerical simulation is performed to demonstrate the effectiveness of the proposed scheme. Actuator saturation and rate limits are considered in the simulation. The initial altitude is 15,000 ft, and the initial Mach number is 0.3. The initial values for the state and input variables are set to the trim values of the corresponding straight- and level-flight condition, and only the pitch rate is deviated -10 deg/s from the equilibrium. The weighing matrices Q and R are set as follows,

$$Q = \mathrm{diag}(2\ \mathrm{m}^{-2}\mathrm{s}^2, 100\ \mathrm{rad}^{-2}, 100\ \mathrm{rad}^{-2}, 10000\ \mathrm{rad}^{-2}\mathrm{s}^2)$$
$$R = \mathrm{diag}(1000, 5000\ \mathrm{rad}^{-2})$$

Simulation results with minimum and maximum span are shown in Fig. 1. It is observed that the transient and steady-state responses are different according to the morphing configuration. Note that the operating angle of attack can be significantly reduced by extending extra span, which is favorable when it comes to reducing the possibility of separation in the high-alpha region.

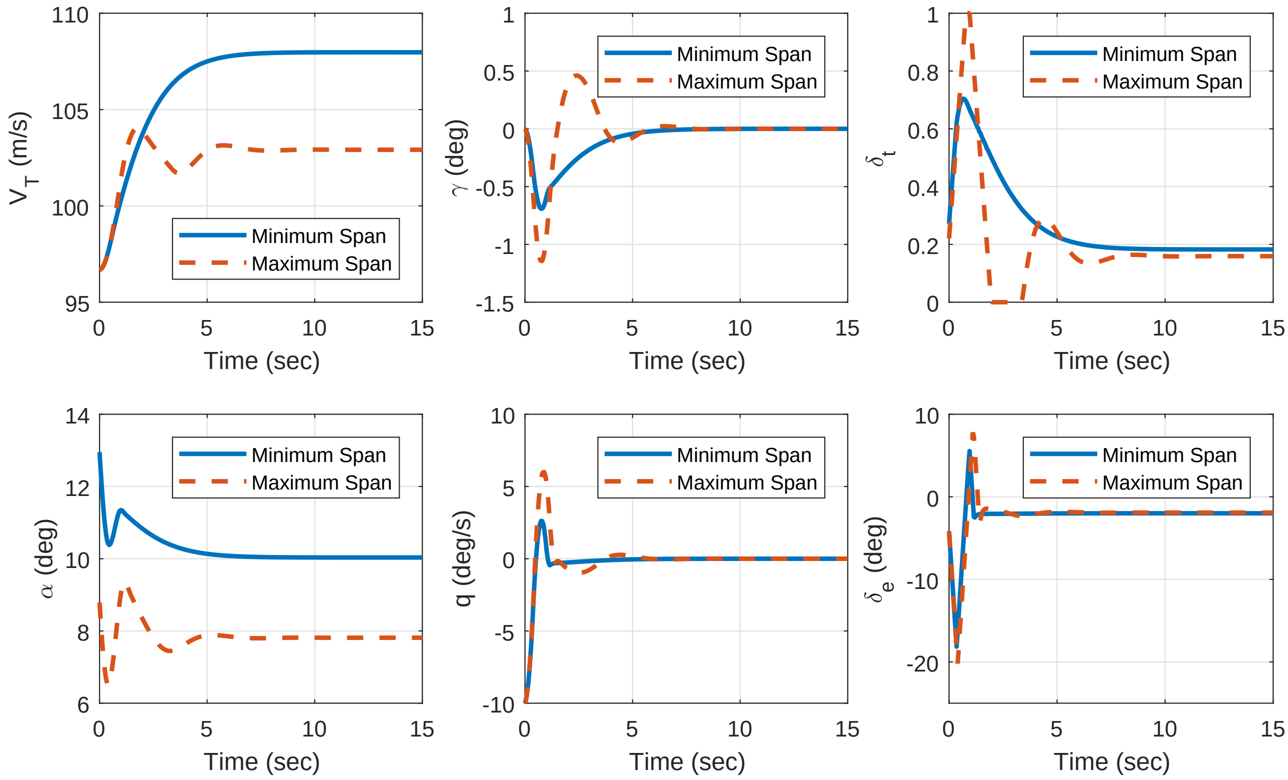

Fig. 1 Simulation Results

6 Conclusion

In this study, nonlinear equations of motion of a morphing aircraft longitudinal dynamics were transformed into a quasi-linear parameter-varying system. At every time step, parameter and state-dependent algebraic Riccati equation is solved to obtain a near-optimal feedback control law. Numerical simulation results show that the proposed controller succeeds in stabilizing the longitudinal dynamics of the morphing aircraft. Furthermore, there is a room for possible improvement of the control performance by utilizing parameter-dependent weighing matrices. Note that the proposed scheme eliminates the need for gain scheduling while covering a wide range of the flight conditions. Furthermore, the unnecessary dependency of the control gain on the non-scheduling variables can be removed. Because there is no assumption other than the small flight path angle, the proposed controller almost fully captures the nonlinearity of the original system.

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References

[1] Wickenheiser AM, and Garcia E. Aerodynamic modeling of morphing wings using an extended lifting-line analysis. *Journal of Aircraft*, Vol. 44, No. 1, pp. 10-16, 2007.

[2] Reich G, and Sanders B. Introduction to morphing aircraft research. *Journal of Aircraft*, Vol. 44, No. 4, p. 1059, 2007.

[3] Valasek J. *Morphing Aerospace Vehicles and Structures*. John Wiley & Sons, Ltd, Chichester, United Kingdom, 2012.

[4] Sun J, Guan Q, Liu Y, and Leng J. Morphing aircraft based on smart materials and structures: a state-of-the-art review. *Journal of Intelligent Material Systems and Structures*, Vol. 27 No. 17, pp. 2289-2312, 2016.

[5] Joshi SP, Tidwell Z, Crossley WA, and Ramakrishnan S. Comparison of morphing wing strategies based upon aircraft performance impacts. *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference*, Palm Springs, CA, Apr. 2004.

[6] Seigler TM, Neal DA, Bae J, and Inman DJ. Flight control of a variable span cruise missile. *ASME International Mechanical Engineering Congress and Exposition*, Anaheim, CA, Nov. 2004.

[7] Bae J, Seigler TM, and Inman DJ, and Lee I. Aerodynamic and aeroelastic considerations of a variable-span morphing wing. *45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference*, Palm Springs, CA, Apr. 2004.

[8] Bae J, Seigler TM, and Inman DJ. Aerodynamic and static aeroelastic characteristics of a variable-span morphing wing. *Journal of Aircraft*, Vol. 42, No. 2, pp. 528-534, 2005.

[9] Valasek J, Tandale MD, and Rong J. A reinforcement learning - adaptive control architecture for morphing. *Journal of Aerospace Computing, Information, and Communication*, Vol. 2, No. 4, pp. 174-195, 2005.

[10] Valasek J, Doebbler J, Tandale MD, and Meade AJ. Improved adaptive-reinforcement learning control for morphing unmanned air vehicles. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, Vol. 38, No. 4, pp. 1014-1020, 2008.

[11] Valasek J, Lampton A, and Marwaha M. Morphing unmanned air vehicle intelligent shape and flight control. *AIAA Infotech@Aerospace Conference*, Seattle, WA, Apr. 2009.

[12] Seigler TM, Neal DA, Bae J, and Inman DJ. Modeling and flight control of large-scale morphing aircraft. *Journal of Aircraft*, Vo. 44, No. 4, pp. 1077-1087, 2007.

[13] Tong L and Ji H. Multi-body dynamic modelling and flight control for an asymmetric variable sweep morphing UAV. *Aeronautical Journal*, Vol. 118, No. 1204, pp. 683-706, 2014.

[14] Seigler TM, and Neal DA. Analysis of transition stability for morphing aircraft. *Journal of Guidance, Control, and Dynamics*, Vol. 32, No. 6, pp. 1947-1954, 2009.

[15] Baldelli DH, Lee D, Pena RSS, and Cannon B. Modeling and control of an aeroelastic morphing vehicle. *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 6, pp. 1687-1699, 2008.

[16] Yue T, Wang L, and Ai J. Gain self-scheduled H_∞ control for morphing aircraft in the wing transition process based on an LPV model. *Chinese Journal of Aeronautics*, Vol. 26, No. 4, pp. 909-917, 2013.

[17] Yue T, Wang L, and Ai J. Longitudinal linear parameter varying modeling and simulation of morphing aircraft. *Journal of Aircraft*, Vol. 50, No. 6, pp. 1673-1681, 2013.

[18] Jiang W, Dong C, and Wang Q. A systematic method of smooth switching LPV controllers design for a morphing aircraft. *Chinese Journal of Aeronautics* Vol. 28, No. 6, pp. 1640-1649, 2015.

[19] He Z, Yin M, and Lu Y. Tensor product model-based control of morphing aircraft in transition process. *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, Vol. 230, No. 2, pp. 378-391, 2016.

[20] Wu Z, Lu J, Rajput J, Shi J, and Ma W. Adaptive neural control based on high order integral chained differentiator for morphing aircraft. *Mathematical Problems in Engineering*, Vol. 2015, Article ID 787931, pp. 1-12, 2015.

[21] Wu Z, Lu J, Zhou Q, and Shi J. Modified adaptive neural dynamic surface control for morphing aircraft with input and output constraints. *Nonlinear Dynamics*, Vol. 87, No. 4, pp. 2367-2383, 2017.

[22] Wen N, Liu Z, Sun Y, and Zhu L. Design of LPV-based sliding mode controller with finite time convergence for a morphing aircraft. *International Journal of Aerospace Engineering*, Vol. 2017, Article ID 8426348, pp. 1-20, 2017.

[23] Wen N, Liu Z, and Zhu L. Linear-parameter-varying-based adaptive sliding mode control with bounded L_2 gain performance for a morphing aircraft, *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*. on-line publication, Mar. 2018. doi: 10.1177/0954410018764472.

[24] Tong L and Haibo J. Multi-body dynamic modelling and roll control of asymmetric variable sweep morphing aircrafts. *10th IEEE International Conference on Control and Automation*, Hangzhou, China, Jun. 2013.

[25] Williams DE, and Friedland B. Modern control theory for design of autopilots for bank-to-turn missiles. *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 4, pp. 378-386, 1987.

[26] Curry M. *Documentation for a dynamic inversion control law proposed for RFCS*. Dryden Flight Research Center, National Aeronautics and Space Administration, 2008.

https://www.nasa.gov/centers/dryden/history/pastprojects/HARV/Work/NASA2/nasa2.html

[27] Lee J, and Kim Y. Neural network-based nonlinear dynamic inversion control of variable-span morphing aircraft. *7th European Conference for Aeronautics and Space Sciences*, Milan, Italy, Jul. 2017.

[28] Gaspar P, Szabo Z, Bokor J, and Nemeth B. *Robust control design for active driver assistance systems: a linear-parameter-varying approach*. Springer International Publishing AG, Cham, Switzerland, 2017.

[29] Shamma JS. Analysis and design of gain scheduled control systems. Ph.D. Dissertation, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, MA, 1988.

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