FLIGHT DYNAMICS MODELING AND TRIM CURVES OF A CONCEPTUAL SEMI-TANDEM WING VTOL UAV

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Abstract

This work focuses in the flight dynamics modeling of a VTOL Semi-Tandem Wing UAV concept and the study of the transition phase, evaluating the trim curves along the flight regime, that is, from hovering to cruise flight condition. The VTOL UAV concept studied has the main feature of tilting both wing and horizontal tail, along with the rotors on both surfaces. Thus, in order to model the aircraft dynamic system the equations of translational and angular motion are presented. For this aircraft configuration it is appropriate to use the multi-body equations of motion, where the aircraft is divided in parts so that the wing, horizontal stabilizer and rotors are independent entities. Additionally, the success of the transition phase from hovering to cruise and from cruise to hovering can be verified if there is the possibility of the aircraft to trim along the flight speed regime, in other words, if there is a combination of states of motion that keep the aircraft stable from hover to cruise condition. So, the trim curves expressing the states are computed using the minimization of a cost function involving the sum of the squares of some of the states of motion, defined through the equations of motion previously mentioned. Such minimization is performed using the Sequential Simplex algorithm. Lastly, the resulted trim curves are presented.

1 Introduction

The control of a VTOL UAV during transition phase from hovering to cruise flight condition and from cruise to hovering is a difficult task, notably in the semi-tandem configuration due to the tilt movement of the wing and horizontal stabilizer, both with spinning propellers, which results in shifting of the center of gravity and gyroscopic moments. Therefore, the traditional modeling involving the 6 degree-of-freedom rigid body equations would be an oversimplification of the system.

In this way, a more complex formulation is required. So, it is appropriate to use the multi-body equations of motion, where the aircraft is divided in parts so that the wing, horizontal stabilizer and rotors are independent entities. This allows the assessment of the linear and angular momentum for each part, which are subsequently derived to obtain the equations of motion.

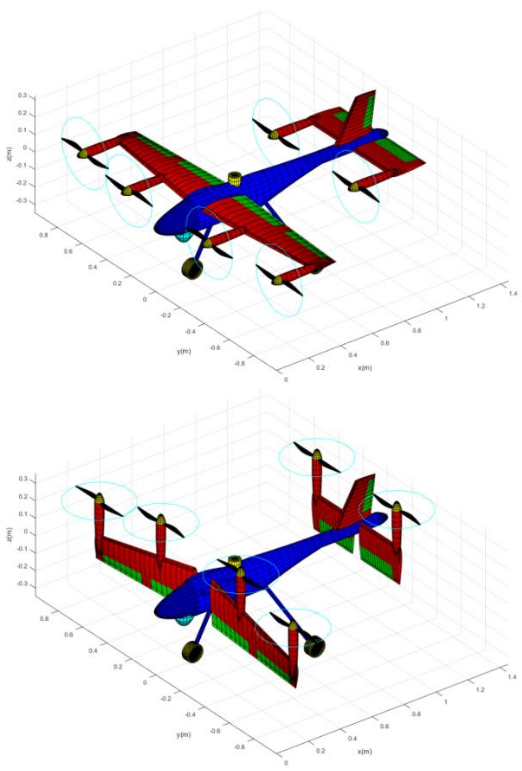

Fig. 1: Concept of VTOL UAV, cruise and hovering condition.

Therefore, a concept of VTOL UAV was designed in order to assess the flight dynamics of such configuration, which can be seen in Fig. 1. Such aircraft would have four propellers at the wing and two propellers at the horizontal tail, and both surfaces would be able to tilt, so that in the cruise configuration both would be horizontally positioned, and in the hover configuration those would be vertically positioned. Such concept is very similar to the configuration studied by Fredericks et al [1].

This aircraft concept has the properties of Table 1.

MTOW (kg)	20
Wing area (m^2)	0.34
Wing span (m)	1.63
Wing aspect ratio	7.8
Wing loading (kg/m^2)	58.8
Horizontal tail area (m^2)	0.15
Vertical tail area (m^2)	0.075
Fuselage length (m)	1.45
Propellers diameter(m)	0.38

Table 1: Aircraft sizing results.

2 Aircraft Dynamic Model

Most flight dynamic analysis uses the hypothesis that the aircraft behaves like a rigid body in the air, with the hypothesis that the mass of such is constant and there are no structural deformations. However, it would be an oversimplification of the system to apply the 6 degree-of-freedom rigid body equations of motion to the concept of aircraft of this work, since the wing and horizontal tail are supposed to tilt along with the spinning rotors, resulting in shifting of the center of gravity and gyroscopic moments.

Therefore, we will present a multi-body equations of motion that are a more appropriate approach, which are much similar to the equations presented in the work of Haixu et al [2]. So, we will be dividing the aircraft in some parts and compute the inertial properties of each. Such parts are: the body, which involves the fuselage, landing gear, vertical tail and all its components; the right and left wing; right and left horizontal stabilizers; and each rotor a separate part. In this way, we also consider that each part has constant mass, even though fuel consumption reduces the body part mass over time, where the fuel tank would be in the fuselage, the weight reduction is too slow to be considered in the dynamic analysis. And finally, no structure deformations are considered, that means that the parts dimensions are constant.

With the previous hypotheses we are able to define the aircraft dynamic system in Fig. 2. In this figure we find the origin of the Earth fixed inertial reference frame O_E, and the origin of the aircraft body coordinate frame O_B, which is the position of the center of gravity of the aircraft body part, that can shift due to the quantity of fuel in the tank, but will not move because of the tilt of the wing or horizontal tail.

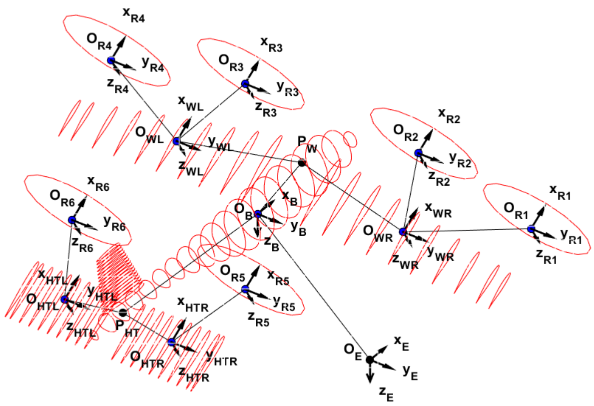

Fig. 2: Reference frames and aircraft dynamic system.

The wing and horizontal tail tilts with respect to the pivot points P_W and P_{HT}, which are fixed, and are positioned on the one quarter chord of the exposed root chords. The wing and horizontal tail are divided in right and left parts, each with its own concentrated mass, positioned in the respective center of gravity, with their own coordinate frame $(O_{WR}, O_{WL}, O_{HTR}, O_{HTL})$, in this manner, when the wing and horizontal tail tilts along the pivot points, their coordinate frames follows. Lastly, for every rotor in the wing and horizontal tail, there is also a coordinate frame $(O_{R1}, \ldots, O_{R6})$, which are fixed with respect to their wing or horizontal stabilizer coordinate frame.

Having the aircraft dynamic system model, we may proceed deriving the translational and angular equations of motion.

3 Equations of Motion

3.1 Translational Motion

We begin by defining the total linear momentum in the Earth fixed inertial reference frame, as the sum of the linear momentum of each individual part. From now on, the subscript *B* will be referring to the aircraft body part, W_i to the right or left, wing or horizontal stabilizer part, R_j to the rotors, and the superscript will be referring to the reference frame of the vector, where in the following equation, *E* means Earth fixed inertial reference frame. So, the aircraft total linear momentum in the Earth fixed inertial reference frame is,

$$\vec{G}^E_{total} = \vec{G}^E_B + \sum_{i=1}^{NW} \vec{G}^E_{W_i} + \sum_{j=1}^{NR} \vec{G}^E_{R_j} \quad (1)$$

The limits in the sum are *NW* the number of aerodynamic surfaces (in our case 4), and *NR* the number of rotors (6 rotors). Expanding, we find,

$$\vec{G}^E_{total} = m_B \vec{V}^E_B + \sum_{i=1}^{NW} m_{W_i} \vec{V}^E_{W_i} + \sum_{j=1}^{NR} m_{R_j} \vec{V}^E_{R_j} \quad (2)$$

Differentiation of the total linear momentum leads to the force equation in the Earth fixed inertial reference frame, where $\vec{F}^E$ is the net applied force vector.

$$\vec{F}^E = \frac{d}{dt}\left(\vec{G}^E_{total}\right) \quad (3)$$

$$\vec{F}^E = m_B \dot{\vec{V}}^E_B + \sum_{i=1}^{NW} m_{W_i} \dot{\vec{V}}^E_{W_i} + \sum_{j=1}^{NR} m_{R_j} \dot{\vec{V}}^E_{R_j} \quad (4)$$

Now, in order to pass the equation to the body coordinate frame, we will use the theorem of Coriolis to compute the acceleration vector from Earth fixed inertial reference frame to the aircraft body coordinate frame, such derivation can be found at Stevens and Lewis [3].

$$\dot{\vec{V}}^E_B = \dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B \quad (5)$$

Where $\vec{\omega}^B_B$ is the angular velocity vector of frame *B* relative to frame *E*. This is also the aircraft body coordinate frame angular velocity vector,

$$\vec{\omega}^B_B = [P \quad Q \quad R]^T \quad (6)$$

The velocity vector for the concentrated masses of the right and left wing and horizontal tail in the Earth fixed inertial reference frame have additional term due to the relative movement with respect to the aircraft body concentrated mass. From Meriam and Kraige [4] we have the equation of relative acceleration of a moving point A with respect to a moving point B, wherein $\vec{r}_{A/B}$ is the position vector of point A in relation to point B, $\vec{V}_{rel_{A/B}}$ is the relative velocity vector of point A in relation to point B, and $\vec{a}_{rel_{A/B}}$ is the relative acceleration vector of point A in relation to point B.

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}}_B \times \vec{r}_{A/B} + \vec{\omega}_B \times \left(\vec{\omega}_B \times \vec{r}_{A/B}\right) + 2\vec{\omega}_B \times \vec{V}_{rel_{A/B}} + \vec{a}_{rel_{A/B}} \quad (7)$$

So, for the concentrated masses of the right and left wing and horizontal tail we have,

$$\dot{\vec{V}}^E_{W_i} = \dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{W_i/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{W_i/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{W_i/B}} + \vec{a}^B_{rel_{W_i/B}} \quad (8)$$

Similarly, for the concentrated masses of the rotors,

$$\dot{\vec{V}}^E_{R_j} = \dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{R_j/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{R_j/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{R_j/B}} + \vec{a}^B_{rel_{R_j/B}} \quad (9)$$

Thereby, we pass the force equation from the Earth fixed reference frame to the aircraft body coordinate frame.

$$\vec{F}^B + m_B B^B_E \vec{g}^E + \sum_{i=1}^{NW}\left\{m_{W_i} B^B_E \vec{g}^E\right\} + \sum_{j=1}^{NR}\left\{m_{R_j} B^B_E \vec{g}^E\right\} = m_B\left(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B\right) + \sum_{i=1}^{NW}\left\{m_{W_i}\left(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{W_i/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{W_i/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{W_i/B}} + \vec{a}^B_{rel_{W_i/B}}\right)\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left(\dot{\vec{V}}^B_B + \vec{\omega}^B_B \times \vec{V}^B_B + \dot{\vec{\omega}}^B_B \times \vec{r}_{R_j/B} + \vec{\omega}^B_B \times \left(\vec{\omega}^B_B \times \vec{r}_{R_j/B}\right) + 2\vec{\omega}^B_B \times \vec{V}^B_{rel_{R_j/B}} + \vec{a}^B_{rel_{R_j/B}}\right)\right\} \quad (10)$$

Note that the terms added in the left side of the equation are vectors of weight of aircraft body, right and left wing or horizontal tail concentrated masses and rotors concentrated masses. Moreover, the vectors of weight use the rotation matrix from Earth fixed referential frame to body coordinate frame B^B_E, which is a function of the Euler angles: roll (ϕ), pitch (θ)

and yaw (ψ). The definition of this matrix is found at Stevens and Lewis [3].

We define now the relative velocity and acceleration vectors in the aircraft body coordinate frame. In the following equations $pivot_i$ and $pivot_j$ means the respective pivot point of the concentrated masses.

$$\vec{V}^B_{rel_{W_i/B}} = \dot{R}^B_{W_i}\vec{r}_{W_i/pivot_i} \quad (11)$$

$$\vec{V}^B_{rel_{R_j/B}} = \dot{R}^B_{R_j}\vec{r}_{R_j/pivot_j} \quad (12)$$

$$\vec{a}^B_{rel_{W_i/B}} = \ddot{R}^B_{W_i}\vec{r}_{W_i/pivot_i} \quad (13)$$

$$\vec{a}^B_{rel_{R_j/B}} = \ddot{R}^B_{R_j}\vec{r}_{R_j/pivot_j} \quad (14)$$

Wherein $\vec{r}^B_{W_i/pivot_i}$: Position vector of right or left wing or horizontal tail concentrated mass relative to respective pivot point. $\vec{r}^B_{R_j/pivot_j}$: Position vector of rotor concentrated mass relative to respective pivot point. Wing and horizontal tail tilt matrix with respect to wing or horizontal tail tilt angle (δ_W, δ_{HT}),

$$R^B_{W,HT} = \begin{bmatrix} \cos(\delta_{W,HT}) & 0 & \sin(\delta_{W,HT}) \\ 0 & 1 & 0 \\ -\sin(\delta_{W,HT}) & 0 & \cos(\delta_{W,HT}) \end{bmatrix} \quad (15)$$

Moreover, $\dot{R}^B_{W,HT}$ and $\ddot{R}^B_{W,HT}$ are the first and second derivatives of $R^B_{W,HT}$ with respect to time. Additionally, we will be using the simplification ($\Omega_B = \vec{\omega}^B_B \times$). Being that,

$$\Omega_B = (\vec{\omega}_B \times\ ) = \begin{bmatrix} 0 & -R & Q \\ R & 0 & -P \\ -Q & P & 0 \end{bmatrix} \quad (16)$$

$$\dot{\Omega}_B = (\dot{\vec{\omega}}_B \times\ ) = \begin{bmatrix} 0 & -\dot{R} & \dot{Q} \\ \dot{R} & 0 & -\dot{P} \\ -\dot{Q} & \dot{P} & 0 \end{bmatrix} \quad (17)$$

Rearranging the equation terms, passing the aircraft body acceleration vector in the aircraft body coordinate frame to the left size we have,

$$\dot{\vec{V}}^B_B = -\Omega_B\vec{V}^B_B + \frac{\vec{F}^B_B}{M} + B^B_E\vec{g}^E - F \quad (18)$$

Note that the only difference from the rigid body equations of motion is the term F defined as follows,

$$F = \frac{1}{M}\sum_{i=1}^{NW}\{m_{W_i}[(\dot{\Omega}_B + \Omega_B\Omega_B)\vec{r}_{W_i/B} + (2\Omega_B\dot{R}^B_{W_i} + \ddot{R}^B_{W_i})\vec{r}_{W_i/pivot_i}]\} + \frac{1}{M}\sum_{j=1}^{NR}\left\{m_{R_j}\left[(\dot{\Omega}_B + \Omega_B\Omega_B)\vec{r}_{R_j/B} + \left(2\Omega_B\dot{R}^B_{R_j} + \ddot{R}^B_{R_j}\right)\vec{r}_{R_j/pivot_j}\right]\right\} \quad (19)$$

In the previous equation we have used the following term for the total aircraft mass, being the sum of the concentrated masses of aircraft body, right and left wing and horizontal stabilizers, and rotors.

$$M = m_B + \sum_{i=1}^{NW} m_{W_i} + \sum_{j=1}^{NR} m_{R_j} \quad (20)$$

3.2 Angular Motion

For the aircraft angular motion equation we start by defining the total angular momentum in the Earth fixed inertial reference frame, again, being the sum of the portions of the aircraft body, right and left wing and horizontal stabilizers, and rotors.

$$\vec{H}^E_{total} = \vec{H}^E_B + \sum_{i=1}^{NW}\vec{H}^E_{W_i} + \sum_{j=1}^{NR}\vec{H}^E_{R_j} \quad (21)$$

The terms $\tilde{I}$ are the inertia matrices of the concentrated masses, with the subscript indicating the part, and the superscript the reference frame. Expanding, we have,

$$\vec{H}^E_{total} = \tilde{I}^E_B\vec{\omega}^E_B + \vec{r}_{B/E} \times \left(m_B\vec{V}^E_B\right) + \sum_{i=1}^{NW}\{\tilde{I}^E_{W_i}\vec{\omega}^E_{W_i} + \vec{r}_{W_i/E} \times \left(m_{W_i}\vec{V}^E_{W_i}\right)\} + \sum_{j=1}^{NR}\left\{\tilde{I}^E_{R_j}\vec{\omega}^E_{R_j} + \vec{r}_{R_j/E} \times \left(m_{R_j}\vec{V}^E_{R_j}\right)\right\} \quad (22)$$

The terms $\vec{\omega}$ are angular velocity vector with subscript indicating the part and superscript the reference frame. Passing the angular motion equation in the Earth fixed inertial reference frame to the aircraft body coordinate frame,

$$\vec{H}^B_{total} = \tilde{I}^B_B\vec{\omega}^B_B + \sum_{i=1}^{NW}\{\tilde{I}^B_{W_i}\vec{\omega}^B_{W_i} + \vec{r}_{W_i/B} \times \left(m_{W_i}\vec{V}^B_{W_i}\right)\} + \sum_{j=1}^{NR}\left\{\tilde{I}^B_{R_j}\vec{\omega}^B_{R_j} + \vec{r}_{R_j/B} \times \left(m_{R_j}\vec{V}^B_{R_j}\right)\right\} \quad (23)$$

From Meriam and Kraige [4] we have the equation of relative velocity of a moving point A with respect to a moving point B.

$$\vec{V}_A = \vec{V}_B + \vec{\omega}_B \times \vec{r}_{A/B} + \vec{V}_{rel_{A/B}} \quad (24)$$

Therefore, we have the velocity of the concentrated masses with respect to the aircraft body coordinate frame,

$$\vec{V}_{W_i}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{W_i/B} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \quad (25)$$

$$\vec{V}_{R_j}^B = \vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{R_j/B} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \quad (26)$$

The net torque $\vec{T}_B^B$ acting at the aircraft body coordinate frame comes from the rate of change of angular momentum. The added terms in the left side of the equation are the weights torques of the concentrated masses with respect to the body coordinate frame.

$$\vec{T}_B^B + \sum_{i=1}^{NW}\{\vec{r}_{W_i/B} \times m_{W_i} B_E^B \vec{g}^E\} + \sum_{j=1}^{NR}\left\{\vec{r}_{R_j/B} \times m_{R_j} B_E^B \vec{g}^E\right\} = \frac{d}{dt}\left(\vec{H}_{total}^B\right) \quad (27)$$

Expanding the derivative of the total angular momentum, separated in aircraft body, right and left wings and horizontal stabilizers, and rotors terms, we find,

$$\frac{d}{dt}\left(\vec{H}_B^B\right) = \tilde{I}_B^B \dot{\vec{\omega}}_B^B + \vec{\omega}_B^B \times \left(\tilde{I}_B^B \vec{\omega}_B^B\right) \quad (28)$$

$$\sum_{i=1}^{NW} \frac{d}{dt}\left(\vec{H}_{W_i}^B\right) = \sum_{i=1}^{NW}\left\{\frac{d}{dt}\left(\tilde{I}_{W_i}^B \vec{\omega}_{W_i}^B\right) + \vec{\omega}_B^B \times \left(\tilde{I}_{W_i}^B \vec{\omega}_{W_i}^B\right) + m_{W_i}\left[\dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} \times \left(\vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{W_i/B} + \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}\right) + \vec{r}_{W_i/B} \times \left(\dot{\vec{V}}_B^B + \vec{\omega}_B^B \times \vec{V}_B^B + \dot{\vec{\omega}}_B^B \times \vec{r}_{W_i/B} + \vec{\omega}_B^B \times \left(\vec{\omega}_B^B \times \vec{r}_{W_i/B}\right) + 2\vec{\omega}_B^B \times \dot{R}_{W_i}^B \vec{r}_{W_i/pivot_i} + \ddot{R}_{W_i}^B \vec{r}_{W_i/pivot_i}\right)\right]\right\} \quad (29)$$

$$\sum_{j=1}^{NR} \frac{d}{dt}\left(\vec{H}_{R_j}^B\right) = \sum_{j=1}^{NR}\left\{\frac{d}{dt}\left(\tilde{I}_{R_j}^B \vec{\omega}_{R_j}^B\right) + \vec{\omega}_B^B \times \left(\tilde{I}_{R_j}^B \vec{\omega}_{R_j}^B\right) + m_{R_i}\left[\dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} \times \left(\vec{V}_B^B + \vec{\omega}_B^B \times \vec{r}_{R_j/B} + \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}\right) + \vec{r}_{R_j/B} \times \left(\dot{\vec{V}}_B^B + \vec{\omega}_B^B \times \vec{V}_B^B + \dot{\vec{\omega}}_B^B \times \vec{r}_{R_j/B} + \vec{\omega}_B^B \times \left(\vec{\omega}_B^B \times \vec{r}_{R_j/B}\right) + 2\vec{\omega}_B^B \times \dot{R}_{R_j}^B \vec{r}_{R_j/pivot_j} + \ddot{R}_{R_j}^B \vec{r}_{R_j/pivot_j}\right)\right]\right\} \quad (30)$$

Because of wing and horizontal tail tilt, the inertia matrices of such surfaces and rotors respective to aircraft body coordinate frame are variables. The inertia matrices of the concentrated masses, with respect to the body coordinate frame, are obtained from the inertia matrices with respect to their own coordinate reference frames by translating and rotating the reference. This operation is demonstrated in the next equations for the concentrated masses of the panels and rotors respectively. There we have $[T]$ and $[T]^T$ the inertia rotation matrix and its transpose, and $\tilde{R}^B$ the translation matrix to the body coordinate frame.

$$\tilde{I}_{W_i}^B = [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^B\right)[T]_{W_i}^T \quad (31)$$

$$\tilde{I}_{R_j}^B = [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^B\right)[T]_{R_j}^T \quad (32)$$

Moreover, the angular velocity vector in the body coordinate frame of the part is the sum of the angular velocity vector of the part with respect to its own reference frame, tilted to adjust the reference orientation, summed with the angular velocity vector of the body part with respect to its own reference frame.

$$\vec{\omega}_{W_i}^B = R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B \quad (33)$$

$$\vec{\omega}_{R_j}^B = R_{R_j}^B \vec{\omega}_{R_j}^{R_j} + R_{W_i}^B \vec{\omega}_{W_i}^{W_i} + \vec{\omega}_B^B \quad (34)$$

Where, $[T]_{W_i}$ and $[T]_{R_j}$: Inertia rotation matrix: rotates the wing, horizontal tail or rotor inertia matrix to the aircraft body coordinate frame. $\tilde{R}_{W_i}^B$ and $\tilde{R}_{R_j}^B$: Inertia translation matrix: transfers the wing, horizontal tail or rotor inertia matrix to the aircraft body coordinate frame. $\vec{\omega}_{W_i}^{W_i}$: Right or left wing or horizontal tail concentrated mass angular velocity vector in respect to its own reference frame. $\vec{\omega}_{R_j}^{R_j}$: Rotors concentrated mass angular velocity vector in respect to its own reference frame.

We can assume from axes alignment that the wing and horizontal tail angular velocity vector is fully aligned with the aircraft body y coordinate, so that,

$$R_{W_i}^B \vec{\omega}_{W_i}^{W_i} = \begin{bmatrix} 0 & \dot{\delta}_{W_i} & 0 \end{bmatrix}^T \quad (35)$$

And for the rotors we have,

$$R_{R_j}^B \vec{\omega}_{R_j}^{R_j} = R_{R_j}^B \begin{bmatrix} \omega_{R_j} & 0 & 0 \end{bmatrix}^T \quad (36)$$

So that, ω_{R_j} is the rotor rotation speed.

Additionally we have,

$$[T]_{W_i} = [T]_{W_i}^T = \begin{bmatrix} \cos(\delta_{W,HT_i}) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos(\delta_{W,HT_i}) \end{bmatrix} \quad (37)$$

$$[T]_{R_j} = [T]_{R_j}^T = \begin{bmatrix} \cos\left(\delta_{W,HT_j}\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos\left(\delta_{W,HT_j}\right) \end{bmatrix} \quad (38)$$

And their derivatives,

$$[\dot{T}]_{W_i} = [\dot{T}]_{W_i}^T = \begin{bmatrix} -\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{\delta}_{W,HT_i}\sin(\delta_{W,HT_i}) \end{bmatrix} \quad (39)$$

$$[\dot{T}]_{R_j} = [\dot{T}]_{R_j}^T = \begin{bmatrix} -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\dot{\delta}_{W,HT_j}\sin\left(\delta_{W,HT_j}\right) \end{bmatrix} \quad (40)$$

Also, considering the position vector between the concentrated mass and the reference frame origin being $\vec{r} = [\mathrm{x} \quad \mathrm{y} \quad \mathrm{z}]^T$, we have the inertia translation matrix from the parallel axis theorem,

$$\tilde{R} = \begin{bmatrix} \mathrm{y}^2+\mathrm{z}^2 & -\mathrm{xy} & -\mathrm{xz} \\ -\mathrm{yx} & \mathrm{x}^2+\mathrm{z}^2 & -\mathrm{yz} \\ -\mathrm{zx} & -\mathrm{zy} & \mathrm{x}^2+\mathrm{y}^2 \end{bmatrix} \quad (41)$$

Additionally, it is necessary to derive the following terms,

$$\frac{d}{dt}\left(\tilde{I}_{W_i}^B\vec{\omega}_{W_i}^B\right) = \frac{d}{dt}\left(\tilde{I}_{W_i}^B\right)\vec{\omega}_{W_i}^B + \tilde{I}_{W_i}^B\frac{d}{dt}\left(\vec{\omega}_{W_i}^B\right) \quad (42)$$

So that,

$$\frac{d}{dt}\left(\tilde{I}_{W_i}^B\right) = [\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + [T]_{W_i}m_{W_i}\frac{d}{dt}\left(\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[\dot{T}]_{W_i}^T \quad (43)$$

$$\frac{d}{dt}\left(\vec{\omega}_{W_i}^B\right) = \dot{R}_{W_i}^B\vec{\omega}_{W_i}^{W_i} + R_{W_i}^B\dot{\vec{\omega}}_{W_i}^{W_i} + \dot{\vec{\omega}}_B^B \quad (44)$$

Similarly for the rotors concentrated masses we find,

$$\frac{d}{dt}\left(\tilde{I}_{R_j}^B\vec{\omega}_{R_j}^B\right) = \frac{d}{dt}\left(\tilde{I}_{R_j}^B\right)\vec{\omega}_{R_j}^B + \tilde{I}_{R_j}^B\frac{d}{dt}\left(\vec{\omega}_{R_j}^B\right) \quad (45)$$

$$\frac{d}{dt}\left(\tilde{I}_{R_j}^B\right) = [\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + [T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[\dot{T}]_{R_j}^T \quad (46)$$

$$\frac{d}{dt}\left(\vec{\omega}_{R_j}^B\right) = \dot{R}_{R_j}^B\vec{\omega}_{R_j}^{R_j} + R_{R_j}^B\dot{\vec{\omega}}_{R_j}^{R_j} + \dot{R}_{W_i}^B\vec{\omega}_{W_i}^{W_i} + R_{W_i}^B\dot{\vec{\omega}}_{W_i}^{W_i} + \dot{\vec{\omega}}_B^B \quad (47)$$

Substituting the terms and rearranging we can write the angular motion equation in the simplified form,

$$\vec{T}_B^B + M_P = A\dot{\vec{\omega}}_B^B + B\vec{\omega}_B^B + C\dot{\vec{V}}_B^B + D\vec{V}_B^B + E \quad (48)$$

And the coefficients are,

$$A = \tilde{I}_B^B + \sum_{i=1}^{NW}\left\{[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T - m_{W_i}\vec{r}_{W_i/B}\times\vec{r}_{W_i/B}\times\right\} + \sum_{j=1}^{NR}\left\{[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T - m_{R_j}\vec{r}_{R_j/B}\times\vec{r}_{R_j/B}\times\right\} \quad (49)$$

$$B = \Omega_B\tilde{I}_B^B + \sum_{i=1}^{NW}\left\{[\dot{T}]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + [T]_{W_i}m_{W_i}\frac{d}{dt}\left(\tilde{R}_{W_i}^B\right)[T]_{W_i}^T + [T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[\dot{T}]_{W_i}^T + \Omega_B[T]_{W_i}\left(\tilde{I}_{W_i}^{W_i} + m_{W_i}\tilde{R}_{W_i}^B\right)[T]_{W_i}^T - m_{W_i}\left[\left(\dot{R}_{W_i}^B\vec{r}_{W_i/pivot_i}\times\vec{r}_{W_i/B}\times\right) + \left(\vec{r}_{W_i/B}\times\Omega_B\vec{r}_{W_i/B}\times\right)\right]\right\} + \sum_{j=1}^{NR}\left\{[\dot{T}]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + [T]_{R_j}m_{R_j}\frac{d}{dt}\left(\tilde{R}_{R_j}^B\right)[T]_{R_j}^T + [T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[\dot{T}]_{R_j}^T + \Omega_B[T]_{R_j}\left(\tilde{I}_{R_j}^{R_j} + m_{R_j}\tilde{R}_{R_j}^B\right)[T]_{R_j}^T - m_{R_j}\left[\left(\dot{R}_{R_j}^B\vec{r}_{R_j/pivot_j}\times\vec{r}_{R_j/B}\times\right) + \left(\vec{r}_{R_j/B}\times\Omega_B\vec{r}_{R_j/B}\times\right)\right]\right\} \quad (50)$$

$$C = \sum_{i=1}^{NW}\left\{m_{W_i}\vec{r}_{W_i/B}\times\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\vec{r}_{R_j/B}\times\right\} \quad (51)$$

$$D = \sum_{i=1}^{NW}\left\{m_{W_i}\left[\left(\dot{R}_{W_i}^B\vec{r}_{W_i/pivot_i}\times\right) + \vec{r}_{W_i/B}\times\Omega_B\right]\right\} + \sum_{j=1}^{NR}\left\{m_{R_j}\left[\left(\dot{R}_{R_j}^B\vec{r}_{R_j/pivot_j}\times\right) + \vec{r}_{R_j/B}\times\Omega_B\right]\right\} \quad (52)$$

$$E = \sum_{i=1}^{NW} \Big\{ \Big([\dot{T}]_{W_i} \big(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \big) [T]_{W_i}^{T} + [T]_{W_i} m_{W_i} \frac{d}{dt} \big(\tilde{R}_{W_i}^{B} \big) [T]_{W_i}^{T} + [T]_{W_i} \big(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \big) [\dot{T}]_{W_i}^{T} \Big) R_{W_i}^{B} \vec{\omega}_{W_i}^{W_i} + [T]_{W_i} \big(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \big) [T]_{W_i}^{T} \big(\dot{R}_{W_i}^{B} \vec{\omega}_{W_i}^{W_i} + R_{W_i}^{B} \dot{\vec{\omega}}_{W_i}^{W_i} \big) + \Omega_B [T]_{W_i} \big(\tilde{I}_{W_i}^{W_i} + m_{W_i} \tilde{R}_{W_i}^{B} \big) [T]_{W_i}^{T} R_{W_i}^{B} \vec{\omega}_{W_i}^{W_i} + m_{W_i} \big[\dot{R}_{W_i}^{B} \vec{r}_{W_i/pivot_i} \times \dot{R}_{W_i}^{B} + \vec{r}_{W_i/B} \times \big(2\Omega_B \dot{R}_{W_i}^{B} + \ddot{R}_{W_i}^{B} \big) \big] \vec{r}_{W_i/pivot_i} \Big\} + \sum_{j=1}^{NR} \Big\{ \Big([\dot{T}]_{R_j} \big(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \big) [T]_{R_j}^{T} + [T]_{R_j} m_{R_j} \frac{d}{dt} \big(\tilde{R}_{R_j}^{B} \big) [T]_{R_j}^{T} + [T]_{R_j} \big(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \big) [\dot{T}]_{R_j}^{T} \Big) \big(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \big) + [T]_{R_j} \big(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \big) [T]_{R_j}^{T} \big(\dot{R}_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{R_j}^{B} \dot{\vec{\omega}}_{R_j}^{R_j} + \dot{R}_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} + R_{W_j}^{B} \dot{\vec{\omega}}_{W_j}^{W_j} \big) + \Omega_B [T]_{R_j} \big(\tilde{I}_{R_j}^{R_j} + m_{R_j} \tilde{R}_{R_j}^{B} \big) [T]_{R_j}^{T} \big(R_{R_j}^{B} \vec{\omega}_{R_j}^{R_j} + R_{W_j}^{B} \vec{\omega}_{W_j}^{W_j} \big) + m_{R_j} \big[\dot{R}_{R_j}^{B} \vec{r}_{R_j/pivot_j} \times \dot{R}_{R_j}^{B} + \vec{r}_{R_j/B} \times \big(2\Omega_B \dot{R}_{R_j}^{B} + \ddot{R}_{R_j}^{B} \big) \big] \vec{r}_{R_j/pivot_j} \Big\} \quad (53)$$

$$M_P = \sum_{i=1}^{NW} \{ \vec{r}_{W_i/B} \times m_{W_i} B_E^B \vec{g}^E \} + \sum_{j=1}^{NR} \{ \vec{r}_{R_j/B} \times m_{R_j} B_E^B \vec{g}^E \} \quad (54)$$

Therefore, we have the aircraft angular motion equation in the body coordinate frame,

$$\dot{\vec{\omega}}_B^B = -A^{-1} B \vec{\omega}_B^B - A^{-1} D \vec{V}_B^B - A^{-1} C \dot{\vec{V}}_B^B + A^{-1} \big(\vec{T}_B^B + M_P - E \big) \quad (55)$$

3.3 Transformation between Reference Axes

We have previously defined the equations of translational and angular motion relative to the body coordinate frame, or body axes, it is now necessary to define the equations of motion with respect to wind axes in order to make it easier the introduction of the aerodynamic forces and moments, which are defined with respect to these axes.

The transformation matrix between body axes to wind axes are defined the same way as in Stevens and Lewis [3], so that the velocity vector in wind axes are given by,

$$\vec{V}_B^W = S \vec{V}_B^B \quad (56)$$

Being that,

$$S = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \quad (57)$$

Also, we define the aircraft velocity vector in the aircraft body coordinates frame $(\vec{V}_B^B)$, and its components, and V_T is the flight speed.

$$\vec{V}_B^B = [U \quad V \quad W]^T \quad (58)$$

$$\vec{V}_B^W = [V_T \quad 0 \quad 0]^T \quad (59)$$

Analogously, we have the aircraft angular velocity vector in the wind axes,

$$\vec{\omega}_B^W = S \vec{\omega}_B^B = [P_W \quad Q_W \quad R_W]^T \quad (60)$$

Now, we define the net force vector in the wind axes,

$$\vec{F}_B^W = S \vec{F}_B^B = \begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + S \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} T_j \\ 0 \\ 0 \end{Bmatrix} \right) \quad (61)$$

And the net torque vector,

$$\vec{T}_B^W = S \vec{T}_B^B = \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + S \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j Q_j \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B \begin{Bmatrix} T_j \\ 0 \\ 0 \end{Bmatrix} \right) \quad (62)$$

Being the aerodynamic force vector composed by drag (D), side force (Y) and lift (L), and the aerodynamic roll ($\bar{L}$), pitch (M) and yaw (N) moments. Additionally, we define each propeller thrust T_j and torque Q_j defined as follows,

$$T_j = k_T \omega_j^2 \quad (63)$$

$$Q_j = k_Q \omega_j^2 \quad (64)$$

Moreover, λ_j is each propeller rotation direction index, being 1 for counter-clockwise and -1 for clockwise.

Then, after handling the equations in a manner very similar as described in Stevens and Lewis [3], but with the concentrated masses terms, we finally get to the equations of motion in the wind axes. First for the translational motion,

$$\begin{Bmatrix} \dot{V}_T \\ \dot{\beta} V_T \\ \dot{\alpha} V_T \cos\beta \end{Bmatrix} = -\Omega_W \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} + S B_E^B \vec{g}^E - SF + \frac{1}{M}\begin{Bmatrix} -D \\ Y \\ -L \end{Bmatrix} + \frac{1}{M} S \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) \quad (65)$$

Wherein,

$$\Omega_W = S\Omega_B = \begin{bmatrix} 0 & -R_W & Q_W \\ R_W & 0 & -P_W \\ -Q_W & P_W & 0 \end{bmatrix} \quad (66)$$

Next we define the angular motion equation in the wind axes, making use of the transformation matrix between body axes to wind axes, we get to the following equation,

$$\begin{Bmatrix} \dot{P}_W \\ \dot{Q}_W \\ \dot{R}_W \end{Bmatrix} = -\left(A_W^{-1} D_W + A_W^{-1} S C \dot{S}^T\right) \begin{Bmatrix} V_T \\ 0 \\ 0 \end{Bmatrix} - (\Omega_R + A_W^{-1} B_W) \begin{Bmatrix} P_W \\ Q_W \\ R_W \end{Bmatrix} + S A^{-1} S^T \begin{Bmatrix} \bar{L} \\ M \\ N \end{Bmatrix} + S A^{-1} \sum_{j=1}^{NR} \left(R_{R_j}^B \begin{Bmatrix} \lambda_j k_Q \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} + \vec{r}_{R_j/B} \times R_{R_j}^B \begin{Bmatrix} k_T \omega_j^2 \\ 0 \\ 0 \end{Bmatrix} \right) + A_W^{-1}\left(M_{P_W} - E_W\right) - A_W^{-1} C_W \begin{Bmatrix} \dot{V}_T \\ 0 \\ 0 \end{Bmatrix} \quad (67)$$

Wherein,

$$\Omega_R = S\dot{S}^T = \begin{bmatrix} 0 & -\dot{\beta} & -\dot{\alpha}\cos\beta \\ \dot{\beta} & 0 & \dot{\alpha}\sin\beta \\ \dot{\alpha}\cos\beta & -\dot{\alpha}\sin\beta & 0 \end{bmatrix} \quad (68)$$

Furthermore, we make use of the following simplifications: $SAS^T = A_W$, $SBS^T = B_W$, $SCS^T = C_W$, $SDS^T = D_W$, $SE = E_W$, $SM_P = M_{P_W}$.

4 Aircraft Trim

The aircraft trim condition, or steady-state condition, is the combination of state variables that make all the state derivatives $\dot{V}_T, \dot{\beta}, \dot{\alpha}, \dot{P}_W, \dot{Q}_W, \dot{R}_W$ identically zero. In our dynamic system we have the following state variables: V_T, β, α, ϕ, θ, ψ, P_W, Q_W, R_W, δ_f (flap angle), δ_e (elevator angle), δ_r (rudder angle), δ_{a_L} (left aileron angle), δ_{a_R} (right aileron angle), δ_W (wing tilt angle), $\dot{\delta}_W$, δ_{HT} (horizontal tail tilt angle), $\dot{\delta}_{HT}$, ω_1^2,..., ω_6^2 (propellers angular speed squared). Therefore for each specific flight condition we must be able to find the combination of state variables that meets with all null state derivatives.

We achieve this goal with a numerical algorithm in the following way. First we define a cost function from the sum of the squares of the state derivatives [3] previously mentioned, which is the following equation.

$$\mathcal{J} = \dot{V}_T^2 + \dot{\beta}^2 + \dot{\alpha}^2 + \dot{P}_W^2 + \dot{Q}_W^2 + \dot{R}_W^2 \quad (69)$$

In the trim condition the cost function should be zero, because the state derivatives must be zero. Thus, if we successively compute the value of the cost function for some chosen state vector $\vec{X}$ using the translational and angular motion equations to compute the state derivatives, in order to gradually approach the cost function to zero, it would be possible to find the state vector for the specific flight condition that nullifies the cost function.

An effective algorithm to solve this problem is the Sequential Simplex, described in Walters et al. [5] and Nelder and Mead [6], which is based on the search of optimum from sequential experimentation and measurement of system outcome from a combination of variables. The algorithm starting procedure implemented was the Corner Initial Method, described in Walters et al. [5], and the stopping criterion used was cost function value less than 1e-15. Moreover, for the hover condition we must use the equations of motion in the aircraft body coordinate frame.

5 Trim Results

Trim curves were computed for the conceptual VTOL UAV previously described, being that the aerodynamic forces and moments were estimated using methods from Datcom [7], Hoerner [8] and Houghton and Carpenter [9].

In this paper we will present results only for the steady-state longitudinal flight, where some states are zeros (β, P_W, Q_W, R_W, ϕ, ψ, δ_r, δ_{a_L}, δ_{a_R}), which are inputs for the algorithm. It was also considered flap angle zero to trim the aircraft, in order to reduce the number of variables.

Additionally, it was established that for every flight condition the wing and horizontal tail tilt angle must be as in Fig. 3, which was obtained using the trim algorithm and smoothing the curves for wing and horizontal tilt angle as functions of flight speed, which were used for ensuing recalculation of the other states, for the condition of full load and flight path angle zero. Therefore, for every other flight condition the wing and horizontal tail tilt angle were inputs to compute the other states.

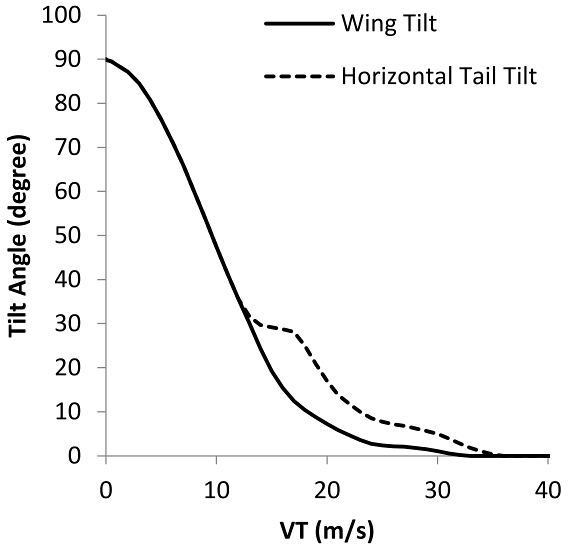

Fig. 3: Wing and horizontal tail tilt angle x flight speed.

The curves express that from 0 to 12 m/s the wing and horizontal tail tilt angle are the same, starting in the hovering condition at 90° where the propellers are all pointing upwards. From 12 m/s forward, the curves differ, being that the wing tilts more until the wing tilt angle becomes zero at 33 m/s, while the horizontal tail zeroes in 36 m/s.

In the Fig. 4 we find the aircraft angle of attack versus flight speed, which is equivalent to the pitch angle for flight angle zero. Next, the elevator angle as a function of flight speed in the Fig. 5. The elevator is only used from 16 m/s forward, since for low flight speeds there is not much dynamic pressure in the aerodynamic controls and the wing and horizontal tail tilt together with propellers thrust is enough to trim the aircraft.

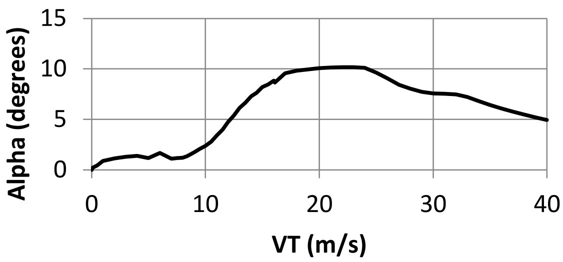

Fig. 4: Aircraft angle of attack x flight speed.

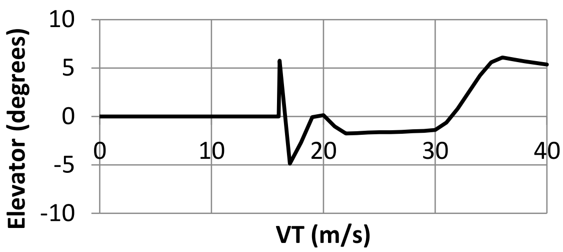

Fig. 5: Elevator angle x flight speed.

For the longitudinal flight it has been determined that the four propellers in the wing will have the same command, thus the same steady power, being the same for the two propellers at the horizontal tail. Propeller performance data was obtained at Brandt, J. B [10], so that we compute the power required. From Fig. 6 we can see that in the hover condition there is high power required from the wing and tail propellers. As the flight speed increases the power required is lowered, since some of the lift force is transferred to aerodynamic lift. Also, from 16 m/s on the tail propellers are turned off and the elevator assumes the role to trim the aircraft. It is necessary to do so because there is a limit on propeller RPM, thus it is not possible to adjust all of the six propellers at the desired advance ratio at high speeds in order to trim the aircraft. In high speeds the aircraft aerodynamic drag increases thus requiring more power from the propellers, therefore, there is a range of flight speed with minimum total power required, which happens between 14 and 16 m/s.

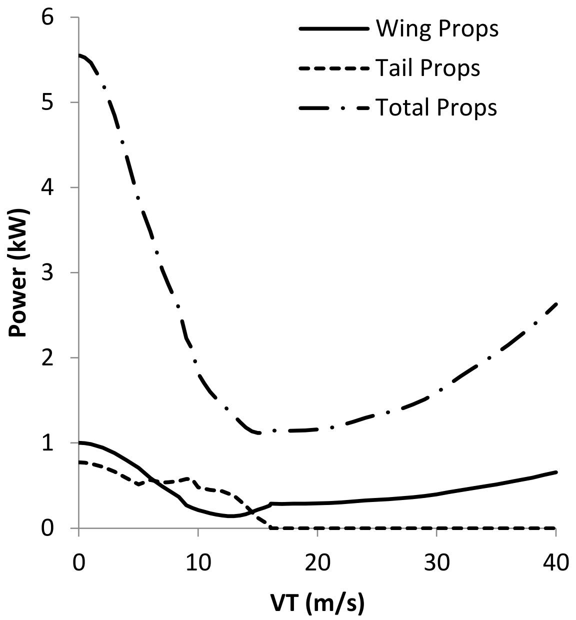

Fig. 6: Total power required and each wing and tail propeller power.

6 Conclusion

The multi-body equations of motion for the proposed VTOL UAV concept were developed which were used to compute the trim curves for the transition phase from hovering to cruise condition. Such computations were performed using the Sequential Simplex algorithm to minimize a cost function.

The results showed that the transition phase is possible for this aircraft concept, provided that the aircraft control system would be able to stabilize the aircraft around the steady-states computed, once subjected to disturbances.

Moreover, the proposed multi-body equations of motions could be applied to various configurations of aircraft with moving aerodynamic surfaces and rotors, in order to study not only the trim conditions, but the flight dynamic performance and control system.

References

[1] Fredericks, W. J., Moore, M. M., Busan, R. C. Benefits of Hybrid-Electric Propulsion to Achieve 4x Increase in Cruise Efficiency for VTOL Aircraft, *AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, 2013.

[2] Haixu, L., Xiangju, Q., Weijun, W. Multi-body Motion Modeling and Simulation for Tilt Rotor Aircraft, *Chinese Journal of Aeronautics*, 2010.

[3] Stevens, B. L., Lewis, F. L. *Aircraft Control and Simulation*, third edition, John Willey and Sons, Inc., 2016.

[4] Meriam, J. L., Kraige, L. G. *Engineering Mechanics Dynamics*, John Willey and Sons, Seventh Edition, 2012.

[5] Walters, F. H.; Parker Jr., L. R.; Morgan, S. L., Deming, S. N. : *Sequential Simplex Optimization: A Technique for Improving Quality and Productivity in Research, Development, and Manufacturing*, CRC Press, Inc., 1991.

[6] Nelder, J. A., Mead, R. A Simplex Method for Function Minimization, *The Computer Journal*, Vol. 7, Issue 4, pp 308–313, 1965

[7] Hoak, D. E. *USAF Stability and Control Datcom*, Douglas Aircraft Company, Inc., 1965.

[8] Hoerner, S. F. *Fluid-Dynamic Drag: Practical Information on Aerodynamic Drag and Hydrodynamic Resistance*, 1965.

[9] Houghton, E. L., Carpenter, P. W. *Aerodynamics for Engineering Students*, Elsevier Butterworth-Heinemann, fifth edition, 2003.

[10] Brandt, J. B. *Small-Scale Propeller Performance at Low Speeds*, M.S. Thesis, Department of Aerospace Engineering, University of Illinois at Urbana-Champaign, Illinois, 2005.

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