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DESTABILIZING EFFECT OF LONGITUDINAL WALL OSCILLATION ON OBLIQUE WAVES IN 2D CHANNEL FLOW

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Abstract

Destabilizing effect of longitudinal wall oscillation of two-dimensional channel flow is investigated numerically and analytically using the direct numerical simulation (DNS) and the Floquet theory. Walls are oscillated in phase and Reynolds number which determined by uniform flow velocity and the half distance between two walls is 10,000. The velocity of the wall oscillation is about 10 % of the uniform flow. Under these condition, DNS analysis shows the earlier transition to the turbulence and this phenomenon is caused by the oblique waves. The analytical results using Floqeut theory which conducted by use of the time-dependent Orr-Sommerfeld equation also support the results mentioned above.

1 Introduction

Laminar-turbulent transition has been studied for a long time and the knowledges obtained from those studies bring many benefits for engineering field, especially for the aircraft industries. In general, the skin friction is large when the boundary layer flow is turbulent. Thus, from the view point of the flow control, our interest is how to decrease the turbulent shear stress, or how to delay the laminar-turbulent transition.

In this context, investigation for the plane Poiseuille flow, so called the channel flow, is a realistic and useful example because it can be described as an exact solution of a linear equation derived from the Navier-Stokes equation, and some theoretical and numerical investigation have revealed its essential futures [1,2]. Thus, there are various studies of the plane Poiseuille flow aiming at the drag reduction. As for the passive control, wavy wall or roughness surface were investigated [3,4]. On the other hand, as active control, wavy walls, vibrating walls, or suction/blowing walls were examined [5-8].

As the study of the active control, Jung et al. [9] firstly pointed out about reduction of the wall shear stress for a turbulent channel flow due to spanwise wall-oscillation. Succeeded study by Quadrio and Ricco [10] numerically demonstrated the friction-drag reduction of 44.7%, which corresponds to the net energy saving of 7.3%. This modified flow not only has the advantage of the amount of the drag reduction, but also has an analogy with simple coupling of the channel flow with the Stokes layer. Thus, many efforts have been devoted to this problem [11,12].

Although this modified channel flow with spanwise wall-oscillation can be simplified based on the Stokes layer, its basic flow is fundamentally three-dimensional flow. Thus, for the purpose of more simplification, modified channel flow with longitudinal wall-oscillation should be studied. From this view point, the authors focused on the stabilizing effect of the longitudinal wall-oscillation on the plane Poiseuille flow. Since the Stokes layer is also an exact solution of the linear equation derived from the Navier-Stokes equation as same as the plane Poiseuille flow, this modified flow can be described a superposition of those two exact solutions. Therefore, the linear stability analysis based on the Floquet theory is applied together with the DNS. That is, the DNS can demonstrate detail character of the flow field and the Floquet analysis can show general feature of the system easily.

In Section 2, the model flow which is dealt at the present study is given. In Section 3, the results of the DNS study are presented. In Section 4, the study using the linear stability analysis based on the Floquet theory is described. Finally conclusions are given in Section 5.

2 Model Flow

Figure 1 shows a model flow considered. Here, Ω and U_w are frequency and amplitude of the longitudinal wall-oscillation. Thus, parameters describing this system are Ω, U_w, and the Reynolds number defined as $Re \equiv h\, U_{max}/\nu$, where U_{max} is the maximum value of the mean flow, ν the kinematic viscosity and h a half distance between two walls. In the present study, Re is fixed as 10000, which is a supercritical one, for convenience.

The coordinate system of (x,y,z) corresponding to the physical space is taken for x in the streamwise direction, y in the direction normal to the wall, z in the spanwise direction.

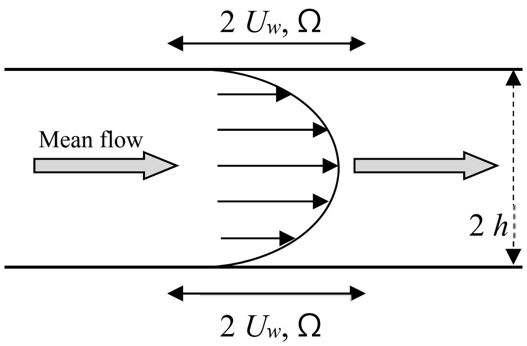

Fig.1 The modified channel flow.

3 DNS Analysis

The numerical space is set as $x \in [0,4\pi]$, $y \in [-1,1]$, $z \in [0,2\pi]$. The flow filed is described as a superimposition of the disturbance $u = \boldsymbol{u}(u,v,w)$ on the basic flow $U(y,t)$. If the pressure can be written as $-2x/Re+p$, the dimensionless equation for u is obtained from the Navier-Stokes equation,

$$\frac{\partial \boldsymbol{u}}{\partial t} + U\frac{\partial \boldsymbol{u}}{\partial x} + v\frac{\partial U}{\partial y}\boldsymbol{e}_x = -\nabla \times \boldsymbol{u} \times \boldsymbol{u} - \nabla p + \frac{1}{R_e}\nabla^2 \boldsymbol{u} \quad (1)$$

here, e_x denotes an unit vector in x the direction. The incompressible condition is adopted. The velocity u is expanded by the Fourier series for x, z directions on the Chebyshev collocation points y_j.

$$\boldsymbol{u}(x,y_j,z,t) = \sum_{k_x,k_z} \boldsymbol{u}(k_x,y_j,k_z,t)\exp[i(k_x x + k_z z)] \quad (2)$$

Then, Eq.(2) is calculated by the Fourier-Chebyshev spectral method [13] for $u(k_x,y_j,k_z,t)$ with the initial disturbances given as,

$$\boldsymbol{u}(k_x,y_j,k_z,0) = \varepsilon \boldsymbol{q}(k_x,y_j,k_z) \quad (3)$$

where ε is a small parameter and q is a random function which satisfied the solenoidal condition. Energy norm for the Fourier modes (k_x,k_z) per unit mass is defined as the follow.

$$E(k_x,k_z) \equiv \frac{1}{4}\int_{-1}^{1} |\boldsymbol{u}(k_x,y,k_z)|^2 dy \quad (4)$$

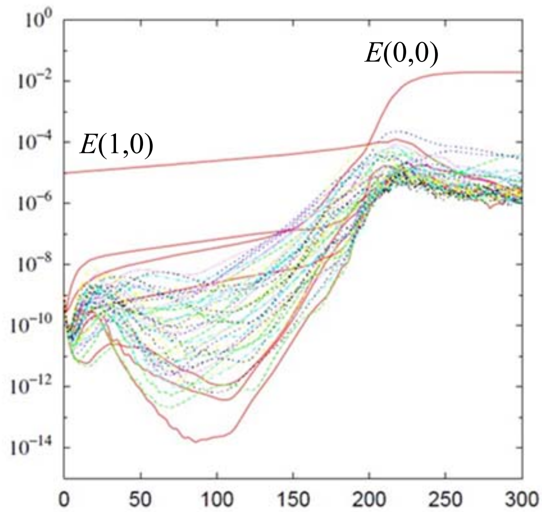

Fig. 2 Variation of energy for each Fourier mode for the case of (Ω, U_w)=(0,0)

A typical results is shown in Fig.2 for the case of (Ω,U_w)=(0.0,0.0) which corresponds to the genuine plane Poiseuille flow. The curves in this figure represent the time variation of energy for each Fourier mode $E(k_x,k_z)$. The solid lines correspond to two-dimensional disturbance, namely $E(k_x,0)$, and the dotted lines correspond to three-dimensional ones. In this calculation, the simulation has been started with the initial disturbances of order 10^9, but a specific disturbance with relatively large amplitude of order 10^5. This large disturbance is a Fourier

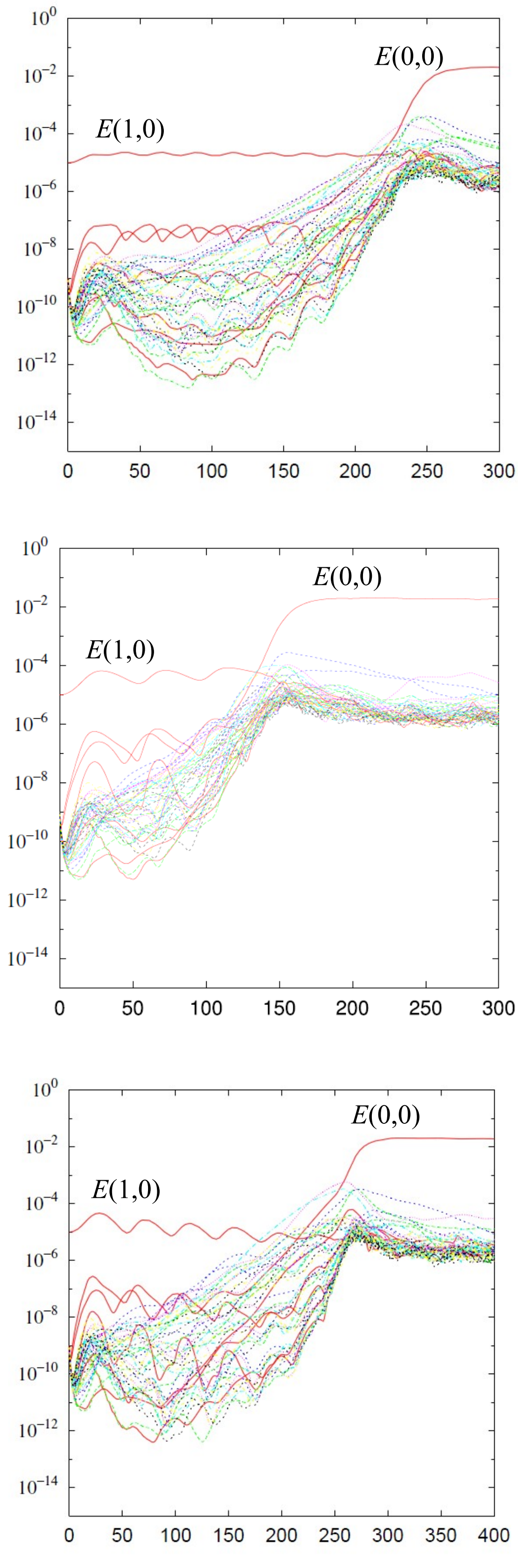

Fig. 3 Variation of energy for each Fourier mode for the case of (a) (Ω, U_w)=(0.25,0.3), (b) (0.15,0.3), (c) (0.15,0.2).

mode of $E(1,0)$, which is called the Tollmien-Schlichting (TS) wave. Because it is well known that the TS mode is dominant and leads to the laminar-turbulent transition under the flow condition considered here, the large TS mode is initially added to the initial disturbance in order to save the computing time-cost. In the present study, all of the simulation examined are including this TS mode.

From this figure, it can be seen that after the transient phase the energy of each mode develop with time and the laminar-turbulent transition occurs at about t=230 in this case.

Some results with wall-oscillation are shown Fig.3 for the case of (Ω,U_w)=(0.25,0.3), (0.15,0.3), and (0.15,0.2). The result of Fig.3 (a) seems to almost same as non-oscillating case of Fig.2 except for oscillation in the time variation of the energy for each Fourier mode. It can be easily supposed that this oscillation is caused by the oscillation of the walls. Actually, it was confirmed that the period of the oscillation appearing on the time variation of the energy coincides with that of the wall-oscillation.

In Fig.3 (b), the laminar-turbulent transition is accelerated and it takes about 150 non-dimensional time for the transition. It seems that the rapidly transition is caused by other than E(1,0) mode, namely the oblique mode. On the other hand, the result of the Fig.3 (c), the transition to turbulent is slightly delayed. In this case, the growth of oblique mode is not strong compared to the case of Fig.3 (b).

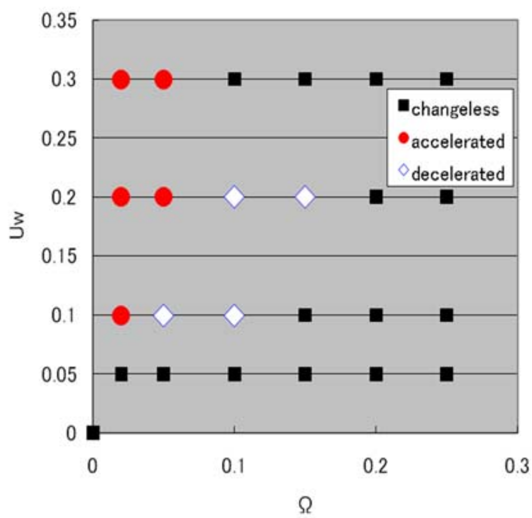

Fig. 4 Results of the parametric study.

It is found from the parametric study that the laminar-turbulent transition of the flow can roughly be grouped in three patterns depending on the wall-oscillation. Result of this parametric study is shown in Fig.4. The circles correspond to the accelerated cases, the diamonds to the decelerated, and square to the less affected cases. It seems that the accelerated cases exist in small Ω region.

4 Floquet Analysis

As mentioned before, the modified flow dealt here can be thought as a superposition of the exact solutions of a linear government equation as the follows,

$$\frac{\partial U}{\partial t} - \nu\left(\frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (5)$$

here ρ is the dencity. This equation is derived from the incompressible Navier-Stokes equation under the parallel flow assumption. In this context, the flow can be represented as $U=U(U(y,t),0,0)$, and $U(y,t)$ is,

$$U(y,t) = 1 - y^2 + U_w \,\mathrm{Re}\left[\frac{\cosh(ky)}{\cosh(k)}\right]\exp(i\Omega t) \qquad (1)$$

here, $k \equiv \sqrt{\Omega/2\nu}$, and i denotes the imaginary unit. The former part of Eq.(6) is contribution of the plane Poiseuille flow, and the latter is the Stokes layer. In the Floquet analysis, Eq.(6) is used as the basic flow.

When the flow field is described by the basic flow $\boldsymbol{U}$ and the small disturbance $\boldsymbol{u}'$ and p', the linearized disturbance equation for $\boldsymbol{u}'$ can be derived from the Navier-Stokes equation as the follows.

$$\frac{\partial \boldsymbol{u}'}{\partial t} + (\boldsymbol{U}\cdot\mathrm{grad})\boldsymbol{u}' + (\boldsymbol{u}'\cdot\mathrm{grad})\boldsymbol{U} = \frac{1}{\rho}\mathrm{grad}\,p' + \nu\nabla^2\boldsymbol{u}' \qquad (7)$$

Now, we assume that the small disturbance can be described as a modal plane wave,

$$\boldsymbol{u}'(x,y,z,t) = \hat{\boldsymbol{u}}(y,t)\exp[i(\alpha x + \gamma z)] \qquad (8)$$

here α, γ are real wave number in x, z direction, respectively. Substituting Eq.(8) into Eq.(7) with the equation of continuity, we obtain time-dependent Orr-sommerfeld equation, which takes the form of

$$\left[\left(\frac{\partial}{\partial t} + i\alpha U(y,t)\right)(D^2-\alpha^2-\gamma^2) - i\alpha D^2U(y,t)\right]\hat{v}(y,t) = \frac{1}{R}(D^2-\alpha^2-\gamma^2)^2\hat{v}(y,t) \qquad (9)$$

where D is the differential operator in y direction.

If Eq.(9) can be rewritten as the form,

$$\frac{\partial}{\partial t}\hat{v}(y,t) = G(y,t)\hat{v}(y,t)\,. \qquad (10)$$

Then using Chebyshec spectral collocation method, Eq.(10) can be shown

$$\frac{d}{dt}\hat{v}(y_j,t) = G_{ij}\hat{v}(y_j,t)\,. \qquad (11)$$

Because of periodicity of the present system, Eq.(11) is rewritten

$$\hat{v}(t) = e^{Qt}\Phi(t) \qquad (12)$$

Here $\Phi(t)$ is an arbitrary periodic function with the period T, and Q consists of N Floquet exponents μ_i. If the real part of μ_i is positive, the system should be unstable.

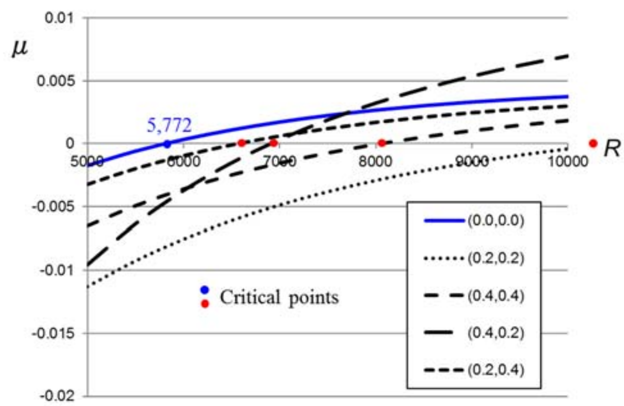

Fig. 5 Variation of μ for E(1,0) with various (U_w,Ω).

Figure 5 shows variation of the Floquet exponent μ_i of 2D TS wave for several cases of (U_w,Ω). The blue line corresponds to the normal 2D cannel flow. The critical Re is 5,772 which is consistent with the result of Orszag [1]. It seems that the effects of wall oscillation are both of stabilize and destabilize depending on the value of (U_w,Ω).

Results of the parametric study is shown in Fig. 6 as a contour map on Ω-U_w plane. The warm color represents the positive area of the Floquet exponent, which corresponds to the unstable region, and the cool corresponds to the stable one. It can be seen that the stable region exists as a deep crevasse along U_w axis. From the comparison with Fig. 4, the bottom of the stable region agrees well with decelerated region estimated by the DNS. Although the unstable region near the U_w axis also corresponds to each other, some portion of the accelerated region by the DNS exists in the stable crevasse. This discrepancy might come from a reason that DNS can reproduce the nonlinear phenomena. From analogy of the Stokes layer, it can easily speculate that the condition with small Ω is equivalent to with large disturbance. Thus some cases estimated as the acceleration by DNS correspond to the transient growth with large disturbances.

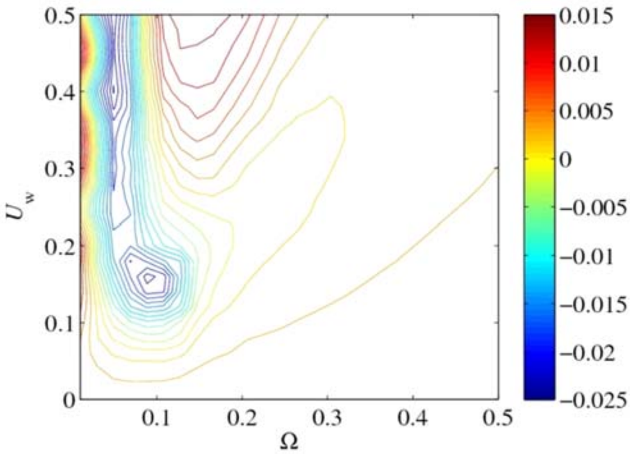

Fig. 6. Results of the parametric study of the Floquet exponents. The warm color corresponds to the unstable, and the cool corresponds to stable.

Figure 7 shows the overtaking of the Floquet exponents of 2D TS mode by the oblique TS mode. There are 4 cases of [(U_w,Ω),(α , γ)]. The overtaking of the oblique TS mode appears no matter whether the two walls oscillates of not. However the wall oscillation suppresses the Floquet exponents and the oblique TS mode can be unstable earlier than the 2D TS mode.

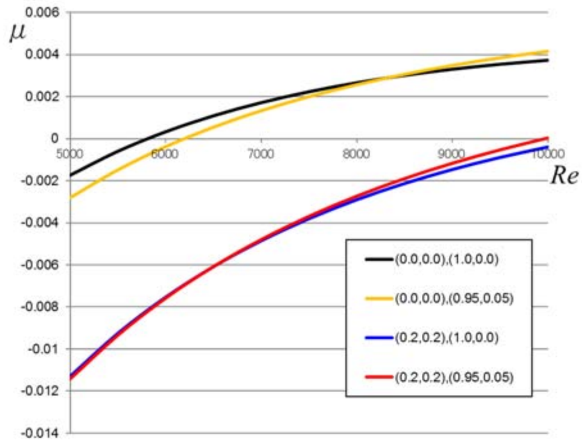

Fig. 7 Variation of μ of 2D:(α,γ)=(1.0,0.0), and 3D:(0.95,0.05) TS modes with/without wall oscillation:(U_w,Ω)=(0.2,0.2).

5 Conclusion

The effect of longitudinal wall-oscillation on the plane Poiseuille flow is studied by direct numerical simulation (DNS) and the Floquet analysis based on the linear stability analysis. From the DNS, the laminar-turbulent transition is accelerated or decelerated depending on the frequency of the wall-oscillation Ω and its amplitude U_w. Also it seems that the acceleration of the transition is caused by the oblique mode. The result obtained by the Floquet analysis agrees well with the DNS analysis and a deep stable crevasse appears in the parameter space. Furthermore, it is shown that the wall oscillation affects more stable and then the oblique TS modes can be rapidly unstable than 2D TS mode.

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