STRUCTURAL DAMPING EFFECTS ON THE ACOUSTIC SCATTERING BY ELASTIC PLATES

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Abstract

In this work we deal with the problem of trailing edge noise scattered by a flat elastic plate. We use a model based on a boundary element method that couples the acoustic problem with the fluid-structure interaction and takes into account structural damping. The solution is obtained using the modal basis of the free vibration problem. The objective of this paper is to expand the acoustic scattering analysis for different damped plates to increase knowledge about the effects of structural damping and to identify potential benefits of using inherently damped structures, such as viscoelastic materials. It is found that there is a range of damping coefficients, capable of reducing peaks in the acoustic spectra associated with structural resonance, while mantaining the reduction of scattered sound due to elasticity. When the damping coefficient is increased above this range, the rigid-plate limit is recovered and acoustic benefits are reduced. The present results allow the selection of optimally-damped structures with respect to acoustic radiation.

1 Introduction

Trailing edges of airfoils and other flow control surfaces are known to be important sources of high frequency sound [1]. When the surface is compliant the turbulent edge-flow also excites structural modes of vibration. In conditions of heavy fluid loading, the energy imparted to the structural motions can be large, and the subsequent scattering of flexural waves at mechanical discontinuities is frequently an important secondary source of sound. The waves scattered by the discontinuities can interfere with the waves that are directly scattered by the trailing edge, and depending on the phase between these waves, this interference can be destructive, which contributes to the reduction of the trailing edge noise; however, when such interference is constructive, as in the case of structural resonance, increases of the radiated sound can be obtained. In our previous work [2] we have developed a model of the acoustic field scattered by a flat poroelastic flexible plate considering the effect of the structural damping, which can potentially alleviate increases of radiated sound due to structural resonance. Calculations were carried out using a boundary element method (BEM) [3]. It was verified that the structural damping modifies the interference patterns through a mechanism of attenuation of the bending waves in the surface, as can be seen in Figure 1, where the plate is represented by a red line and in grayscale we observe the normalized pressure field, which has a harmonic temporal dependence.

Since damping attenuates plate displacements, it may contribute to reduce the significant changes that occur in acoustic scattering when a plate is excited near the resonance conditions. In our previous work [2], we verified that the structural damping can reduce and even eliminate the

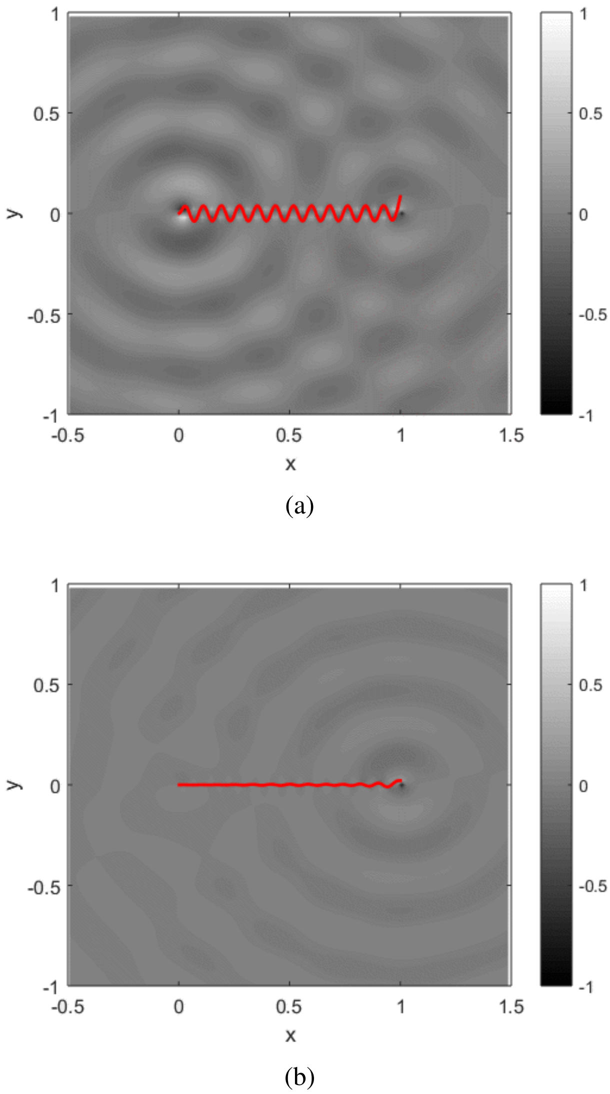


Fig. 1 Scattered pressure fields for an aluminum plate immersed in water ($\varepsilon = 0.135$), with $k_0 = 20$: (a) elastic plate without damping, (b) elastic plate with damping. [2]

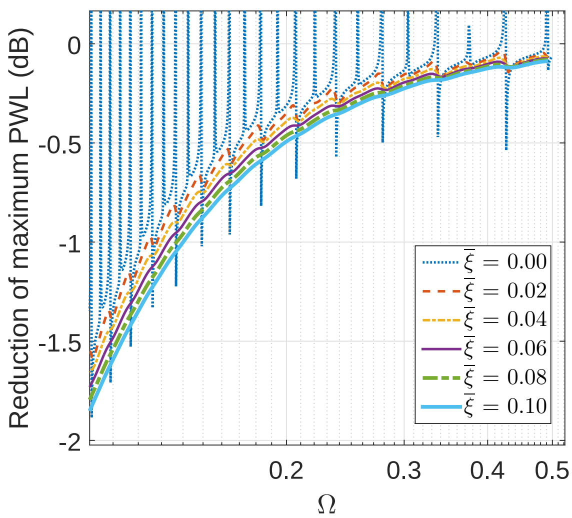


Fig. 2 Change in power level radiated by elastic plates relative to the rigid limit as a function of bending wave Mach number Ω for $k_0 = 10$ and different values of damping. [2]

peaks in far-field spectra, as shown in Figure 2. For the non-compact limit in which the acoustic wavelength is much smaller than the plate, another important conclusion that can also be observed in Figure 2 is that in addition to attenuating the peaks of sound radiation, the damping contributes with a modest reduction of the off-resonance radiated sound (between peaks in spectra).

The objective of this work is to apply the previously developed model, with the proposed structural damping parameter, for different fluid-structure interaction cases and to identify general characteristics of a system that can be represented by this model, such as a potentially critical damping condition above which plate vibrations decrease amplitude. This would potentially allow a selection of optimally damped structures with respect to their acoustic radiation.

2 Mathematical model

The problem of acoustic scattering by a flexible plate is modeled using a point quadrupole source S positioned near the trailing edge of an elastic plate, which has finite chord and infinite span, as shown in Figure 3. The plate has the leading edge clamped and trailing edge free, but other combinations of structural boundary conditions are also possible.

An incident quadrupolar sound field is generated by a turbulent eddy in the vicinity of the trailing edge. A quadrupole in free field is mainly characterized by a near-field pressure that does not propagate to the far acoustic field. However, the proximity of the source and the edge makes the acoustic scattering by the plate considerable. Thus, the near-field pressure is now scattered by

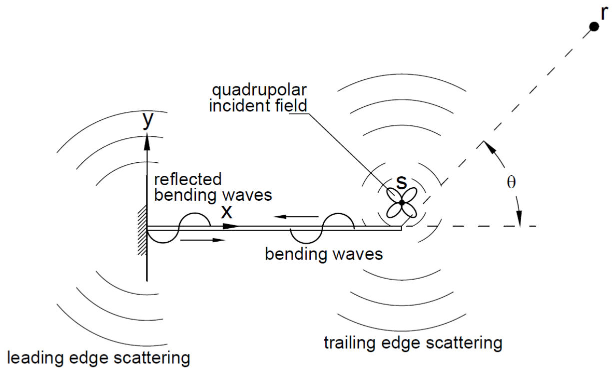


Fig. 3 Schematic of a rectangular elastic plate with finite chord and infinite span, where one edge is clamped along the z axis and the other edge is free, subject to acoustic radiation from source S. [3]

the plate and radiates to the far-field. The far-field noise is increased because the scattered sound radiates as a more efficient field, whose directivity can be a dipole or a cardioid according to the incident frequency [4, 5].

Structural bending waves along the plate are also excited by the source; these waves propagate along the surface, hit the clamped leading edge of the plate and are reflected towards the trailing edge. Furthermore, the leading edge is a structural discontinuity that generates scattered acoustic waves from the incident bending waves. The addition of structural damping can mitigate this phenomenon, as shown in reference [2].

As in [2, 3], to obtain the scattered sound, we solve an inhomogeneous Helmholtz equation,

$$\nabla^2 \tilde{p} + \tilde{k}_0^2 \tilde{p} = -\tilde{S}, \tag{1}$$

where $\tilde{S}$ is the acoustic source function, $\tilde{k}_0$ is the acoustic wavenumber, given as $\tilde{\omega}/\tilde{c}_0$ for angular frequency $\tilde{\omega}$ and speed of sound $\tilde{c}_0$. The overhead tildes indicate dimensional terms, and an $\exp(-i\tilde{\omega}\tilde{t})$ time dependence is implicitly assumed. At the fluid-structure interface, the velocity of the vibrating plate is equal to that of the fluid in its vicinity, so the plate displacement ($\tilde{\eta}$) and fluid displacement are related to the pressure at the plate surface by the linearised Euler equation as

$$\tilde{\rho}_f \tilde{\omega}^2 \tilde{\eta} = \left. \frac{\partial \tilde{p}}{\partial \tilde{y}} \right|_{\tilde{y}=0}, \tag{2}$$

where $\tilde{\rho}_f$ is the fluid density. The equation for a harmonic load applied to a thin elastic damped plate is

$$\tilde{B}\nabla^4 \tilde{\eta} - \tilde{m}\tilde{\omega}^2 \tilde{\eta} - i\tilde{\omega}\tilde{c}\tilde{\eta} = \Delta \tilde{p} \tag{3}$$

where $\tilde{B}$ is the effective bending stiffness of the plate, $\tilde{m}$ is the mass per unit area, $\tilde{c}$ is the damping per unit area, and $\Delta \tilde{p}$ is the applied pressure load in the positive $\tilde{y}$ direction.

In order to obtain non-dimensional equations, we follow the procedure adopted by Jaworski & Peake [6] and Crighton & Innes [7] by defining the coincidence frequency,

$$\tilde{\omega}_c = \sqrt{\frac{\tilde{m}\tilde{c}_0^4}{\tilde{B}}}, \tag{4}$$

the vacuum bending wave Mach number,

$$\Omega = \sqrt{\frac{\tilde{\omega}}{\tilde{\omega}_c}} = \frac{\tilde{k}_0}{\tilde{k}_B}, \tag{5}$$

the intrinsic fluid loading parameter,

$$\varepsilon = \frac{\tilde{\rho}_f \tilde{k}_0}{\tilde{m}\tilde{k}_B^2} = \frac{\tilde{\rho}_f \tilde{c}_1}{\tilde{\rho}_s \tilde{c}_0} \sqrt{\frac{1-2\nu}{12(1-\nu)^2}}, \tag{6}$$

where $\tilde{k}_B$ is the bending wavenumber, $\tilde{c}_1$ is the speed of longitudinal compression waves in the solid, $\tilde{\rho}_s$ is the plate density, related to the mass per unit area as $\tilde{\rho}_s = \tilde{m}/\tilde{h}$, and $\tilde{h}$ is the thickness of the plate. In our previous work [2] we introduced the damping coefficient,

$$2\xi = \frac{\tilde{c}}{\tilde{m}\tilde{\omega}_c}, \tag{7}$$

written in a convenient form as

$$\bar{\xi} = \frac{\xi}{\Omega^2}. \tag{8}$$

To identify the remaining non-dimensional variables, we use the reference chord length of the finite elastic plate, $\tilde{\ell}$,

$$\eta = \tilde{\eta}/\tilde{\ell}, \tag{9a}$$

$$k_0 = \tilde{k}_0 \tilde{\ell}, \tag{9b}$$

$$p = \tilde{p}/(\tilde{\rho}_f \tilde{c}_0^2), \tag{9c}$$

$$S = \tilde{S}/(\tilde{\rho}_f \tilde{c}_0^2 \tilde{\ell}^2), \tag{9d}$$

$$(x,y,z) = (\tilde{x},\tilde{y},\tilde{z})/\tilde{\ell}, \tag{9e}$$

leading to the non-dimensional equations

$$\nabla^2 p + k_0^2 p = -S, \tag{10}$$

$$\nabla^4 \eta - \frac{k_0^4}{\Omega^4}\eta(1+2i\xi) = \varepsilon \frac{k_0^3}{\Omega^6}\Delta p \tag{11}$$

$$k_0^2 \eta - = \frac{\partial p}{\partial y}\bigg|_{y=0}. \tag{12}$$

The acoustic problem is represented by Equation 10 subject to boundary conditions 11 and 12, which relate the pressure and its normal derivative on the plate surface. It is necessary to provide two boundary conditions for the vibration problem 11 at each end of the plate to close the problem. For the cantilever plate configuration of figure 3, these conditions are

$$\eta(0) = \frac{\partial \eta(0)}{\partial x} = \frac{\partial^2 \eta(1)}{\partial x^2} = \frac{\partial^3 \eta(1)}{\partial x^3} = 0. \tag{13}$$

2.1 Solution of problem using a structural modal basis

In order to solve the fluid-structure interaction problem, we rewrite equation 11 as

$$\mathcal{L}(\eta) - \frac{k_0^4}{\Omega^4}\eta(1+2i\xi) = \varepsilon \frac{k_0^3}{\Omega^6}\Delta p. \tag{14}$$

where $\mathcal{L} = \nabla^4$. Now let us consider the auxiliary eigenvalue problem

$$\mathcal{L}(\eta) = \beta^4 \eta, \tag{15}$$

subject to the same boundary conditions of equation 13. Solutions of this eigenvalue problem lead to a complete orthonormal basis ϕ_i for functions satisfying the boundary conditions of the problem, such that

$$\mathcal{L}(\phi_i) = \beta_i^4 \phi_i, \qquad \text{where} \qquad \langle \phi_i, \phi_j \rangle = \delta_{ij}. \tag{16}$$

ϕ_i is called the *modal basis*; these modes are used as an auxiliary basis to solve the fluid-loaded plate problem. The eigenvalues of this problem are real and positive, and β_i is identified as the bending wavenumber of a vibration mode ϕ_i of the plate.

Since the modal basis is a complete orthonormal set for functions satisfying the boundary conditions of the problem, we can use it to write the plate displacement η and from equation 12 obtain the following equation, that relates the pressure difference between the two sides of the plate Δp with the transverse pressure gradient evaluated at the plate surface $\partial p/\partial y|_{y=0}$,

$$\frac{\partial p}{\partial y}\bigg|_{y=0} = \frac{\varepsilon k_0^5}{\Omega^6} \frac{\langle \Delta p, \phi_j \rangle}{\beta^4 - \frac{k_0^4}{\Omega^4}(1+2i\xi)} \phi_j. \tag{17}$$

3 Numerical procedure

3.1 Vibration problem

The eigenvalue problem 16 is solved using a pseudo-spectral method [8]. We employ a discretisation using 321 Chebyshev polynomials. The numerical solution gives values of the modes on the Chebyshev grid; the modes are then sampled using barycentric interpolation at a grid x_i of N points, with locations chosen to be appropriate for the application of the boundary element method for the acoustic problem.

3.2 Boundary element formulation

The problem of acoustic scattering is solved by a boundary element method. A fundamental solution for the Helmholtz equation (eq.1) is the free space Green function, G(**x**,**y**), written for a two-dimensional formulation as

$$G(\mathbf{x},\mathbf{y}) = \frac{i}{4} H_0^{(1)}(k_0|\mathbf{x}-\mathbf{y}|). \tag{18}$$

Here, $H_0^{(1)}$ stands for the Hankel function of the first kind and order zero. Using Green's second identity, one can write the following boundary integral equation

$$T(\mathbf{x})p(\mathbf{x}) = \int_{\Gamma}\left[\frac{\partial p(\mathbf{y})}{\partial n_y}G - \frac{\partial G}{\partial n_y}p(\mathbf{y})\right]d\Gamma - \frac{\partial^2 G}{\partial \mathbf{z}_{i_m}\partial \mathbf{z}_{i_n}}S(\mathbf{z}_i) \quad (19)$$

where $T(\mathbf{x}) = 1/2$ when $\mathbf{x}$ is on a smooth boundary surface Γ, and $T(\mathbf{x}) = 1$ when $\mathbf{x}$ is a field point anywhere in the fluid region. The derivatives with respect to the inward normal direction of the boundary surface are represented by $\partial/\partial n$ and n is an inward unit normal. The i^{th} source location is z_i and the incident quadrupolar field are computed as the second derivative of the Green's function.

The scattering surface, Γ, is discretized into a finite number of elements with polynomial reconstructions for the unknowns in each element. Then, equation 19 is solved for each of these elements through the solution of a linear system of equations, written in compact form as

$$[H]\{p\} - [G]\{\partial p/\partial n\} = \{S\}. \quad (20)$$

The coefficients of matrix $[H]$ are given by $h_{ij} = 1/2$ for $i = j$ and $h_{ij} = \int_{\Gamma}\frac{\partial G(\mathbf{x}_i,\mathbf{x}_j)}{\partial n_j}d\Gamma$ for $i \neq j$. The coefficients of matrix $[G]$ are written as $g_{ij} = \int_{\Gamma}G(\mathbf{x}_i,\mathbf{x}_j)d\Gamma$. The boundary conditions specified on the surface of the plate are calculated using equation 17. One can rewrite the system of equations 20 in terms of the acoustic pressure and obtain a direct solution of the coupled problem. Hence, the new linear system is given by

$$([H] - [G][D])\{p\} = \{S\}, \quad (21)$$

where $[D]$ is a matrix relating $\{p\}$ and $\{\partial p/\partial n\}$.

4 Results

To study the acoustic scattering problem we have calculated the sound radiated by the free edge of an elastic plate near a lateral point quadrupole source of unit intensity. The free edge is located at $(x,y) = (1,0)$, and the quadrupole source is placed at $(x,y) = (1,0.004)$. We calculate the change in acoustic power due to effects of structural damping for observers in the acoustic far-field located 50 chords from the plate trailing edge; the polar angle θ is measured according to the schematic in Figure 3. In this work we have worked with a plate with thickness h equals to 0.2% of its chord. When considering elasticity, we are referring to an aluminium plate immersed in the air, and under these conditions, the fluid loading parameter $\varepsilon = 0.0021$, according to the literature [9]. Following the procedure of Cavalieri *et al* [3] we used one hundred in vacuo bending modes and 802 boundary elements in the plate discretisation for all simulations.

We consider first the scattering of an acoustic quadrupole in the vicinity of the trailing edge of a rigid plate, which can be seen in Figure 4, this case is obtained when the transverse pressure gradient evaluated at the plate surface is null, $\partial p/\partial y|_{y=0} = 0$. Figure 4 also shows results for an elastic undamped plate. An apparent feature is the presence of sharp peaks, which correspond to resonances of the fluid-loaded plates. These ressonance conditions are relevant since they can lead to significant changes to acoustic scattering, with increases or decreases of the radiated sound depending on the phase between acoustic excitation and structural response. Reduction of these sharp increases was our motivation to study the effects of the structural damping on the problem at hand. In what follows some sample values of the acoustic wavenumber will be chosen from the graph of Figure 4, and we will analyze what happens when we add damping by means of the damping coefficient of the equation 7.

The first value chosen for the acoustic wave number is $k_0 = 0.1$, number that represents the compact limit, i.e. the acoustic wavelength is much larger than the characteristic length of the plate. In order to evaluate the damping effect, a sweep in the damping coefficient was performed, and the results are shown in Figure 5. For this value of k_0 it is possible to observe that the damping does not bring any benefit in terms of sound radiation reduction. This is explained by the fact that $k_0 = 0.1$ does not correspond to a resonance point, as can be seen in Figure 4. For $k_0 \ll 1$, it is

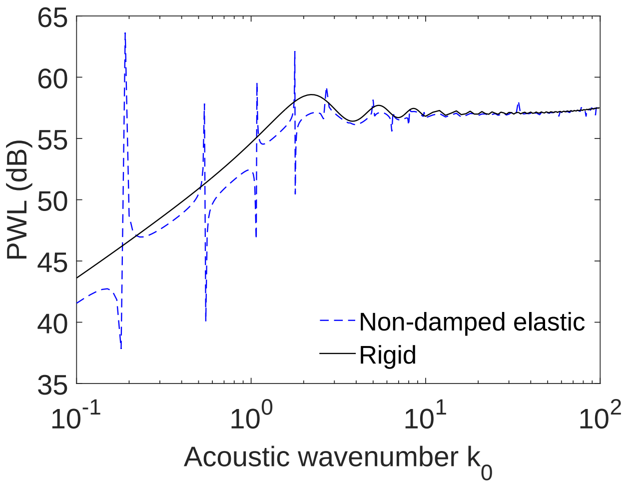


Fig. 4 Sound Power radiated by rigid and non-damped elastic plates as a function of k_0

seen that the damping acts exclusively by attenuating the peaks corresponding to the resonance conditions.

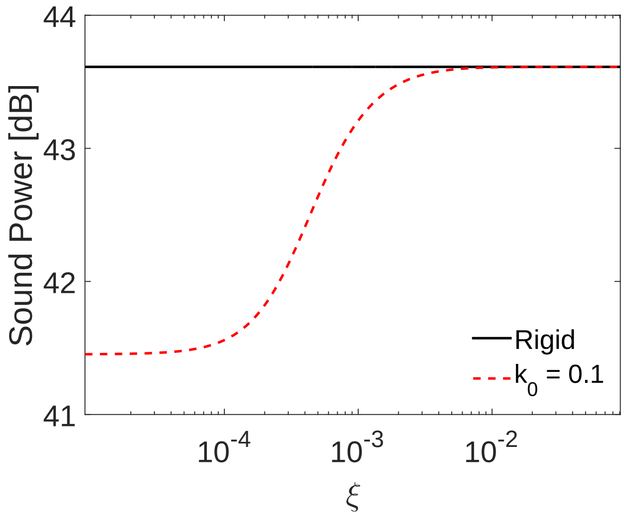


Fig. 5 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 0.1$

Now consider $k_0 = 0.2$, which is in the region of the first resonance peak shown in Figure 4. Results for the damped case are shown in Figure 6. Let us analyze the graph starting from the left; at low values of ξ, we observe that the sound radiation surpasses the rigid case, as we are dealing with a resonance. As we increase ξ, the acoustic scattering reduces to values that are smaller than the rigid case until reaching a minimum value, and as we continue to increase ξ, the behavior of the curve approaches the rigid limit.

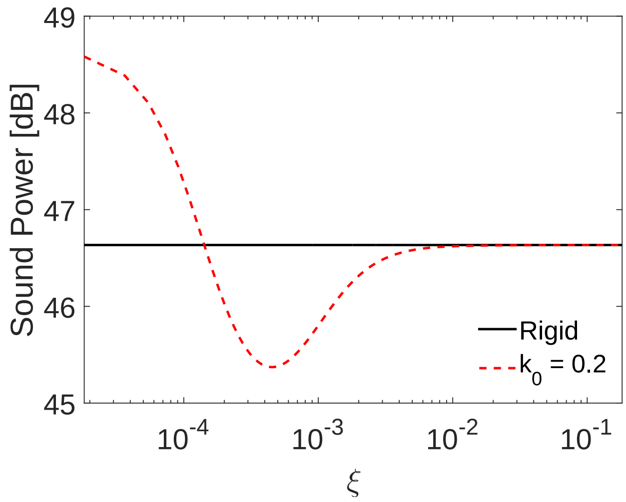


Fig. 6 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 0.2$

To represent the non-compact limit (plate chord considerably larger than the acoustic wavelength), consider $k_0 = 10$, a value that does not correspond to a region of resonance and for which the results are shown in Figure 7. What

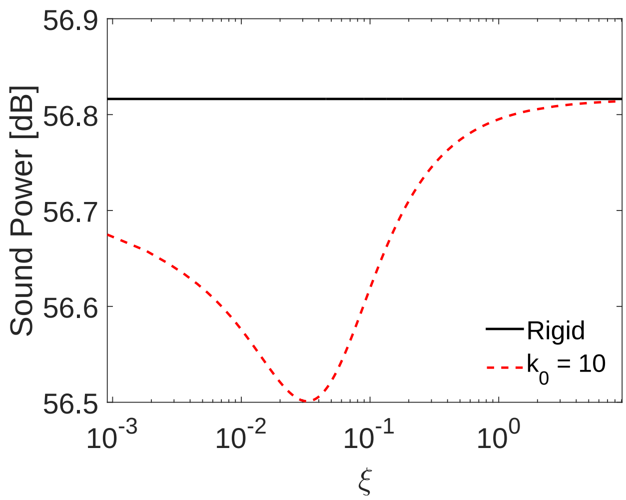


Fig. 7 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 10.0$

is possible to observe is that there is a range of damping values for which the acoustic scattering is lower than that of the purely elastic plate even though it is not a resonance condition. Within this range, there is a damping value for which the reduction of the sound radiation is maximum. Therefore, besides the acoustic benefit given by the elasticity, there is a small contribution given exclusively by the damping. We examine directivities for the three values of k_0 in Figure 8. As the Helmholtz number is increased, the directivity shape moves from that of a compact dipole to a cardioid; backscattering by the leading edge [5, 10] is related to additional lobes in the directivity in Figure 8(c).

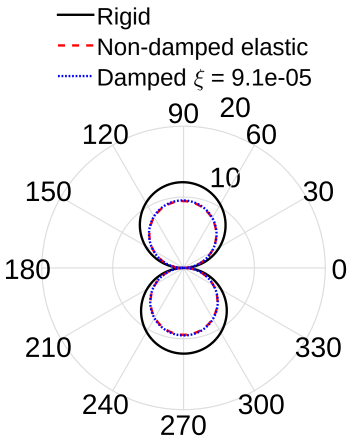
Another feature that can be observed by analyzing the graphs of Figures 5, 6, 7 and also of the figures presented in the Appendix with other values of k_0 is that the rigid limit is always recovered for values of $\xi \geq 1$. To understand why this happens, let us rewrite equation 17 using equation 8,

$$\left.\frac{\partial p}{\partial y}\right|_{y=0} = \frac{\varepsilon k_0^5}{\Omega^6} \frac{\langle \Delta p, \phi_j \rangle}{\beta^4 - \frac{k_0^4}{\Omega^4}(1 + 2i\frac{\xi}{\Omega^2})} \phi_j, \qquad (22)$$

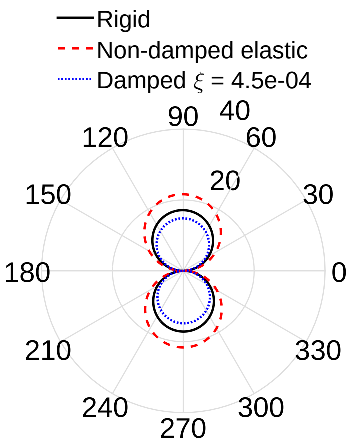
analyzing only the term in parentheses $(1 + 2i\frac{\xi}{\Omega^2})$, we observe that if $\xi \gg \Omega^2$ the absolute value of the denominator in the equation 22 becomes large, and therefore $\partial p/\partial y$ at plate surface approaches zero, which corresponds exactly to the rigid limit.

5 Conclusions

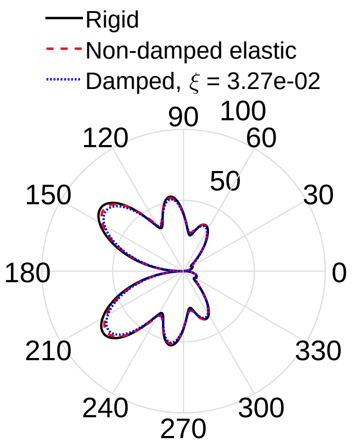
Structural damping is found to produce significant changes in the scattered field of elastic plates. There is a range of damping coefficients, capable of reducing peaks in the acoustic spectra associated with structural resonance, while mantaining the reduction of scattered sound due to elasticity. This is the main effect for $k_0 \ll 1$. However, when analyzing the other extreme, $k_0 \gg 1$, we observe that in addition to the reduction of acoustic scattering due to elasticity there is also a slight reduction that is due exclusively to damping. Another conclusion that has been observed is that the introduction of damping above a certain threshold can cause the behavior of the elastic plate to approximate that observed for a rigid plate, in terms of sound radiation.



(a) $k_0 = 0.1$, compact limit



(b) $k_0 = 0.2$, resonance condition



(c) $k_0 = 10.0$, non-compact limit

Fig. 8 $|p'|$) of the scattered sound by damped plates, compared to the reference non-damped plate and to the rigid limit

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Appendix

Sound power radiated by damped elastic plates with different values of k_0

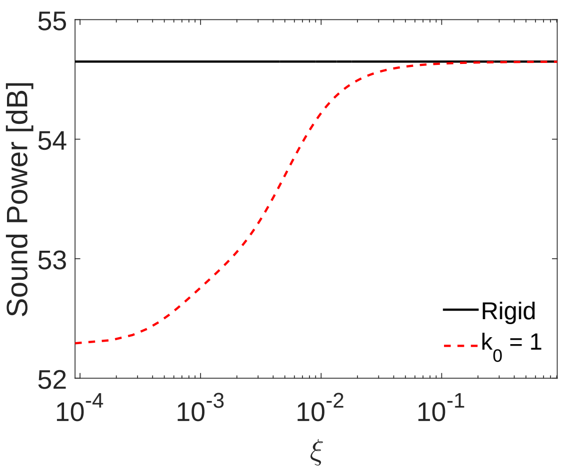

Fig. 9 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 1.0$

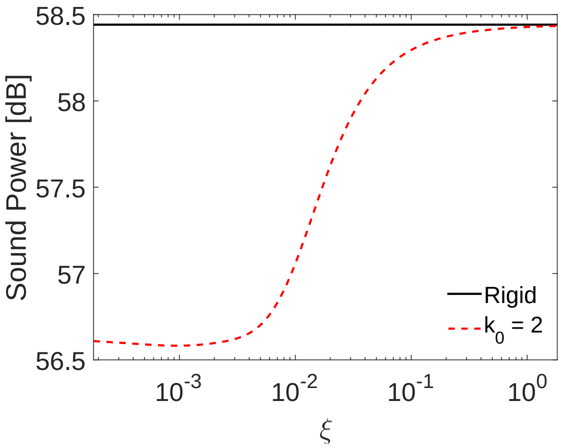

Fig. 10 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 2.0$

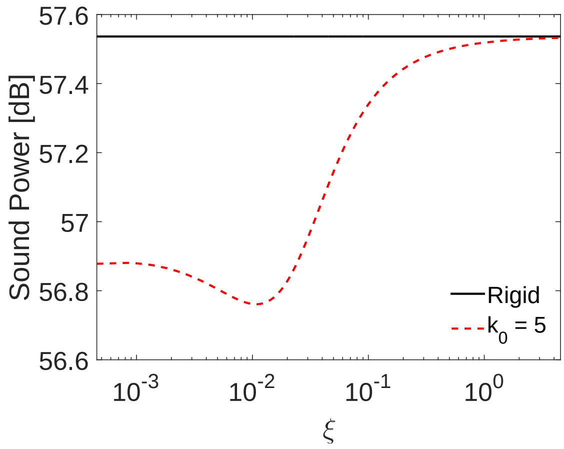

Fig. 11 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 5.0$

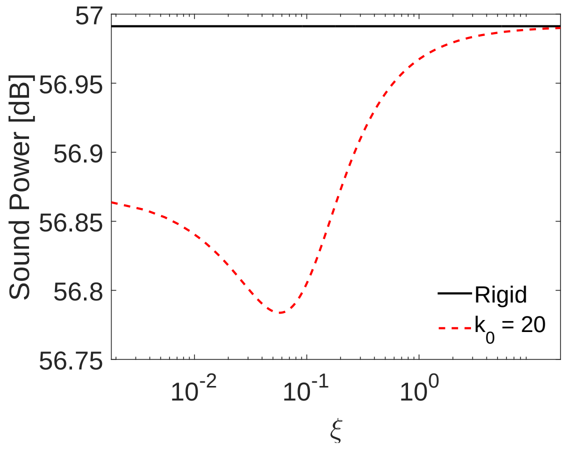

Fig. 12 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 20.0$

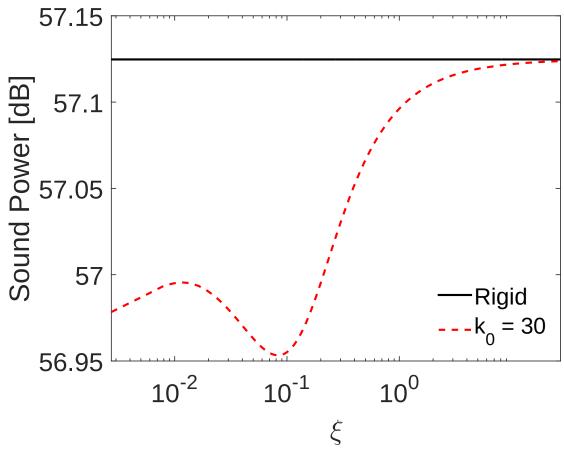

Fig. 13 Sound Power radiated by damped elastic plates as a function of ξ and $k_0 = 30.0$

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