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MODIFIED COMPOSITE GUIDANCE LAW AGAINST MOVING TARGET CONSIDERING NARROW FIELD-OF-VIEW LIMIT

Seokwon Lee* , Sungjun Ann * , Youdan Kim**
*Seoul National University

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Abstract

A missile guidance law is proposed to intercept a non-maneuvering target considering seeker's field-of-view limit . The guidance law is based on the composite structure consisting of look-angle control and pure proportional navigation. The composite guidance law is modified to make the guidance law applicable to air-to-air engagement. Command of the look angle is properly corrected to achieve specified impact angle effectively. Numerical simulation is performed to demonstrate the performance of the proposed algorithm.

1 Introduction

Recently, field-of-view (FOV) constraint in missile guidance has received a lot of interest. For missiles with strapdown seeker, lock-on condition should be satisfied for consistent detection of the target. Moreover, the maneuver of the missile is strictly restricted, because the orientation of the strapdown seeker is fixed to the missile body platform. To deal with the requirement, the guidance law should be designed to maintain the lock-on condition as well as to satisfy terminal objectives. The restricted maneuverability of the missile with strapdown seeker makes the design of guidance law difficult.

To deal with the FOV limit of the missile, a lot of research on guidance schemes have been proposed [1, 2, 3, 4]. In the guidance structure, composite guidance laws consisting of two different guidance commands with switching logic have been proposed. Usually, the first guidance command is generated to make the missile seeker maintain within FOV limit, and the second guidance law is to achieve the interception of the target. Most of the previous studies, however, have focused on stationary target interception.

To treat a moving target, hybrid guidance law was proposed for air defense missile[5], but the impact angle at the terminal interception was not considered. To impose the impact angle constraint, composite guidance law [6] and biased proportional navigation [7] were proposed. Although these works can be well applied to the surface-to-surface engagement, it is required to modify the guidance law to intercept the fast-moving target within a narrow FOV limit.

Aforementioned, it is difficult to intercept the target in air-to-air engagement, because the target is moving with fast speed. It is not easy to design a guidance law considering both the FOV limit and impact angle constraints, and therefore it is desired to extend the guidance algorithm to deal with a moving target. In this study, the composite guidance law is modified to make the guidance law applicable to air-to-air engagement. Look-angle command is automatically adjusted to achievable switching value to satisfy the prescribed impact angle. Using the proposed guidance law, desired trajectory can be obtained as a closed-form solution.

The remainder of the paper is organized as follows. Section 2 presents the problem formulation and engagement kinematics. Modified composite guidance law is proposed in Sec 3. Numerical simulation results are discussed in Sec. 4, and concluding remark is given in Sec. 5.

2 Problem Statement

In this study, a planar engagement between a missile and a target is considered. The missile and the target are moving in an engagement plane as shown in Fig. 1. From the geometry, the engagement kinematics are given by

$$\lambda = \sigma + \gamma_m \tag{1}$$

$$\begin{aligned} \dot{r} &= V_T \cos(\gamma_T - \lambda) - V_m \cos\sigma \\ \dot{\lambda} &= \frac{V_T}{r}\sin(\gamma_T - \lambda) + \frac{V_m}{r}\sin\sigma \\ \dot{\sigma} &= \frac{V_T}{r}\sin(\gamma_T - \lambda) + \frac{V_m}{r}\sin\sigma - \frac{a_m}{V_m} \\ \dot{\gamma}_m &= \frac{a_m}{V_m} \end{aligned} \tag{2}$$

where λ denotes a line-of-sight (LOS) angle, σ is the look-angle of the missile, r represents the distance between the missile and the target, (V_T, γ_T) are the speed and the flight-path angle of the target, respectively, (V_m, γ_m) are the speed and the flight-path angle, respectively, and a_m is the normal acceleration of the missile. The following assumptions are used in this study.

AS 1: The speed of the missile is faster than that of the target, $\eta = V_T/V_m < 1$.

As 2: FOV is narrow enough to confine the collision geometry such that $\sigma_{\lim} < \sin^{-1}\eta$.

The objective of the guidance law is to intercept the retracting and approaching target while achieving a specified impact angle. For the lock-on condition, it is also required that the target is detected by the seeker.

3 Modified Composite Guidance Law

3.1 Guidance Law

Composite guidance law consisting of look-angle control and pure proportional navigation (PPN) has the following switching framework[6].

$$a_{composite} = \begin{cases} V_m\dot{\lambda} + kV_m(\sigma_c - \sigma) & \text{Stage 1} \\ NV_m\dot{\lambda} & \text{Stage 2} \end{cases} \tag{3}$$

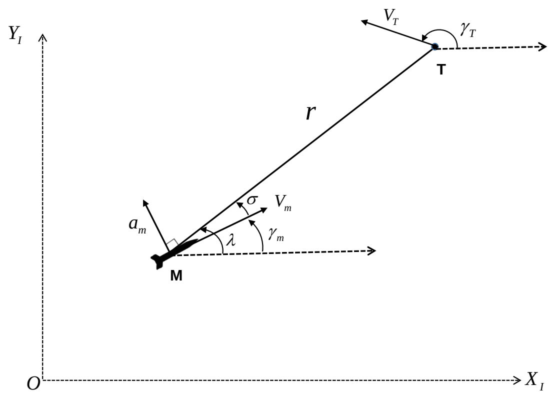

Fig. 1 Engagement Kinematics

where σ_c denotes a look-angle command, and (k, N) are feedback gain and navigation constant, respectively. Figure 2 shows the concept of the composite guidance scheme. In the composite guidance law, turn maneuver is performed by controlling the look-angle. The guidance command is switched to PPN to capture the target in a stable manner during the homing phase. To satisfy the terminal impact angle, LOS angle corresponding to the terminal impact angle can be calculated as

$$\lambda_f = \tan^{-1}\left(\frac{\sin\gamma_f - \eta\sin\gamma_T}{\cos\gamma_f - \eta\cos\gamma_T}\right) \tag{4}$$

where $\eta = V_T/V_m$. Considering the relationship between the angles by PPN, the terminal LOS angle can be rewritten as

$$\begin{aligned} \lambda_s &= \lambda_f - \frac{\gamma_f - \gamma_0}{N} \\ &= \left(\frac{N}{N-1}\right)\left(\lambda_f - \frac{\gamma_f + \sigma^*}{N}\right) \end{aligned} \tag{5}$$

Suppose that the look angle command is properly selected as $\sigma_c = \sigma^*$ and the look angle converges to the command. Then, the transition occurs when the LOS meets the condition of the transition criteria $\lambda = \lambda_s$. After transition, the missile is guided by PPN to intercept the target and eventually achieves terminal impact angle condition.

In Ref. [6], the guidance law is applied to surface-to-surface engagement where the target

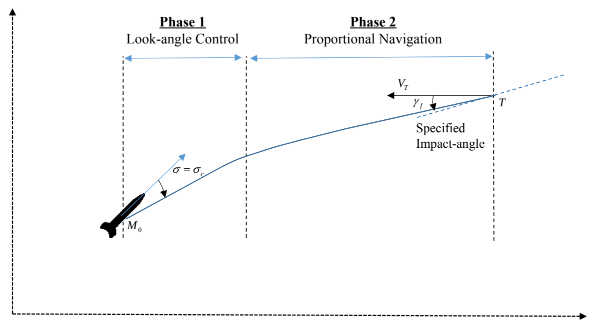

Fig. 2 Concept of the Composite Guidance Scheme

moves along the surface ($\gamma_T = 0, \pi$) and the missile is initiated from the launch point placed at the same altitude with the target such that $\lambda_0 = 0$. In stage 1, the desired look angle is selected as $\sigma_d < -\sin^{-1}\eta$, where the FOV limit is wider than the desired value. In this study, let us consider more general engagement scenario, i.e., $\lambda_0 \in [-\pi, \pi]$. Note that the FOV limit is restricted as $\sigma_{\lim} < \sin^{-1}\eta$.

3.2 Modification of Look-angle Command

To achieve the performance of the guidance scheme, look-angle command should be properly selected. Suppose that the desired look angle is selected within the FOV limit such that $-\sigma_{\lim} < \sigma_c < \sigma_{lim}$. In stage 1, the trajectory is governed by deviated pure pursuit (DPP), which has two equilibrium points in λ kinematics.

$$\lambda_1^* = \gamma_T + \sin^{-1}\left(\frac{V_m}{V_T}\sin\sigma_c\right) \tag{6}$$

$$\lambda_2^* = \gamma_T + \pi - \sin^{-1}\left(\frac{V_m}{V_T}\sin\sigma_c\right) \tag{7}$$

Note that λ_1^* in Eq. (6) is a stable equilibrium point located in the tail-chase engagement, and λ_2^* in Eq. (7) is an unstable equilibrium point in the head-on engagement. Based on the characteristics of DPP, let us modify look angle command.

3.2.1 Tail-chase Engagement

In this domain, the trajectory in stage 1 has stable phase characteristics such that LOS angle converges to the equilibrium point λ_1^*. To finish the engagement with PPN (Stage 2), the LOS angle at the switching time should be between λ_1^* and λ_0. The switching condition in tail-chase engagement can be represented as

$$\lambda_0 \le \lambda_s \le \lambda_1^* \quad \text{or} \quad \lambda_1^* \le \lambda_s \le \lambda_0 \tag{8}$$

If the above condition does not hold, then the desired look angle σ_c should be corrected. Suppose that $\lambda_0 \le \lambda_1^* < \lambda_s$. Then, the condition is satisfied if the look angle is corrected such that $\lambda_1^* - \lambda_s \ge 0$. Using Eqs. (5) and (6), we have

$$\lambda_1^* - \lambda_s = \left(\frac{N}{N-1}\lambda_f - \frac{\gamma_f}{N}\right) - \gamma_T - \left(\frac{\sigma_c}{N-1} + \sin^{-1}\left(\frac{V_m}{V_T}\sin\sigma_c\right)\right) \tag{9}$$

From Eq. (9), $\frac{\partial}{\partial\sigma_c}(\lambda_1^* - \lambda_s) < 0$, and therefore σ_c is corrected to be decreased until the condition satisfies. The proposed algorithm of the look-angle modification logic in tail-chase engagement is summarized in Algorithm 1.

Algorithm 1 Modification of Look-angle command (Tail-chase engagement)

```
procedure LOOK-ANGLE COMMAND CORRECTION
    Initialize : σ_{c,old} = σ_0
    while (1) do
        λ_1^* = γ_f + sin^{-1}((V_m/V_T) sin σ_{c,old})
        λ_{s,σ_{c,old}} = (N/(N-1)) (λ_f - (γ_f + σ_{c,old})/N)
        if (λ_{s,σ_{c,old}} - λ_0)(λ_{s,σ_{c,old}} - λ_1^*) ≥ 0 then
            σ_{c,old} = σ_{c,old} - k sign(λ_f - λ_s)
        end if
        λ_f = obtained from Eq. (4)
        λ_s = λ_{s,σ_{c,old}}
        Return (σ_c, λ_s)
    end while
end procedure
```

3.2.2 Head-on Engagement

In the head-on engagement, the characteristics of the trajectory in stage 1 is unstable, and LOS angle gets away from the equilibrium point λ_2^*. To succeed in the engagement while achieving impact angle γ_f, guidance phase should be switched to the phase 2. The switching condition in head-on engagement can be represented as

$$\lambda_2^* \leq \lambda_0 \leq \lambda_s \quad \text{or} \quad \lambda_s \leq \lambda_0 \leq \lambda_2^* \qquad (10)$$

Suppose that $\lambda_s \leq \lambda_2^* \leq \lambda_0$. Then, the LOS should increase to make λ_2^* greater than λ_0. Taking partial derivative of λ_2^* and λ_s with respect to σ_c yields

$$\frac{\partial \lambda_2^*}{\partial \sigma_c} < \frac{\partial \lambda_s}{\partial \sigma_c} < 0 \qquad (11)$$

Therefore, σ_c is corrected to be decreased to satisfy the condition. The look-angle modification logic in the head-on engagement is summarized in Algorithm 2.

Algorithm 2 Modification of Look-angle command (Head-on engagement)

```
procedure LOOK-ANGLE COMMAND CORRECTION
    Initialize : σ_{c,old} = σ_0
    while (1) do
        λ_2^* = γ_T + π − sin^{−1}( (V_m/V_T) sin σ_{c,old} )
        λ_{s,σ_{c,old}} = ( N/(N−1) ) ( λ_f − (γ_f + σ_{c,old})/N )
        if (λ_{s,σ_{c,old}} − λ_0)(λ_{s,σ_{c,old}} − λ_2^*) ≤ 0 then
            σ_{c,old} = σ_{c,old} − k sign(λ_0 − λ_2^*)
        end if
        λ_f = obtained from Eq. (4)
        λ_s = λ_{s,σ_{c,old}}
        Return (σ_c, λ_s)
    end while
end procedure
```

4 Numerical Simulation

To demonstrate the effectiveness of the proposed guidance law, numerical simulation is performed. Considering air-to-air engagement, the speeds of the missile and the target are selected as $V_m = 2,000 m/s$ and $V_T = 1,000 m/s$, respectively. The acceleration and the FOV limit are chosen as $a_{\max} = 20g$ and $\sigma_{\lim} = 5deg$, respectively. In this study, two simulation scenarios, tail-chase and head-on engagements, are considered to demonstrate the performance. For comparative study, the existing composite guidance [6] law is performed.

Table 1 Simulation results: Tail-chase engagement

	Miss distance (m)	Impact angle error (deg)
Composite guidance	0.98	0.0929 deg
Proposed method	0.31	0.0567deg

4.1 Tail-chase engagement

It is required that the missile intercepts a retracting target with $\gamma_f = 177.5deg$ for tail-chase engagement. Initial condition for the engagement is set as $\lambda_0 = 180$, $\sigma_0 = 4.9deg$, and $r_0 = 20,000m$.

Figures 3-5 show the simulation results for the tail-chase scenario. Both guidance schemes achieve the guidance objectives. The missile maintains the look angle within the FOV limit and reaches the final LOS angle as $\lambda_f = 175deg$. Consequently, the final impact angle γ_f is achieved with small error as shown in Table 1. The composite guidance law steers the look-angle to the prescribed command (maximum value), and therefore the missile performs a maneuver with large turn radius in stage 1. Using the proposed algorithm, on the other hand, the corrected look-angle command makes the missile perform the maneuver with less turn radius while intercepting the target with specified impact angle. The miss distance and the error of the impact angle are summarized in Table 1.

4.2 Head-on engagement

For head-on engagement, it is required that the missile intercepts an approaching target with $\gamma_f = -2.5deg$. Initial condition for the engagement is set as $\lambda_0 = 0$, $\sigma_0 = -4.9deg$, and $r_0 = 25,000m$.

Figures 6-8 show the simulation results for the head-on scenario. As shown in Fig. 8, the existing composite guidance law cannot intercept

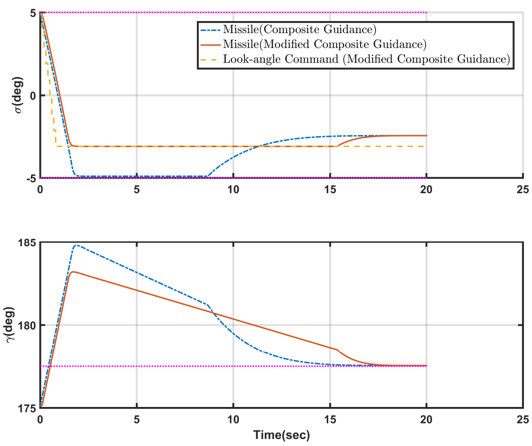

Fig. 3 Time histories of look-angle and flight-path angle (Tail-chase engagement)

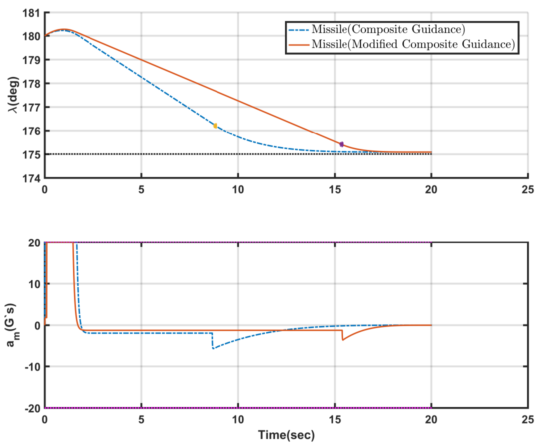

Fig. 4 Time histories of LOS angle and acceleration of the missile (Tail-chase engagement)

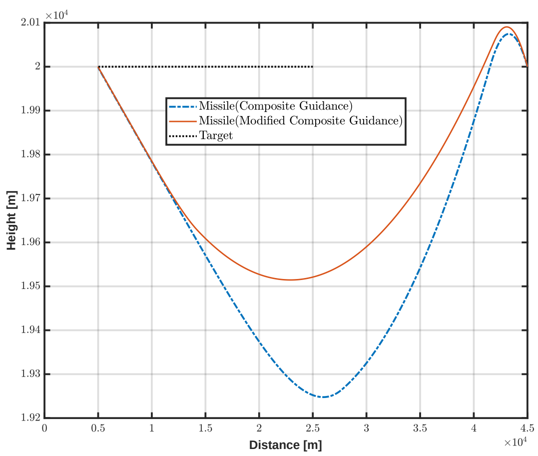

Fig. 5 Trajectories (Tail-chase engagement)

the target. The guidance command steers the look-angle to the prescribed command in stage 1, and therefore the switching of the guidance phase does not occur. Consequently, the guidance law is only done by look angle control, which yields a large miss distance. Using the proposed algorithm, on the other hand, the look-angle command is properly corrected. As a result, the missile intercepts the target while achieving specified impact angle. Table 2 summarizes the miss distance and the error of impact angle.

5 Conclusion

In this study, a modified composite guidance law considering seeker's field-of-view was proposed. Based on the existing guidance law, the look angle command was modified to improve the performance of the existing guidance law, which is applicable in air-to-air engagement. Using the proposed method, the interception with specified impact angle can be completed especially in the case of narrow field-of-view limit, thus the method can be approapriately applied to the anti-air missile.

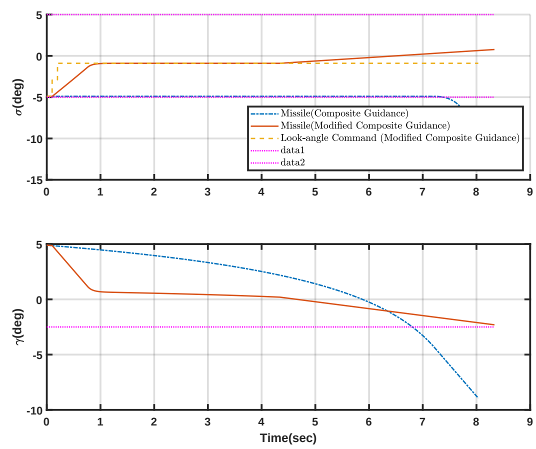

Fig. 6 Time histories of look-angle and flight-path angle (Head-on engagement)

6 Contact Author Email Address

mailto: blueswl@snu.ac.kr

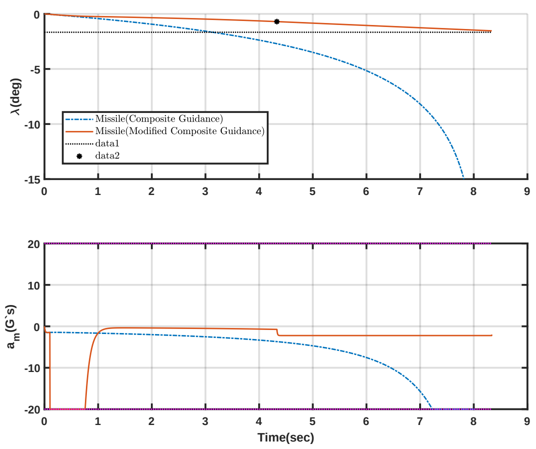

Fig. 7 Time histories of LOS angle and acceleration of the missile (Head-on engagement)

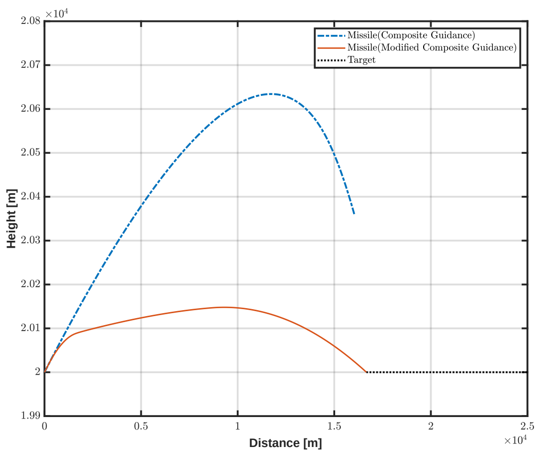

Fig. 8 Trajectories (Head-on engagement)

Table 2 Simulation results: Head-on engagement

	Miss distance (m)	Impact angle error (deg)
Composite guidance	998.9	-6.44
Proposed method	0.05	0.19

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