APPLICATION RESEARCH OF ZIMMERMAN-WEISSENBURGER-YANG METHOD FOR PREDICTING FLUTTER BOUNDARY

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Abstract

In aircraft flutter design, flutter characteristics are usually obtained by flutter analysis, flutter model wind tunnel test and flutter flight test. In this paper, the wind tunnel test data of aircraft T-tail high speed flutter model and the flutter flight test data of civil aircraft are combined. The flutter boundary is predicted by the damping method and the Zimmerman-Weissenburger Flutter Margin method, and the prediction effect of the different flutter types is studied by the damping method and the Zimmerman-Weissenburger method. It is found that the damping method cannot predict the flutter boundary near the flutter boundary, But the Zimmerman-Weissenburger method can predict the flutter boundary when the damping does not change significantly. An improved Zimmerman-Weissenburger method is proposed for multimode coupled flutter phenomena, and the method is named Zimmerman-Weissenburger-Yang method. This new method is applied to predict the modal coupling flutter boundary with multiple degrees of freedom using the aircraft flutter test data, and the results show that the Zimmerman-Weissenburger-Yang method can improve the flutter prediction accuracy.

1 Introduction

In aircraft flutter design, flutter characteristics can be obtained through flutter model wind tunnel test and flutter flight test. The flutter boundary prediction is usually carried out by using damping method, flutter margin method, displacement method and power spectral analysis method in wind tunnel flutter model test and flutter flight test.

Mc Donnell Aircraft Company's Zimmerman and Weissenburger proposed a method for predicting the critical velocity of subcritical flutter in a flutter test, which is known as Zimmerman-Weissenburger (Z-W) method[1]. In order to verify the validity of the Z-W method, Zimmerman calculated the stability criterion forthe wing flutter model and the flight test of a fighter. Based on the Z-W method, Bennett[2], Heeg[3] and Zeng[4]carried out wind tunnel test. Katz[5], Lee[6] and Ju [7] applied the Z-W method to study the flight flutter test data. These studies show that the method can predict the flutter boundary.

In this paper, the wind tunnel test of aircraft T-tail high speed flutter model and the flutter flight test data of civil aircraft are combined, and the flutter boundary is predicted by the damping method and the Z-W method. The prediction effect of the different flutter types is studied by the damping method and the Z-W method. After that, an improved Z-W method is proposed for multi-mode coupled flutter phenomena, and the method is named Zimmerman-Weissenburger-Yang method to predict the modal coupling flutter boundary with multiple degrees of freedom. The prediction capability is validated by the flight flutter test data. The results show that the Zimmerman-Weissenburger-Yang method can improve the accuracy of flutter prediction.

2 Flutter Prediction Method

In aircraft flutter design, the flutter boundary is estimated by damping method and flutter margin Zimmerman-Weissenburger (Z-W) method, and the flutter boundary is predicted.

2.1 Damping Method

Damping method is a common method of flutter boundary prediction, the modal parameters of structural response data are identified, modal damping is obtained, and the flutter boundary is predicted according to the variation of damping with velocity. The relationship between damping and velocity as shown in Fig. 1. the speed at which the damping is zero is determined to be the critical flutter velocity. The main flutter types are explosive flutter, moderate flutter, hump small damping type flutter. For the explosive flutter, the modal damping varies significantly with the velocity, the damping of the flutter point decreases rapidly and at the flutter point the damping is zero. The modal damping changes slowly with the increase of the velocity, but the change is not significant. Hence it is difficult to predict the flutter point accurately. For small damping hump flutter, the modal damping is less than 0.03. As the modal damping first increases and then decreases with the increase of velocity, the modal damping of flutter point does not change obviously, so the damping method cannot predict the flutter type accurately. In the critical flutter point, the explosive flutter damping changes significantly, while the small damping hump-type flutter and moderated flutter damping changes are not significant. Therefore, the damping method is most suitable for predicting the explosive type flutter.

Damping calculation can be written as

$$G = \frac{1}{n\pi} ln\left(\frac{A_0}{A_n}\right)$$

where, n is the cycles, A_0 is the initial amplitude, A_n is the nth periodic amplitude, G is the structural damping of amplitude A_0 over An.

The key to the successful application of damping method is whether the estimation of main modal damping is accurate. It is easy to obtain stable frequency and damping data in wind tunnel test of low-speed flutter model, but it is difficult to obtain stable frequency and damping data in wind tunnel test of high-speed flutter model and of aircraft flight flutter test due to low signal-to-noise ratio. The damping method has two defects in the actual test. First of all, the flutter test cannot avoid the strong noise interference, closely spaced modes and mixing and data processing errors and other factors, the modal damping is difficult to be accurately estimated, and this method only apply damping as the criterion of instability, which is easy to lead to distortion in the estimation of critical velocity. Secondly, generally speaking, the more measured points, the nearer the critical point is, the higher the precision of the velocity damping method is, but in fact, because of the safety and test conditions, the wind tunnel test of the high-speed flutter model and the aircraft flutter test usually have a certain distance from the flutter point, so it is very difficult to get the data near the flutter boundary, The damping method is suitable for predicting the explosive type flutter by subcritical test points.

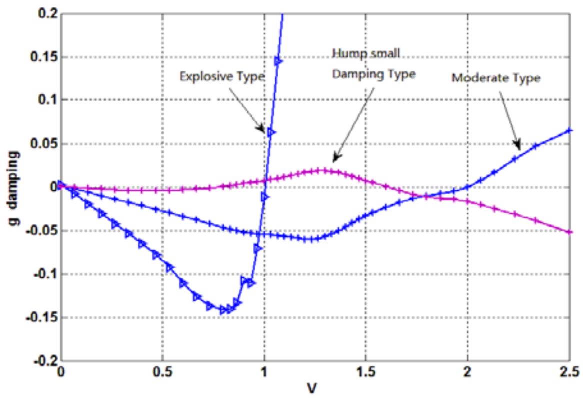

Fig.1 Damping as an indication of flutter onset

2.2 Flutter Margin Z-W Method

The Z-W method is introduced in the paper [1,2,9,10], and the flutter margin function is calculated by using damping and frequency to determine the flutter boundary of the classical bending mode and torsion modal coupling of lifting
surface.

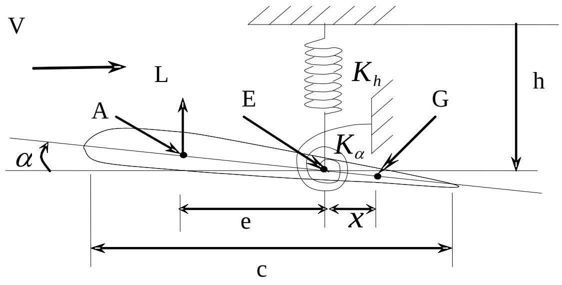

Fig.2 Schematic of simple bending/torsion idealization.

A bending-torsion coupling of two-degree of freedom wing is shown Fig. 2. Where the center of gravity is G, elastic center is E, aerodynamic center is A, h is the plunging displacement, α is pitching displacement. The distance between the center of gravity and the elastic center is x, the chord length is c, K_h is bending stiffness, K_α is torsion stiffness. The lift forces is positive down, and torques about the center of gravity is positive leading edge up.

The plunging (bending) equation of motion is written as

$$m\ddot{h} + k_h h - (k_h x)\alpha + L = 0$$

The pitching (torsion) equation of motion is written as

$$I_c\ddot{\alpha} + (k_\alpha + k_\alpha x^2)\alpha - (k_h x)h - M_c = 0$$

where m is mass, L is aerodynamic force, I_c is rotational inertia represented inertia, M_c is aerodynamic moment about the c.g. $C_{L\alpha}$ is lift curve slope, q is dynamic pressure, V is velocity, a_1, a_2, a_3 is the L force constants, b_1, b_2, b_3 is M_c moment constants.

$$L = C_{L\alpha} q \left[a_1\alpha + a_2\left(\frac{\dot{h}}{V}\right) + a_3\left(\frac{\dot{\alpha}}{V}\right)\right]$$

$$M_c = C_{L\alpha} q \left[b_1\alpha + b_2\left(\frac{\dot{h}}{V}\right) + b_3\left(\frac{\dot{\alpha}}{V}\right)\right]$$

By substituting the aerodynamic force and moment expression into the previous flutter equation as follows

$$m\ddot{h} + C_{L\alpha} q a_2\left(\frac{\dot{h}}{V}\right) + k_h h + C_{L\alpha} q a_3\left(\frac{\dot{\alpha}}{V}\right) + (C_{L\alpha} q a_1 - k_h x)\alpha = 0$$

$$I_c\ddot{\alpha} - C_{L\alpha} q b_3\left(\frac{\dot{\alpha}}{V}\right) + (k_\alpha + k_\alpha x^2 - C_{L\alpha} q b_1)\alpha - C_{L\alpha} q b_2\left(\frac{\dot{h}}{V}\right) - (k_h x)h = 0$$

Using P representing $\frac{d}{d_t}$, they can be rewritten as

$$\left\{P^2 + \left(\frac{a_2}{m}\right)\left(\frac{C_{L\alpha} q}{V}\right)P + \left(\frac{k_h}{m}\right)\right\}h + \left\{\left(\frac{a_3}{m}\right)\left(\frac{C_{L\alpha} q}{V}\right)P + \left(\frac{a_1}{m}\right)(C_{L\alpha} q) - \left(\frac{k_h x}{m}\right)\right\}\alpha = 0$$

$$\left\{\left(\frac{b_2}{I_c}\right)\left(\frac{C_{L\alpha} q}{V}\right)P + \left(\frac{k_h x}{I_c}\right)\right\}h - \left\{P^2 - \left(\frac{b_3}{I_c}\right)\left(\frac{C_{L\alpha} q}{V}\right)P + \left[\left(\frac{k_\alpha + k_h x^2}{I_c}\right) - \left(\frac{b_1}{I_c}\right)(C_{L\alpha} q)\right]\right\}\alpha = 0$$

By proper operation, these two equations can be reduced to an equation with one α variables by eliminating h, such as

$$(P^4 + A_3P^3 + A_2P^2 + A_1P + A_0)\alpha = 0$$

$$A_0 = K_{01}(C_{L\alpha} q) + K_{02}$$

$$A_1 = K_{11}\frac{(C_{L\alpha} q)^2}{V} + K_{12}\frac{C_{L\alpha} q}{V}$$

$$A_2 = K_{21}\frac{(C_{L\alpha} q)^2}{V} + K_{22}\, C_{L\alpha} q + K_{23}$$

$$A_3 = K_{31}\left(\frac{C_{L\alpha} q}{V}\right)$$

where the K's represent configuration, the expressions for the K's are given below :

$$K_{01} = \frac{k_h(a_1 x - b_1)}{mI_c}$$

$$K_{02} = \frac{k_h k_\alpha}{mI_c}$$

$$K_{11} = \frac{a_1 b_2 - a_2 b_1}{mI_c}$$

$$K_{12} = \frac{a_2(k_\alpha + k_h x^2) + k_h x(a_3 - b_2) - k_h b_3}{mI_c}$$

$$K_{21} = \frac{a_2 b_3 + a_3 b_2}{mI_c}$$

$$K_{22} = \frac{-b_1}{I_c}$$

$$K_{23} = \left(\frac{k_h}{m}\right) + \frac{k_\alpha + k_h x^2}{I_c}$$

$$K_{31} = \frac{a_2}{m} - \frac{b_3}{I_c}$$

For the above equation, the aircraft configuration will define the K, and A depends on aerodynamic factor $C_{L\alpha}q$. It is expected that the resulting pitch motion will vary with speed. Equation represents a linear differential equation with constant coefficients whose solution is of form

$$\alpha = \sum_{j=1}^{j=4} \alpha_{0j} e^{s_j t}$$

where α_{oj} is pitch motion constant, s_j are the four roots of the corresponding characteristic equation.

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + A_0 = 0$$

The roots s_j may be expressed in the complex form $S_j = \beta_j + i\omega_j$, β is real parts corresponding to damping; ω is imaginary part corresponding to frequency. Since the coefficients of the characteristic equation are all real, these roots must occur in the conjugate pairs

$$S_{1,2} = \beta_1 \pm i\omega_1 \text{ , } S_{3,4} = \beta_2 \pm i\omega_2$$

Thus the characteristic equation may be written in the alternate form

$$[s - (\beta_1 + i\omega_1)][s - (\beta_1 - i\omega_1)][s - (\beta_2 + i\omega_2)][s - (\beta_2 - i\omega_2)] = 0$$

Expanding this equation and compare similar coefficients in the equation, we have

$$A_3 = -2(\beta_1 + \beta_2)$$
$$A_2 = (\beta_1^2 + \omega_1^2) + (\beta_2^2 + \omega_2^2) + 4\beta_1\beta_2$$
$$A_1 = -2[\beta_1(\beta_2^2 + \omega_2^2) + \beta_2(\beta_1^2 + \omega_1^2)]$$
$$A_0 = (\beta_1^2 + \omega_1^2)(\beta_2^2 + \omega_2^2)$$

At the flutter stability boundary, the damping β is zero, and after performing some algebraic manipulations, we have

$$\left[\left(\frac{A_2}{2}\right)^2 - A_0\right] - \left[\frac{A_2}{2} - \frac{A_1}{A_3}\right]^2 = 0$$

It will be convenient to define a variable function of $C_{L\alpha}q$ by the relation.

$$F = F(C_{L\alpha}q) = \left[\left(\frac{A_2}{2}\right)^2 - A_0\right] - \left[\frac{A_2}{2} - \frac{A_1}{A_3}\right]^2$$

At flutter onset point, $F_f = F(C_{L\alpha}q)_f = 0$. A more direct indication of how F varies with is readily obtained by introducing the factor A. The flutter margin FM can be expressed in terms of frequencies and decay rates which could be measured in flight or wind tunnel test.

The expression for the Flutter Margin then becomes

$$FM = \left[1 - \left(\frac{\beta_2 - \beta_1}{\beta_2 + \beta_1}\right)^2\right]\left\{\left(\frac{{\omega_2}^2 - {\omega_1}^2}{2}\right)^2 + (\beta_2 + \beta_1)^2\left[\left(\frac{{\omega_2}^2 + {\omega_1}^2}{2}\right)^2 - \left(\frac{\beta_2 + \beta_1}{2}\right)^2\right]\right\}$$

At the Flutter point, some modal damping β_1 or β_2 is zero , Flutter margin of aircraft is zero. The Flutter margin FM curve can be interpolated to predict flutter velocity V or flutter pressure with the change of velocity or (dynamic pressure q, coefficient $C_{L\alpha}q$).

For wind tunnel test of low-speed flutter model and high speed flutter model of constant Mach change pressure, the Mach number is basically unchanged, $C_{L\alpha}$ the effect is small, the $F(q)$ prediction is suitable. For flutter flight test, the variation range of Mach number is large and the $C_{L\alpha}$ influence is large, the $F(C_{L\alpha}q)$ prediction is better.

3 Application Example

In the high speed flutter model wind tunnel test and flutter flight test, the Z-W method and the damping method are used to predict the flutter boundary.

3.1 Wind tunnel test of high speed flutter model

The two kinds of T-tail flutter models (Model A and model B) with different scales in the high speed flutter wind tunnel test of aircraft T-tail are studied. Figure 3 shows the model structure with aluminum alloy wing spar and ribs covered by carbon fiber skin and filled with low-density foam. The airfoil of the wing has flat main part with diamond leading edge and trailing edge. Figure 4 shows the Model B structure using aluminum alloy wing spar and rib, also covered by carbon fiber skin, filled with low-density foam. the profile is symmetric airfoil.

Fig.3 T-tail flutter model A

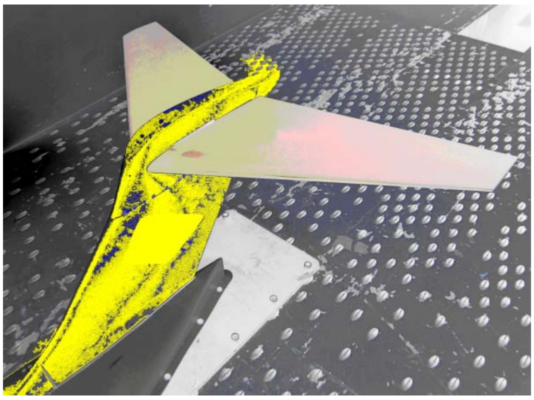

Fig.4 T-tail flutter model B

The flutter characteristics of transonic model were obtained by adopting fixed Mach number, changing wind tunnel density, and increasing the air blowing method of dynamic pressure gradually. The Z-W method and the damping method are used to predict model A and a model B horizontal stabilizer bending-torsion coupled flutter boundary of Mach number 0.82. The results of Z-W method and damping method for model Aare $1.043q_0$ and q_0 , respectively, as shown in Fig. 5 and Fig. 6. Z-W method and damping method results for model B are $1.202q_0$ and $1.032q_0$, respectively, as depicted in Figure 7 and Figure 8. The analysis shows that the flutter boundary which is interpolated by the damping method is more conservative than the interpolation result of the Z-W method. Flutter margin Z-W method can get regular curve, while damping method has only one key flutter mode damping gradually reduced with dynamic pressure.

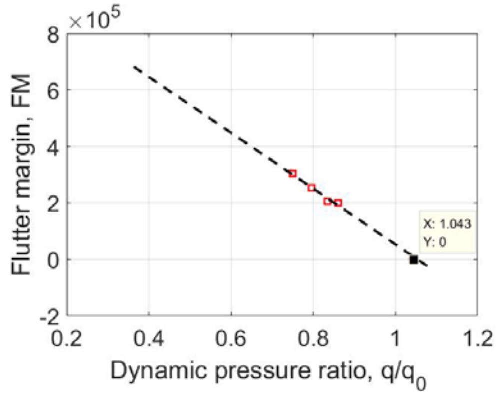

Fig.5 The Z-W method for model A flutter boundary prediction

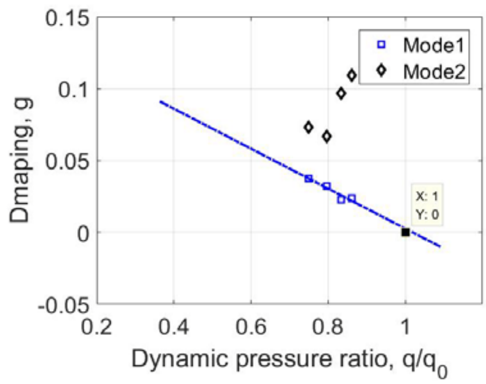

Fig.6 The damping method for model A flutter boundary prediction

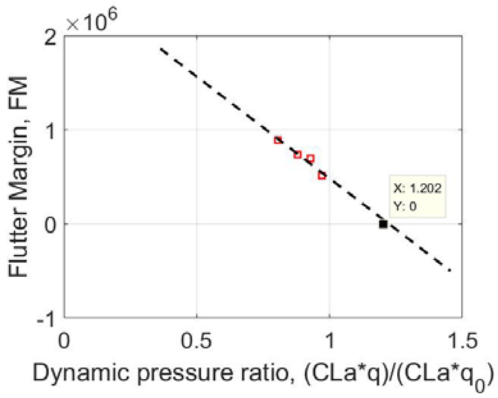

Fig.7 The Z-W method for model B flutter boundary prediction

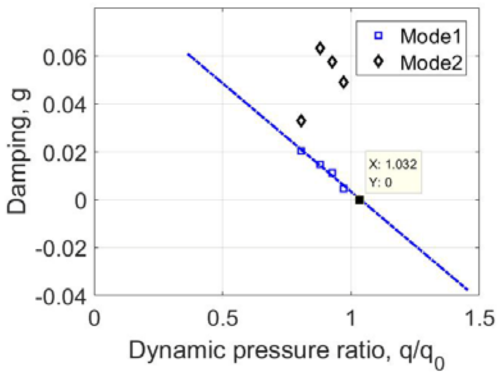

Fig.8 The damping method for model B flutter boundary prediction

3.2 Flutter flight test

In the Flutter flight test, the constant altitude test flight method is used to increase the speed and Mach number, and the flight envelope is approached carefully. Flutter test flight altitude keeps at 3500m, and the flight Mach number varies from 0.4 to 0.7. Both the lift coefficient slope and dynamic pressure change at the same time. Using $F(C_{L\alpha}q)$ prediction function, the damping prediction results are shown in Figure 9. The damping curve is not valid and can't be inserted outside the flutter boundary. The *FM* prediction results are shown in Figure 10. The Flutter margin FM curve can converge to the flutter boundary and can predict $C_{L\alpha}q$ as $C_{L\alpha}$ q_0.The flutter pressure of a given Mach number is obtained. The comparison shows that the damping method cannot predict the flutter boundary by linear interpolation. And the Z-W method can predict the flutter boundary. The damping method cannot predict the flutter boundary near the flutter boundary, and the Z-W method can predict the flutter boundary without any significant change in damping.

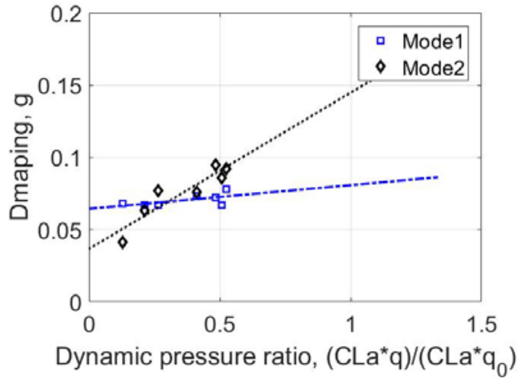

Fig.9 The damping method for flight flutter test flutter boundary prediction

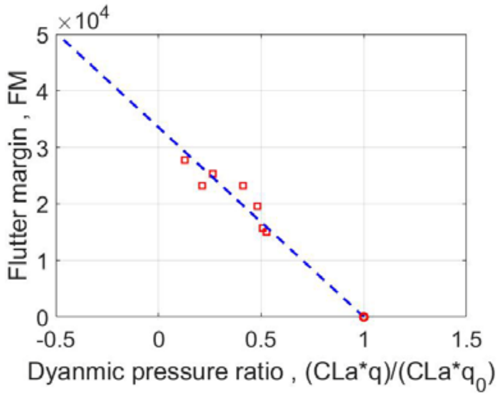

Fig.10 The Z-W method for flight flutter test flutter boundary prediction

4 Zimmerman-Weissenburger-Yang method

Traditional Z-W methods usually use $F(q)$ functions, and the flutter margin is obtained by selecting a bending mode and a torsion mode that participate in the flutter coupling *FM*. When the Mach number changes greatly, $C_{L\alpha}$ will also change a lot. This is required $F(C_{L\alpha}q)$ to be used for flutter prediction. If there are more than two modes involved in the flutter coupling, the influence of the frequency and damping of the multimode modes should be considered in the prediction formula. For example, the bending modes of wing flutter with under wing engines include the first bending, the second bending and the third bending of the wing, and the first-order vertical bending of the fuselage. The torsion modes include the first torsion of the wing and the engine nacelle pitching.

Considering the limitation of Z-W method, a new method of flutter prediction with multiple freedom or single freedom is proposed in this paper. Based on the linear combination theory, the frequency ω_B and damping β_B of the bending modes participating in the flutter coupled modes, the frequency ω_T and damping β_T of the torsion modes are expressed as:

Bending modal frequency

$$\omega_B = f_{B1}\omega_{B1} + f_{B2}\omega_{B2} + \cdots + f_{Bn}\omega_{Bn}$$

Bending modal damping

$$\beta_B = f_{B1}\beta_{B1} + f_{B2}\beta_{B2} + \cdots + f_{Bn}\beta_{Bn}$$

Torsion modal Frequency

$$\omega_T = f_{T1}\omega_{T1} + f_{T2}\omega_{T2} + \cdots + f_{Tn}\omega_{Tn}$$

Torsion modal damping

$$\beta_T = f_{T1}\beta_{T1} + f_{T2}\beta_{T2} + \cdots + f_{Tn}\beta_{Tn}$$

Coefficient f_{Bi} represents the percentage factor of the i-th bending mode, Coefficient f_{Ti} represents the percentage factor of the i-th torsion mode.

The percentage factor f indicates the degree to which the modal participates in the flutter mode coupling, which can be determined by the characteristic vector ratio or the flutter engineering experience.

The improved flutter prediction Z-W method is defined as Zimmerman-Weissenburger-Yang method, expressed as

$$FM_F = \left[1 - \left(\frac{\beta_T - \beta_B}{\beta_T + \beta_B}\right)^2\right]\left\{\left(\frac{{\omega_T}^2 - {\omega_B}^2}{2}\right)^2 + (\beta_T + \beta_B)^2\left[\left(\frac{{\omega_T}^2 + {\omega_B}^2}{2}\right)^2 - \left(\frac{\beta_T + \beta_B}{2}\right)^2\right]\right\}$$

When the airplane gradually develops from the steady state to the flutter state, the frequency of the bending mode increases gradually, and the torsion modal frequency decreases gradually, and the same flutter frequency is gradually developed. $\omega_T = \omega_B = \omega_{Flutter}$ and $\left(\frac{{\omega_T}^2 - {\omega_B}^2}{2}\right)^2 = 0$.

The closer the flutter point, the damping of the bending mode and the torsion mode decreases, and the damping of the mode is close to 0. $\beta_B = 0$ or $\beta_T = 0$, and $\left(\frac{\beta_T - \beta_B}{\beta_T + \beta_B}\right)^2 = 1$. When the flutter occurs, $FM_F = 0$.

Using the flutter test data of a certain aircraft, flutter prediction is conducted by multi-modal flutter margin Z-W method using Zimmerman-Weissenburger-Yang method. For the swept wing with large aspect ratio, the modes involved in flutter mainly include the first bending mode, the second bending mode and the first torsion mode of the wing. The wing bending mode includes the first bending of the wing and the second bending of the wing. The coefficient of participation in flutter is defined as 0.35 and 0.65, respectively. f_{B1} is 0.35, f_{B2} is 0.65. The wing torsion mode is the first torsion mode of the wing, f_{T1} is 1.0.

The Z-W method prediction result is shown in Figure 10. prediction result of $C_{L\alpha} q$ is $C_{L\alpha}\ q_0$. The Zimmerman-Weissenburger-Yang method prediction result is shown in Figure 11. The prediction result of $C_{L\alpha} q$ is 1.073 $C_{L\alpha}\ q_0$. The prediction of flutter pressure of Z-W method is less than the Zimmerman-Weissenburger-Yang method, which shows that the prediction results of Z-W method are more conservative.

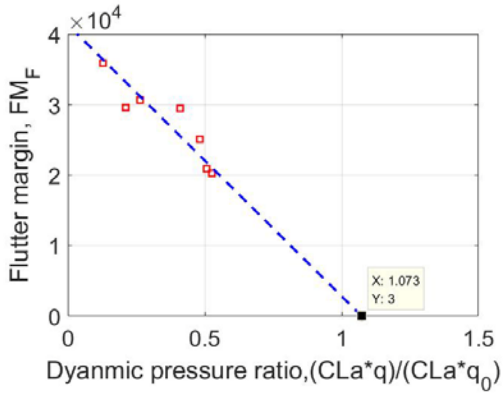

Fig.11 Result of the Zimmerman-Weissenburger-Yang method for flight flutter test flutter boundary prediction

4 Conclusion

(1) This paper summarizes the usual flutter prediction methods in aircraft flutter design, and analyses the prediction mechanism of damping method and Z-W method.

(2) According to the high-speed flutter model wind tunnel test and flutter flight test data of aircraft, the prediction characteristics of damping method and Z-W method are studied, and the analysis shows that the Z-W method can predict the flutter boundary well, even for the high speed flutter model wind tunnel test which is difficult to recognize.

(3) A Zimmerman-Weissenburger-Yang flutter prediction method is proposed for predicting the modal coupling flutter of multiple degrees of freedom, which can improve the accuracy of flutter prediction.

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