A 7D TRADE

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Abstract

The markets for military aircraft and air-to-ground missiles and bombs relate to each another. Customers respond to the features and prices in each market in unique, but quantifiable ways. Both markets face budget limitations. New methods described here demonstrate how to find and work within these mutually limiting constraints.

1 Related Dependencies

Many markets unavoidably tie to others. Such is the case for air-to-ground missiles and bombs and the military aircraft that carry them. If the missile or bomb is too large, it won't fly. Conversely, pilots of military aircraft must abide by the projectile's range. While all military planners broadly understand these principles, the implications of the constraints facing interlocked markets elude many.

This paper examines how the restrictions on the military aircraft market impinge upon those for the market for air-to-ground missiles and bombs, and vice versa. The approach used here generalizes to any pair or group of markets that share mutual dependencies.

1.1 Prompt Global Strike Delivery Options

Over ten years ago, the United States (US) began development of a system that could deliver a conventional warhead anywhere in the world within an hour, in a program called Prompt Global Strike (PGS) [1]. Delivery methods entertained for PGS have included launches from land, or sea, or space. Each of these potential methods is fraught with problems. System launches from land or sea would look much like those of Intercontinental Ballistic Missiles (ICBMs), which could, in turn, incite dangerous unwanted responses from parties other than the desired target. Releasing the weapon from space, while from the United States point of view is likely not a strict violation of international law, nonetheless upsets the balance of power thus forms a strong reason against doing it.

An air-launched PGS is far less likely to suffer such potentially disastrous responses, as such releases do not mimic ICBMs or drops from space. We will examine what must happen considering the markets for both air-to-ground missiles and military aircraft for the PGS to be successful in an economic sense. That is, we must understand how budget limits determine the feature constraints available to military planners if they pair PGS to an aircraft as part of an air-launched system; this work forms our 7D trade.

2 Paired Markets: Missiles/Bombs & Aircraft

2.1 US Missiles and Bombs

If the United States develops the PGS as a weapon launched from a fighter, bomber, or attack aircraft, the PGS becomes an air-to-surface weapon. The US market for publicly acknowledged devices of this sort is a matter of public record. We observe the US purchases of such devices over the 20-year period beginning January 1, 1997, and ending on December 31, 2016, in Figure 1 [2], [3]. There, glide bombs form the first six entries (BLU-109, -110, -111, -117 along with the Small Diameter Bombs (SBD) 1 and II) while missiles make up the rest.

Glide Bomb/ Missile	1997 - 2016 Qty	2016 $K	Max V kph	Pay-load Kg	Max Rng Km	Launch Kg
BLU-109	18,556	$51	1,605	240	28	924
BLU-110	6,565	$36	1,605	202	28	447
BLU-111	33,330	$32	1,605	87	28	227
BLU-117	24,506	$40	1,605	429	28	948
SBD I	16,577	$59	1,200	93	111	129
SBD II	2,417	$78	1,200	93	72	129
AGM-158	635	$1,352	1,200	450	370	1,021
AGM-158-1	275	$1,912	1,200	450	1,000	1,021
AGM-88E	643	$892	2,280	66	150	355
AGM-84	4,152	$528	855	221	270	675
AGM-130	102	$804	1,200	907	75	1,323
AGM-154A	1,742	$557	1,200	42	110	450
AGM-154B	3,893	$501	1,200	177	110	450
AGM-154C	6,599	$442	1,200	250	110	450
AGM-142	46	$1,773	1,482	350	80	1,361
AGM-114P	14,886	$113	1,591	9	8	47
AGM-114N	8,741	$123	1,591	9	8	47
AGM-114R	25,238	$161	1,591	9	8	47

Fig. 1. US market for air to ground glide bombs and missiles, 1977-2016, in 2016$K

2.1.1 US Missile and Bomb Demand

The second and third columns from Figure 1 contain the quantities purchased and the average prices, respectively, for the US air-to-ground missile and bomb purchases from 1997 to 2016. We plot these ordered pairs as Figure 2.

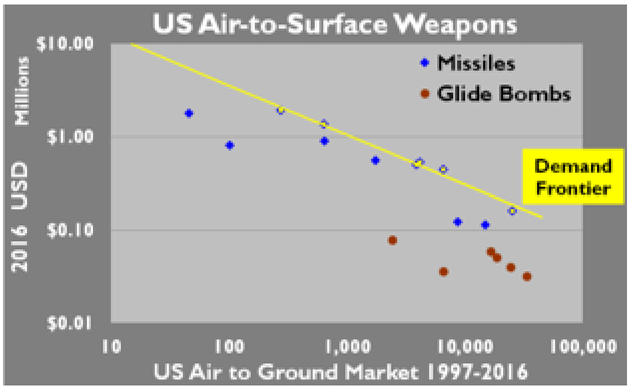


Fig. 2. US missile demand and Demand Frontier

The blue points in Figure 2 are for missiles; the brown ones are for glide bombs. Note that not only do the missiles uniformly sell for more than the bombs, there also appears to be an outer limit for sales in this market. These six outermost models (for the AGM-84, AGM-114R, AGM-84C, AGM84D, AGM-158, and AGMA-158-1), marked with yellow markers over the blue points, form a statistically significant boundary called the Demand Frontier, defined by Equation (1).

$$Price = \$4.17E10^{7} * Quantity^{-0.533} * \epsilon \quad (1)$$

Where:

Price = estimated projectile price, in 2016$
Quantity = missiles/bombs sold, 1957-2016
ϵ = the error for this equation

Equation 1, an unbiased estimator using the Ping Factor [4], has an adjusted R^2 of 98.2%, a P-value of 0.01 and a standard error of $91,800 (we remove the recurring multiplicative error term, ϵ, in subsequent equations for convenience). The Demand Frontier reveals limiting quantities given prices. For an air-launched Prompt Global Strike system, the Frontier limits quantities given target prices of the weapon system. We discover a pair of these limits in Figure 3.

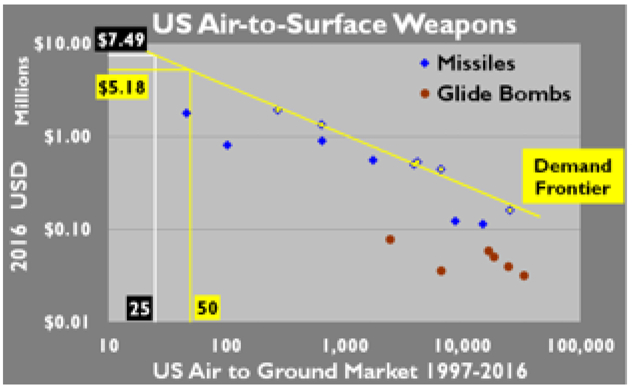


Fig. 3. US missile prices set quantity limits

Figure 3 reveals that if the United States government were willing to spend up to $7.49 million per PGS system in 2016$, might be able to afford up to 25 of these devices. If instead, they limited themselves to $5.18 million (again, in 2016$), they could buy up to 50 such systems.

Given that this market forces constraints upon its buyers, we might ask ourselves what it is we would get for such prices, this notion addresses the idea of value, which we address in the next section.

2.1.2 US Missile and Bomb Value

We hypothesize that the features of the missiles and bombs we have in Figure 1 might have something to do with their sustainable value. We find we can estimate the value of missiles and bombs using their features, as in calculated in Equation (2) and displayed in Figure 4.

$$Price = 977 * R\,Km^{0.452} * Grv1, Pow2^{3.00} * MV^{0.167} \quad (2)$$

Where:

Price = estimated projectile price, in 2016$
RKm = range in kilometers
Grv1, Pow2 = gravity bombs have a value of 1; missiles have a value of 2
MV = launch mass in kilograms times maximum speed in kilometers per hour

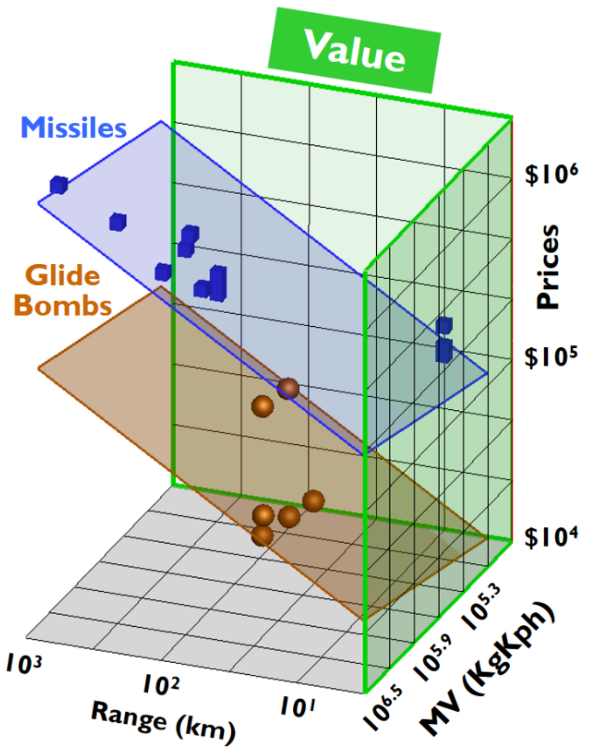


Fig. 4. US missile 3D Value Space

Equation 2 (which excluded AGM-142 as an outlier) adjusted by the Ping Factor (as are all equations that follow), has an adjusted R^2 of 96.9%, P-values of 1.84E-06, 5.69E-10 and 3.06% for range, grv1pwr2, and mv, respectively, and a standard error of $147,000.

The term grv1, pow2 is a step function term that discovers powered (pow) projectiles (missiles) have eight times (2^3) more value than do gravity (grv) bombs. The mv term is launch mass times maximum velocity (KgKph), which is an expression of momentum.

We can discover the graphical meaning of Equation (2) in its associated chart. Log-linear in all three directions; Figure 4 shows us how the market rewards (indicated by their demonstrated willingness to pay) added momentum (the mv term, KgKph) and range (in kilometers). Note that the curvilinear responses (range raised to the 0.452 power, momentum raised to the 0.167 power) appear planar in a three-dimensional log-linear environment. Observe that the market readily pays for the added flexibility missiles offer, at a rate eight times that for glide bombs.

Let's consider a subset of the missile and bomb database in Figure 5.

Name	Max R km	MV KgKph	2016 Price	1997-2016 Qty
BLU-111	28	364,389	$32,000	33,330
AGM-158-1	1,000	1,225,200	$1,912,000	275
AGM-84	270	577,125	$528,000	4,152

Fig. 5. Four variables for three observations

In Figure 5, we have four variables. Three of them appear in Figure 4, range (a horizontal dimension), mv (another horizontal axis, momentum, the product of launch mass in kilograms times the maximum speed of a missile or a bomb) and price (our vertical axis). Two of the Figure 5 variables appear in Figures 2 and 3, quantity (a horizontal dimension), and, once again, price (which we know to be vertical).

We can plot these four variables as ordered quads, which we do in Figure 6. In a market setting, the general format for ordered quads is (valued feature 1, valued feature 2, price and quantity). The left-hand side of such plots is green and depicts Value Space. The red-hand side of the graph is red, revealing the Demand Plane. Value Spaces and Demand Planes connect in dual states across four dimensions; all markets work and have always worked in this way.

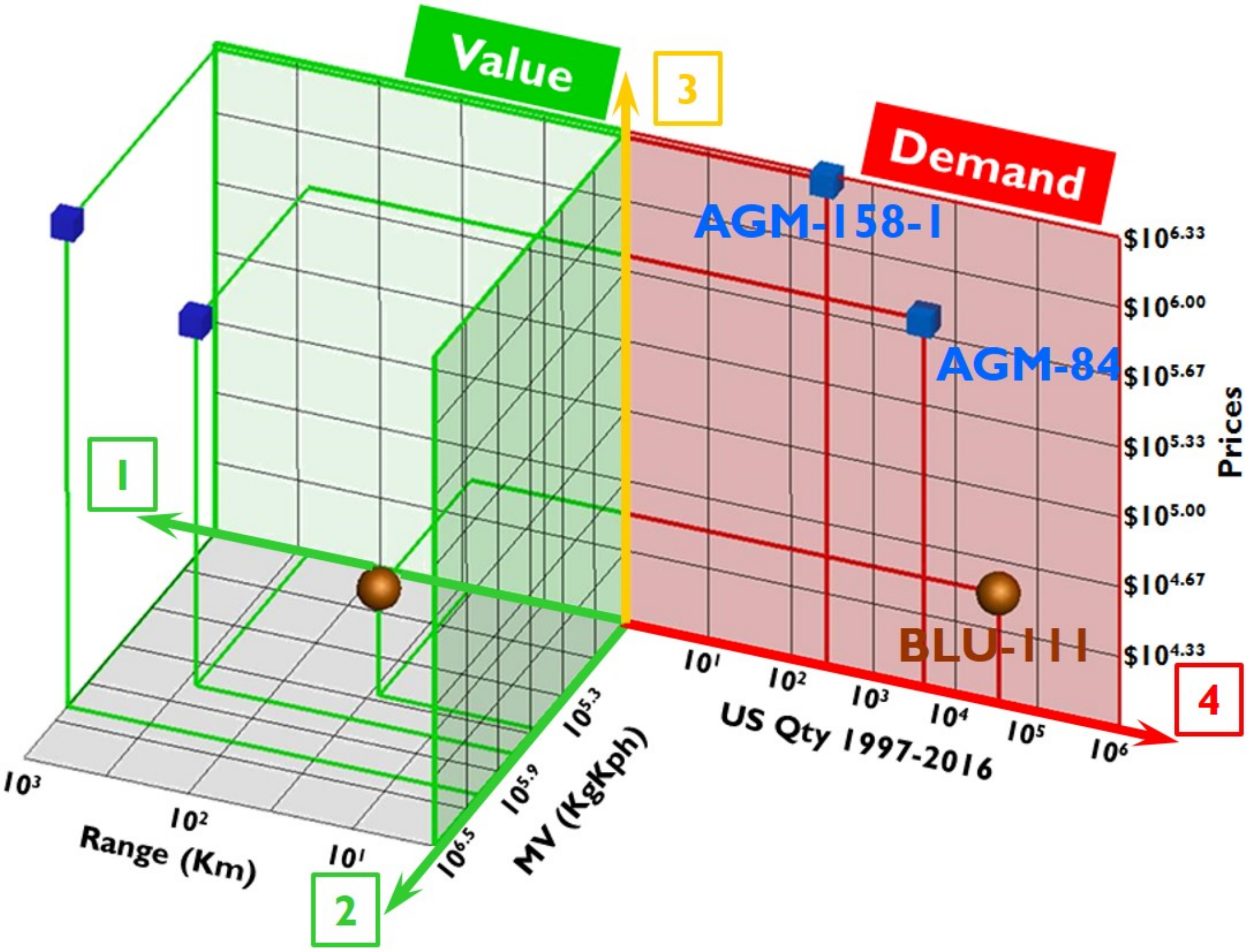


Fig. 6. Three models in the US missile/bomb market: They plot as ordered quads in dual states

Aircraft	1957-2016 Qty	2016 $M	Max Kph	Mas PL Kgs	Max Range Km	Aircraft	1957-2016 Qty	2016 $M	Max Kph	Mas PL Kgs	Max Range Km
B-52H	740	$80.2	1,046	36,287	16,237	F-14	712	$57.1	2,485	6,577	2,951
B-1B	100	$424.6	1,336	56,599	11,999	F-111	563	$106.0	2,655	14,288	6,759
AV-8B	323	$42.4	1,064	6,003	1,101	F-4	5,195	$18.7	2,369	8,459	2,599
F/A-18AD	1,480	$50.9	1,915	6,337	2,012	A-7	1,569	$13.3	1,123	6,804	4,603
F/A-18EF	563	$64.9	1,915	8,051	2,443	F-8	1,219	$11.9	1,971	1,814	2,792
F-15AE	1,415	$51.6	3,018	11,113	2,543	A-4	2,960	$7.2	1,083	4,491	3,219
F-117A	64	$85.3	993	1,814	1,721	F-5	2,246	$9.7	1,706	3,175	3,718
F-16CD	4,540	$28.2	2,128	7,711	4,216	F-35A	205	$322.3	1,713	8,165	2,221
F-22	195	$171.9	2,414	9,539	2,961	F-35B	36	$185.0	1,712	6,804	1,666
B-2	21	$1,143.0	1,014	22,680	11,104	F-35C	63	$221.9	1,712	8,165	2,221
A-6	693	$64.6	1,043	8,165	5,222	MQ-9	313	$16.7	483	1,851	2,851
A-10	716	$19.6	707	7,257	4,152	MQ-1	158	$18.6	217	204	1,239

Fig. 7. The US market for fighters, bombers and attack aircraft from 1/1/1957 to 12/31/2016

2.2 US Fighter, Bomber and Attack Aircraft

The market for unclassified fighter, bomber and attack aircraft in the United States is well known and fully documented. There are only a couple of dozen of such planes were or have been in wide use over a sixty-year period, as shown in Figure 7.

The US used many of these planes for roles other than bombing. However, we need to have them all incorporated into this study not only to provide us with a sufficient number of data points but also to remind us that there are many ways to deliver ordnance from the air – to be thorough, we include them all [5].

2.2.1 Military Aircraft Demand

If we plot the quantity column from Figure 7 as the horizontal amounts, and the prices as the vertical amounts, we get Figure 2, which is the United States market for bomb-dropping aircraft over 60 years. The outermost points, highlighted in yellow, form this market's Demand Frontier, described by Equation 3, depicted in Figure 8.

$$Price = \$1.14E10^{10} * Quantity^{-0.731} \quad (3)$$

Where:

Price = aircraft price, in 2016$
Quantity = number of aircraft sold, 1957-2016

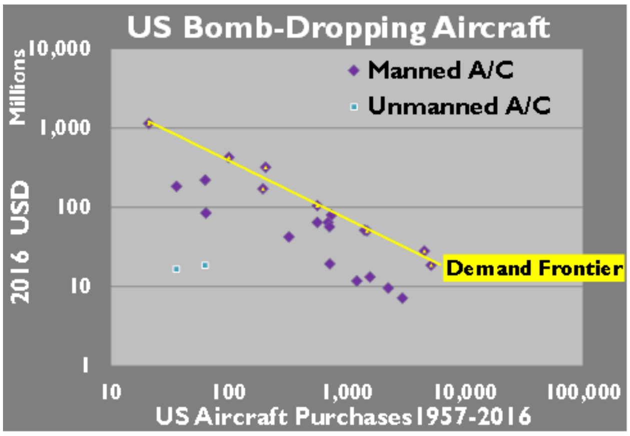


Fig. 8. US bomb-dropping aircraft demand

Equation 3, an unbiased estimator using the Ping Factor, has an adjusted R^2 of 97.6%, a P-value of 2.68E-06 and a standard error of $59.9 million.

The P-factor indicates the chance of this equation coming about due to chance is very low – in other words; we can feel confident in using it for forecasting.

Recently, the US began work on their B-21 Raider bomber, with the expectation that they will buy a minimum of 100 of them [6] at a unit cost of $550 million in FY 2010 dollars [7]. Given we know Equation 1, we might want to confirm that quantity-price combination of 100 units at $550M in 2010 dollars is feasible for the B-21 bomber. We examine this in Figure 9.

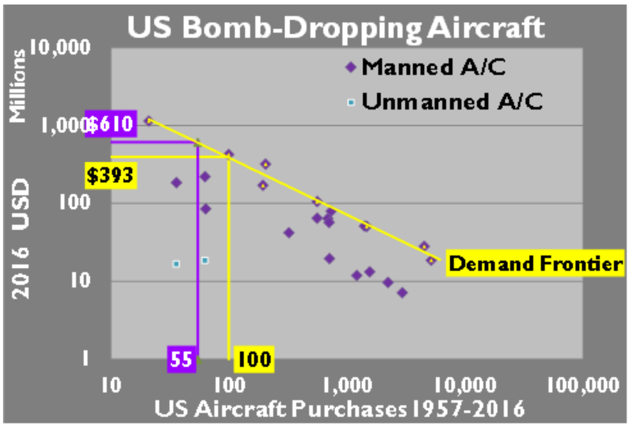


Fig. 9. US bomb-dropping aircraft limits

Figure 9 reveals that at $610 million in 2016 (the inflated value of the $550 million in 2010) the Demand Frontier only supports 55 units (the ordered pair (55 units, $610 million)). Conversely, if the US government sets its requirements for 100 units, it will have to get the price of the B-21 down to $393 million each. Given the standard error of $59.9 million, the target price is over 3.6 standard deviations away from the predicted limit price at 100 units ((target price of $610 million – limit price of $393 million)/standard error of $59.9 million). Unless there is a large change in the procurement approach to military planes capable of carrying bombs, the chance of getting 100 vehicles at the posted price is low.

Given this, we may want to see what is possible regarding the features we would like to get compared to those for which the US government has sufficient monies. We do this in the next section.

2.2.2 Military Aircraft Value

We hypothesize that bomber features support their value. Equation 4 shows we can predict their sustainable prices from their features.

$$Price = \$1.01E10^5 * Quantity^{-0.768} * R^{0.547} * Kph^{0.93} \quad (4)$$

Where:

Price = aircraft price, in 2016\$
Quantity = number of aircraft sold, 1957-2016
R = max range, in kilometers
Kph = max speed, in kilometers per hour

Equation 4 is an unbiased estimator adjusted by the Ping Factor. Its adjusted R^2 is 83.9%, with P-values of 3.72E-09, 0.33% and 0.01% for quantity, max range and max speed, respectively, and it has a standard error of \$76.2 million.

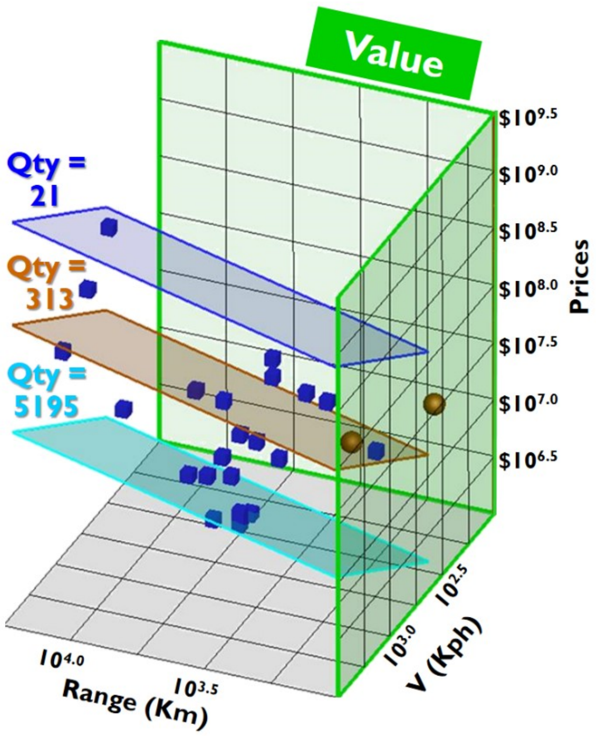


Fig. 10. Value in US bomb-dropping aircraft

Figure 10 shows the results from Equation (4). Maintainable price goes up with range and speed but falls quickly with added quantities. The upper dark blue surface shows the value using the value of 21 as the quantity term in Equation (4), the number of B-2s sold. Note that if we instead use 313 as the quantity term in Equation 4 (which matches the number of MQ-9s sold), the sustainable quantity falls dramatically, as it does when we set the quantity value to 5195 (the quantity of F-4s the US purchased) for the lowest, light blue surface. There is another useful and important implication of the quantity term from Equation 4, as we see in Figure 11.

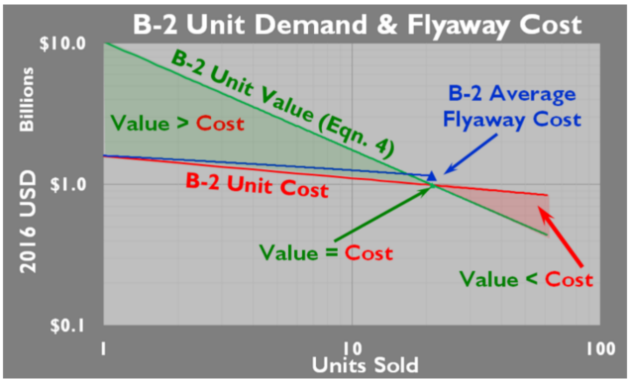


Fig. 11. When cost exceeds value, programs stop

Figure 11 shows us that the B-2 Unit Value, using Equation 4, started at over \$10 billion for the first unit and instantly began to fall. In this market, the quantity exponent for incremental value, at -0.768, is very steep; this means that recurring aircraft costs, which likely have much flatter slopes (that is, learning curves with slopes less negative exponents) can eventually catch up to aircraft value. In Figure 11, the recurring value of the B-2 exceeded its cost until it reached the 21st unit, at which point the two curves equated to one another. Beyond this point, costs would be greater than value; that is why the B-2 program stopped. In any industry, product lines cease to run when cost exceeds value.

Combining the 4D structure we discovered in Figure 6, the bomber Demand Frontier we found in Figure 9 and the Value response surfaces we portrayed, we derive bomber market trade possibilities in Figure 12. The brown plane, the bomber value at 100 units, crosses the yellow plane (at \$393 million in 2016\$), forming a straight line in log space, but a curvilinear production possibility curve in linear space in the left-hand picture of Figure 13. At the same time, bomber value at 55 units, the light blue plane intersects the purple plane (\$610 million in 2016\$), offering the higher production possibility curve in the left side of Figure 13.

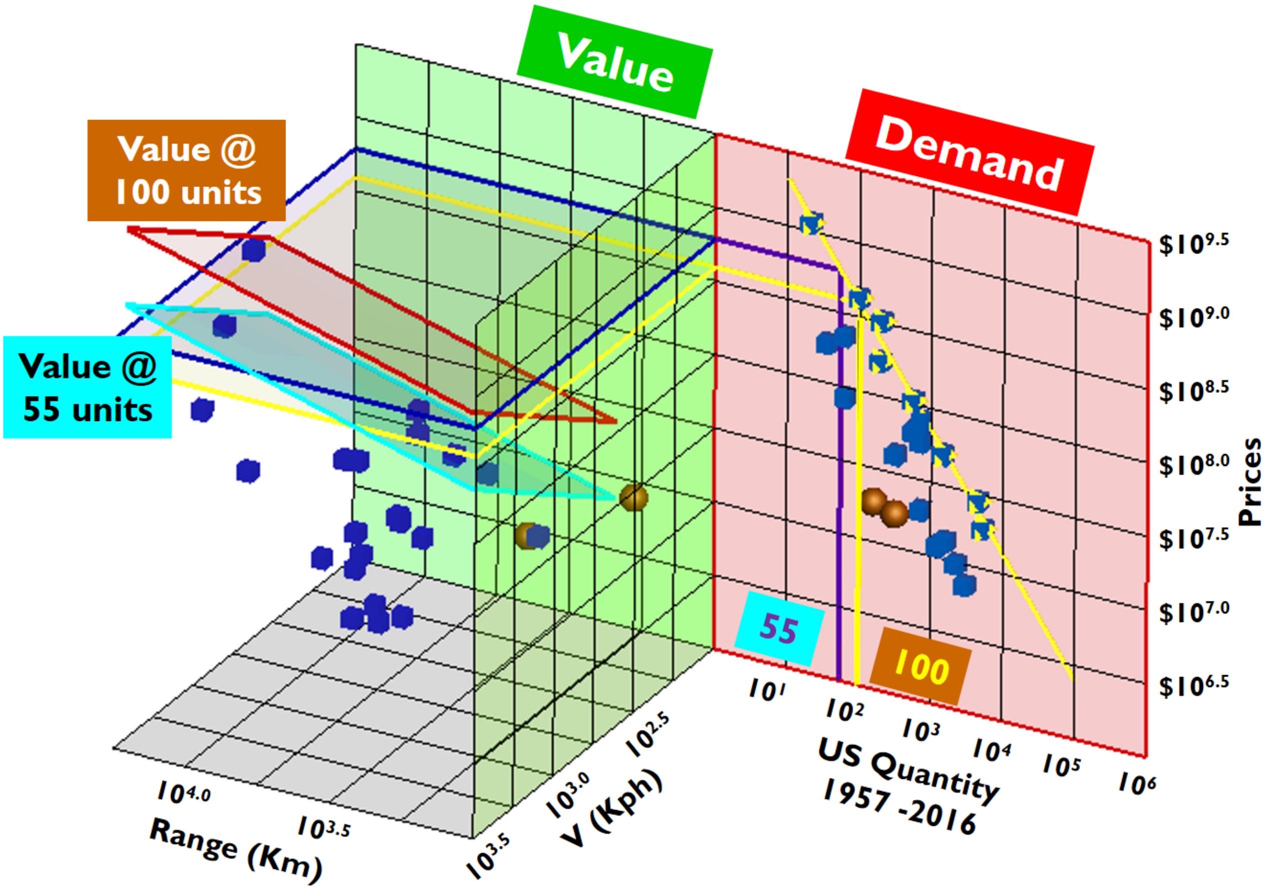
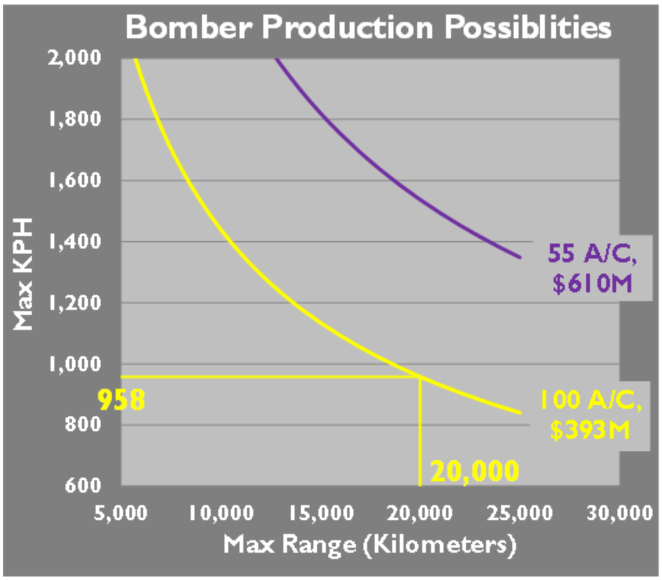


Fig. 12. The US market for fighters, bombers and attack aircraft has varying possibilities for differing quantities



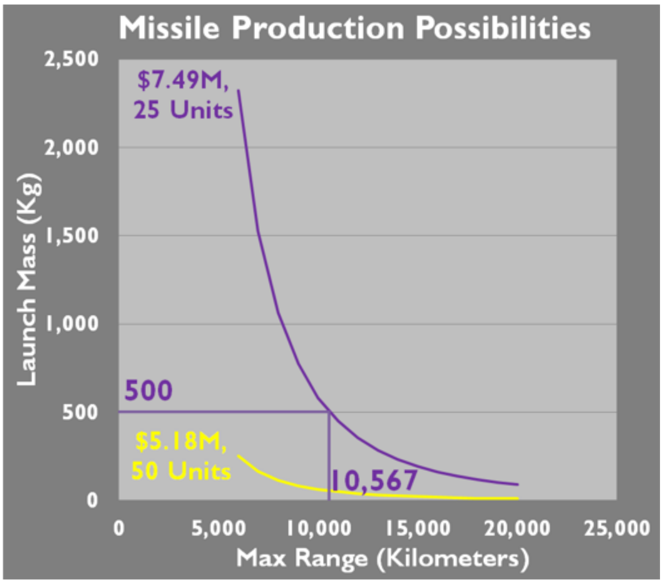


Fig. 13. If we set the price, we determine not only maximum quantity, but also production possibility curves

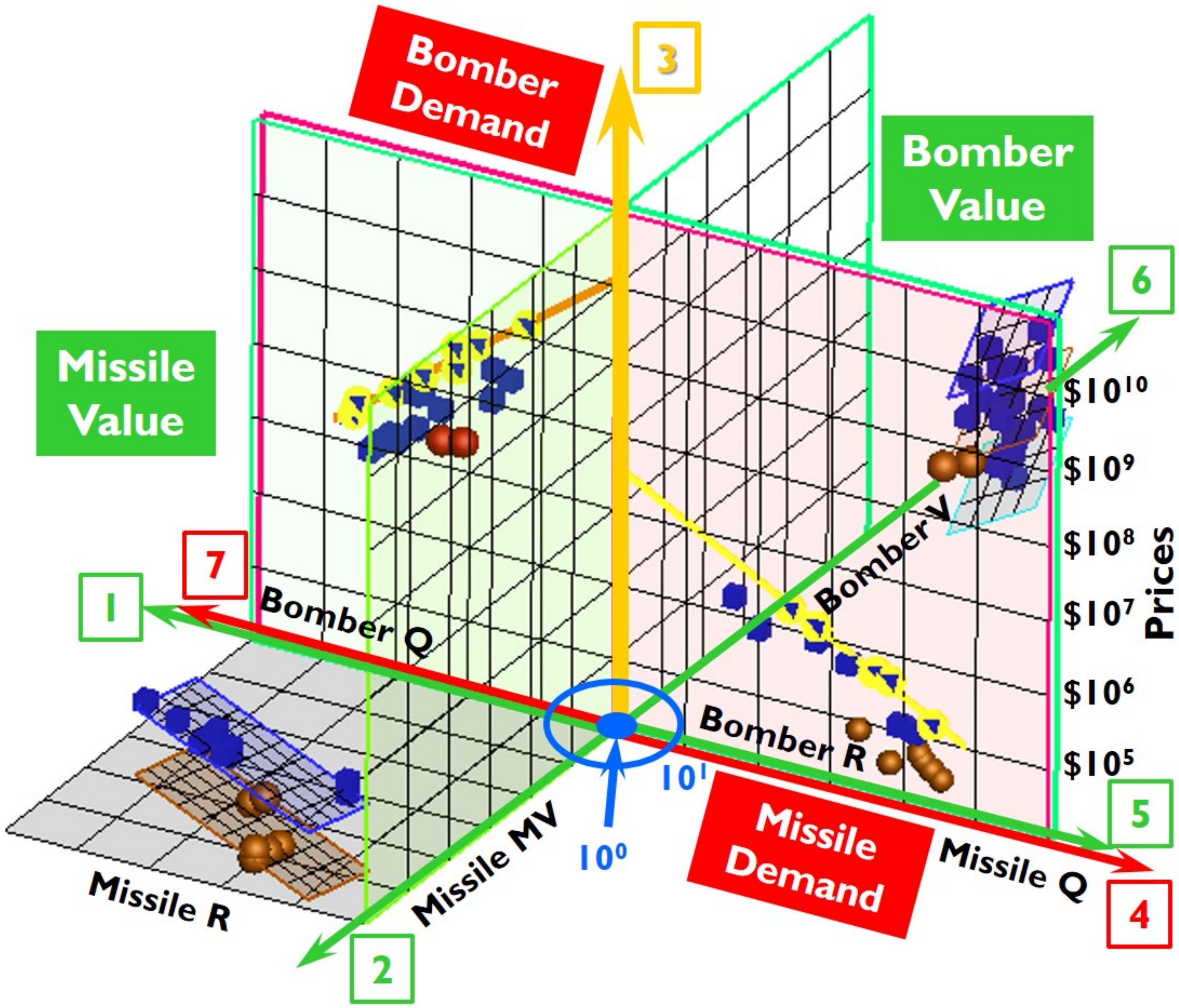


Fig. 14. 7D trades have 1 common price axis, 4 value axes (2 for each market) and 2 demand axes (1 for each market)

In the right-hand portion of Figure 13, we see the results of doing the type of exercise we did in Figure 12 for air-to-ground missiles and bombs. We get the same general types of production possibility curves, with added possibilities as we increase the target price for the system.

3 Conclusions and Summary

3.1 Trading Missiles and Bombers

The left-hand part of Figure 13 makes it clear that if the United State Air Force abided by their limit of $393 million for the B-21, the price that might allow them to buy up to 100 units. Then they could obtain sufficient range (20,000 kilometers) at a high-subsonic speed (958 kilometers per hour) for a wide variety of missions, including Prompt Global Strike. Importantly this feature set does not refer to other important features in modern bombers, such as payload, radar signature or crew rest facilities. We could address of all of these characteristics may as in the fashion above, that is, by fixing some features and letting the others vary (in this case, by not addressing them, those specifications are fixed),

The other side of Figure 13 tells a different story. Here, we've allowed launch mass to vary as we adjust range, given certain prices and maximum possible quantities. We cannot go a long distance with only spending $5.18 million per missile, and our launch mass quickly approaches zero as we try to add range. However, if allow the missile price to climb to \$7.49 million per missile, we can obtain much longer ranges. As Figure 13 indicates, we could have a launch mass that might be able to have a range over 10,000 kilometers.

Importantly, though, this maximum range is well less than half the distance around the world. The preceding means that to reach all parts of the world inside of one hour, the original vision for Prompt Global Strike if the system is to be air-launched, there must be several bases across the world in which wings of the B-21s operate.

In large measure because of its huge overruns, the United States Air Force (USAF) could only afford to buy 21 B-2 bombers. Given their sparse numbers, they only operate out of a single spot, Whiteman Air Force Base [8]. In the USAF, there must be sufficient numbers of aircraft to form squadrons.

The analysis leads us the conclusion that if the B-21 is to launch PGS, its price must be low enough to enable the USAF to buy several squadrons of it. With as many as100 aircraft, the USAF, based on its experience with the B-2, might be able to form five squadrons of B-21s, thus allowing dispersed basing across multiple sites, greatly enhancing the efficacy of the PGS. The demand limitations argue strongly for the lower price limit as shown above. Sixty years of purchase history in this market reveal its long-lasting boundaries.

3.2 7D Trade Format

In Figure 14, we see the specific 7D trade at hand. Here we have the missile/bomb market towards the front of Figure 14 and the market for bombers butting up against it in the back. As in Figure 6, missile range (km) is still dimension 1, missile momentum (KgKph) is still dimension 2, missile price is still dimension 3 (2016\$), and missile demand (as quantities sold) is still dimension 4. Note, however, that the scales have changed. All horizontal scales (1, 2 and 4 for missiles), still in log format, begin at 10^0 and go outward from the origin in multiples of ten. Meanwhile, the vertical axis for the prices (dimension 4) begins at 10^4. Missile value points are in the lower right-hand part of Figure 14, while missile demand points lie against their Demand Plane. Observe the value and demand points all appear lower – this is because while the bottom of the vertical price scale still starts at the same place (that is, 10^4), it now extends upward to 10^{11} instead of $10^{6.33}$. We need to do this to accommodate the companion bomber market, which we'll do presently.

The bomber market abuts the missile market in this example. It is as if the axes and points in Figure 12 had been rotated counterclockwise 180° about the vertical, or price axis. Thus, the bomber range (in kilometers) axis takes on dimension 5, while maximum bomber velocity (V, in kilometers per hour) accounts for dimension 6. The vertical axis for bombers addresses price, as it for missiles – thus, there is no need to duplicate it, and remains as dimension 3. Finally, we measure bomber quantities with dimension 7. As with missiles, each of the horizontal axes begins at 10^0 and goes outward from the origin in multiples of ten. Here, too, the vertical scaling for bombers in Figure 14 is different than it was in Figure 12. Dimensions 5 and 7 of the bomber market immediately about dimensions 1 and 4 of the missile market.

3.3 Using the Analysis and the Method

In the case studied here, certainly the buyers of bombers and perhaps their counterparts for missiles failed to study the purchase histories in their markets and the economic implications of them. In the process, some people have made decisions that the data analysis of it does not support for financial reasons – there simply is not enough money to get the quantities of bombers they want for the prices they imagine they can afford. In the United States, the USAF makes requests, but the US Congress ultimately makes the budget determinations. The USAF needs to recognize and abide by the limits the US Congress places upon it.

While this paper addressed a specific trade, analysts can use this approach when making financial decisions between any pair of related markets. For trades involving more markets, their concurrent depiction in one view (as in Figure 14) requires necessarily requires some data compression, but the author has some well-established techniques addressing that issue [9].

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