STUDIES ON THE APPLICATION OF MOEA/D TO AERODYNAMIC OPTIMIZATION DESIGN

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Abstract

Aerodynamic multi-objective optimization problem (AMOP) is common in aerodynamic optimization design (AOD). Multi-Objective Evolutionary Algorithms (MOEAs) are popular to solve AMOPs since they can obtain a set of solutions called Pareto optimal in a single run. Since numerically evaluating the objective of AOD is generally computationally expensive and time consuming, improving the convergence performance of MOEAs and focusing the computational resource on the solutions that the decision maker (DM) most interested in are two important aspects to solve AMOP better. An improved multi-objective evolutionary algorithm based on decomposition using a combined operator is used to improve the convergence performance. Several methods of solving AMOP considering the DM's preference are also discussed which focus the computational efforts on the Pareto solutions that suit the DM's preference better. Multi-objective test functions and aerodynamic optimization design of airfoil is tested. The results show that the improved algorithm is capable to solve AMOPs with user preference, and converge faster than the original algorithm.

1 Introduction

To get better aerodynamic performance, aerodynamic optimization design has to handle multi-disciplinary and multi-objective design cases. Very often, the objectives contradict each other. For such multi-objective optimization problems (MOP), one is expecting to get a set of solutions that are the best tradeoffs among the objectives, called Pareto optimality [1].

Recently, Multi-Objective Evolutionary Algorithms (MOEAs) [1][7] have become one of the most popular approaches for solving multi-objective optimization problems. They have been successfully applied to aerodynamic multi-objective optimization problem (AMOP) [8][10]. One of the best known shortcomings of MOEAs is the low convergence speed. A large population and large number of iteration are needed to obtain satisfactory results, which means high computational cost. Moreover, the decision maker (DM) may be only interested in part of the Pareto front, while MOEAs always try to approximate the whole Pareto front, which means the computational resources are wasted on the effort to get the Pareto front parts that the DM may not care about. By focusing the optimization only on the part that the DM wants, the computational resources will be used with high efficiency and mort rational, the optimization is expected to find the solutions preferred more quickly [11]. Many works have been done to handle MOPs with user preferences [12]-[14]. For AMOPs, the computational cost to get high-fidelity aerodynamic performance data is very high. Hence, algorithm with high efficiency and rational usage of computational resources is needed.

Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) [7] decomposes the MOP into N scalar optimization subproblems and solves them simultaneously by evolving a population of solutions. MOEA/D based on differential evolution (MOEA/D-DE) [15] is proposed to deal with MOPs with complicated Pareto Set shapes. Compared with traditional Pareto dominance based MOEAs, MOEA/D has superiorities [7]: (1) MOEA/D shows better performance using small population size; (2) MOEA/D is able to obtain different

Pareto solutions by altering the weight vectors used. These properties make MOEA/D suitable for aerodynamic optimization design.

This paper studies the application of MOEA/D to AMOPs. Two aspects of contribution are made in this paper: (1) a hybrid operator of covariance matrix adaptation evolution strategy (CMA-ES) [16] and differential evolution (DE) [17] is proposed to improve the convergence performance; (2) Several methods of solving AMOP considering the DM's preference based on MOEA/D are discussed.

2 Related concepts

2.1 MOP and Pareto Optimal

A multi-objective optimization problem can be generally stated as follows:

$$\begin{aligned} &\min F(x) = \left(f_1(x), \cdots, f_m(x)\right)^T \\ &\text{s.t. } x \in \Omega \end{aligned} \tag{1}$$

where $x = (x_1, \cdots, x_n)^T$ is the decision variable vector, Ω is the variable space. $F(x)$ is the objective function vector and $f_i(x)$ is the ith objective function, $i = 1,2, \cdots, m$.

Pareto dominance is one of the most popular methods used to consider tradeoff between objectives contradicting with each other. The best tradeoffs among the objectives can be defined in terms of Pareto front (PF) [1].

2.2 Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D)

MOEA/D decomposes a MOP into N scalar optimization sub-problems and solves them simultaneously by evolving a population of solutions. The primary part of MOEA/D is the decomposition of multi-objective problem. Tchebycheff approach [7] is the most popular one to decompose a MOP. The scalar optimization problem obtained by Tchebycheff approach is in the following form:

$$\begin{aligned} &\min g(x|w, z^*) = \max_{i=1,\ldots,m} w_i |f_i - z_i^*| \\ &\text{s.t. } x \in \Omega \end{aligned} \tag{2}$$

where z^* is the ideal reference point, i.e. $z_i^* = \min\{f_i(x) | x \in \Omega\}$. w is the weight vector and $w_i > 0, \sum_{i=1}^{m} w_i = 1$. The optimal of Eq. (2) corresponds to one of the Pareto optimal of Eq. (1). Therefore, one is able to obtain different Pareto optimal solutions by altering the weight vector.

2.3 Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES)

CMA-ES utilizes the information of the previous search steps to adjust the covariance matrix and the step-size adaptively in the procedure of the optimization. The new individuals at generation are sampled as:

$$x^{(t+1)} = \bar{x}^{(t)} + \sigma^{(t)} \mathcal{N}(0, C^{(t)}) \tag{3}$$

where t is the iteration number, $\mathcal{N}(0, C^{(t)})$ means a multivariate normal dist-ribution with mean 0 and covariance matrix $C^{(t)}$, $\sigma^{(t)}$ is the step size, and $\bar{x}^{(t)}$ is the weighted mean vector of previous iteration. The details of CMA-ES can be found in [16].

3 CMA-ES Enhanced MOEA/D-DE (MOEA/D-DE+CMA)

3.1 Proposed hybrid operator

CMA-ES is incorporated into MOEA/D-DE obtaining MOEA/D-DE+CMA, trying to make use of the learning and self-adapting ability of CMA-ES to improve the convergence quality of MOEA/D-DE. The motivation behind this idea is based on the major motivation of MOEA/D that if two weight vectors are close to each other, the optimal solutions of their sub-problems will be similar (or close) to each other. Thus, when optimizing the ith sub-problem, the information of the neighboring sub-problem can be helpful. Hence, the information of the neighboring sub-problms is used by DE and CMA-ES in their own ways utilizing different information. Through this, no additional function evaluation is needed by the improved algorithm, which means that computational burden with respect to function evaluation is just the same with MOEA/D-DE.

The evolution operator used in the improved algorithm is a combination of DE and CMA:

$$y^{(i)} = c_1^{(i)} y_{DE}^{(i)} + c_2^{(i)} y_{CMA}^{(i)} \tag{4}$$

where the superscript i indicates the ith sub-problem, $y^{(i)}$ is the trail solution in the next iteration, $y_{DE}^{(i)}$ and $y_{CMA}^{(i)}$ are the individuals obtainned by DE and CMA-ES respectively, $c_1^{(i)}$ and $c_2^{(i)}$ are the coefficient used to adjust the proportion of $y_{DE}^{(i)}$ and $y_{CMA}^{(i)}$ that can be reserved in $y^{(i)}$, and $c_1^{(i)} + c_2^{(i)} = 1$. Two aspects are considered to determine the coefficients $c_1^{(i)}$ and $c_2^{(i)}$:

1) They should be adjusted with the processing of the evolution. The probability that the neighbors have similar optimal search directions is high at the beginning of the evolution, hence $c_2^{(i)}$ should be larger to make full use of the learning ability of CMA-ES. Then $c_2^{(i)}$ should decrease with the process of the evolution;
2) They should be adjusted according to the degree of objective improvement of the scalar problem. Higher improvement indicates larger potential room for improvement, which suggests that the solution of the sub-problem is still far from the optimal, and the information from the neighboring sub-problems is still valuable for the CMA-ES operator, so a large $c_2^{(i)}$ is wanted; otherwise the sub-problem either reaches a local optimal or is near the global optimal, both situations need the more global search algorithm DE part to refine the solution.

   Thus, c_1 and c_2 are updated as follows:

$$\begin{cases} c_1^{(i)} = 1 - c_2^{(i)} \\ c_2^{(i)} = \pi^{(i)}(1-p) \end{cases} \tag{5}$$

where $p = 0.5 * (1 + iter/iter\_max)$ is used to adjust the parameter with the processing of the evolution, $iter$ is the current iteration number, $iter\_max$ is the maximum iteration number. And

$$\pi^{(i)} = \begin{cases} 1, \text{if } \Delta^{(i)} > T \\ \Delta^{(i)}/T, \text{otewise} \end{cases} \tag{6}$$

$$\Delta^{(i)} = \frac{g^{(t-1,i)} - g^{(t,i)}}{g^{(t,i)}} \tag{7}$$

$g^{(t,i)}$ is the ith scalar problem objective at the tth iteration, T is a predetermined threshold value. $\Delta^{(i)}$ is the relative improvement degree of ith sub-problem. The parameters are designed such that the learning ability of CMA-ES is used when the sub-problem converges fast, and global search ability of DE is used when the solutions are near the optimal point. The update of $c_1^{(i)}$ and $c_2^{(i)}, i = 1,2, \ldots, N$ is conducted after the evolution of all the population.

3.2 Test function cases

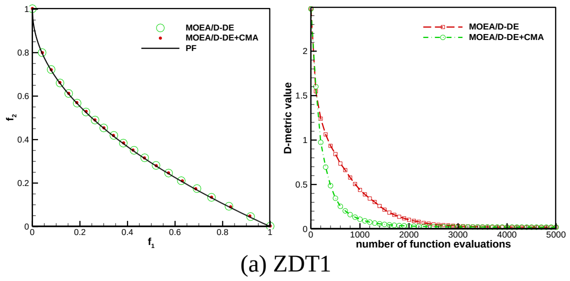
In order to test the performance of the proposed algorithm, the most widely employed suite of benchmark multi-objective problems, the four real-valued ZDT problems [18], ZDT1-3 and 6, are tested in this section. Each of the test cases has been run for 30 times.

Distance from representative in the PF (D-metric) is used to assess the performance of the algorithm. Let P^* be a set of uniformly distributed points along the PF, and A an approximation to the PF, then the D-metric is defined as

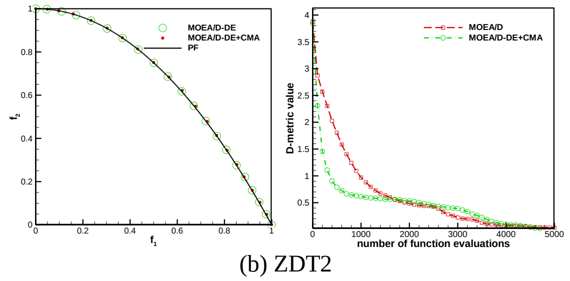
$$\text{D-metric} = \frac{\sum_{v \in P^*} d(v, A)}{|P^*|} \tag{8}$$

where $d(v, A)$ is the minimum Euclidean distance between v and the points in A. If P^* contains enough points to represent the PF well, the D-metric can measure both the diversity and convergence of A in a sense. To have a small value of D-metric, A must be very close to the PF and can't miss any part of the whole PF.

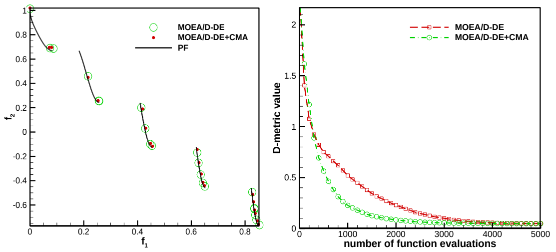
As has been demonstrated at the previous part the advantage of using small population size, population size of 20 is used, neighborhood size set to 20, update size 5, threshold 0.05 and the maximum iteration number is 500.



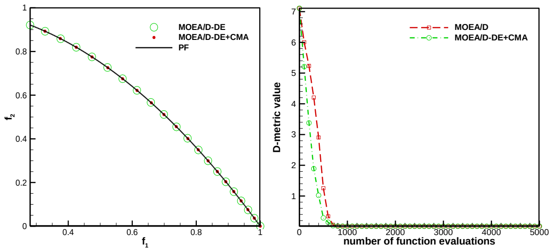
(a) ZDT1



(b) ZDT2



(c) ZDT3



(d) ZDT6

Fig. 1 Pareto front obtained with the median D-metric value and the evolution of average D-metric for each function.

Fig. 1 shows the PF obtained with the median D-metric value and the evolution of average D-metric for each function. We can see that the obtained PF is almost the same, while the proposed algorithm convergences faster than MOEA/D-DE.

3.3 Multi-objective aerodynamic optimization of airfoil

RAE2822 foil is chosen as the base foil. The optimization objectives are to maximize the lift-to-drag radio and minimize the absolute value of moment coefficient simultaneously at the constraint that the thickness of the new foil can't be smaller than the original thickness. Mach number $M = 0.729$, Reynolds number $Re = 6.5 \times 10^6$ and angle of attack $AoA = 2.31^o$. The airfoil is parameterized by Class Shape Transformation (CST) method [19] and the total design variables are 12. The numerical simulation method used is based on the RANS equations, turbulence model using the S-A model, and thin layer approximation to discrete the viscous term. The optimization problem is defined as

$$\min F(x) = (f_1(x), f_2(x))^T$$
$$f_1(x) = \frac{25 * C_d}{C_l}, f_2(x) = 10 * |C_m| \quad (9)$$
$$\text{s.t.}\, d_{new} - d_0 \geq 0$$

where C_l, C_d, C_m are the lift, drag and moment coefficient respectively, d_0 is the thickness of the original foil and d_{new} of the new foil. The penalty function used to handle the thickness constrain was proposed by Jan, M. A. and Qingfu, Zhang in 2010 [20]. Parameter setting is the same as previous subsection except that the maximum iteration number is set to 100.

Since the analytical or real PF cannot be obtained before or even after the optimization, D-metric cannot be used in this case as the performance index. Instead, hypervolume (HV) indicator [21] is used as the performance index. Fig. 2 gives an example of how the HV is calculated in 2-D case. It is obvious that the larger the HV, the better the approximation is. The lift-to-drag radio and absolute value of moment coefficient of the base foil are 53 and 0.088 respectively, and the corresponding initial objectives using Eq. (15) is (0.472,0.88), so in this section, the reference point used is (0.56,0.9), which is a biased value from the initial objectives.

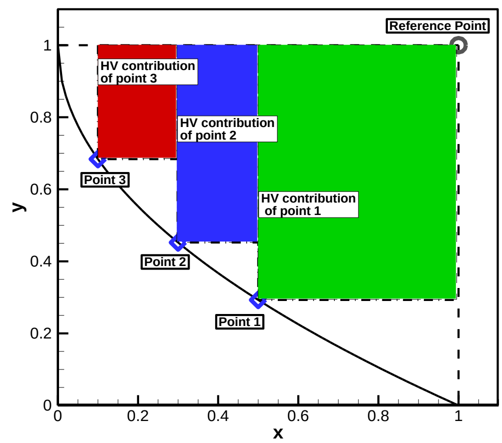
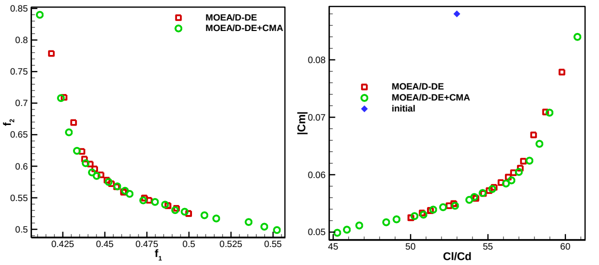


Fig. 2 Introduction of HV indicator. The solid line is the PF. And the colored area is the HV of point 1-3 with respect to the reference point used.

Fig. 3 gives the comparison of the obtained PF by the two algorithm. It can be seen that the PF obtained by MOEA/D-DE+CMA has a wider distribution, which is desired in multi-objective optimization. Fig. 4 gives the comparison of the evolution of HV versus iteration number. It can be seen that MOEA/D-DE+CMA converges faster than MOEA/D-DE. Also, MOEA/D-DE+CMA obtained larger HV than MOEA/D-DE at the end of the optimization, indicating a better result.



(a) PF obtained (b) PF of the real objectives

Fig. 3 Pareto front obtained of the optimization.

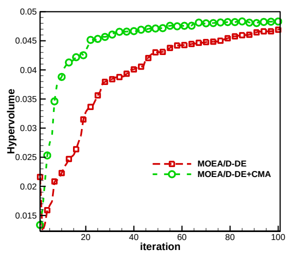
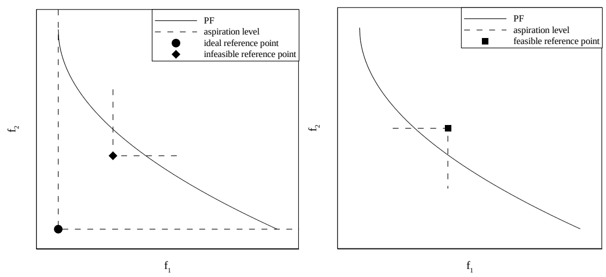


Fig. 4 The evolution of HV versus iteration number.

4 Optimization with user preference

4.1 Reference point based MOEA/D for MOP with preference (rpMOEA/D)

Based on the concept of achievement scalarizing function [22], the original MOEA/D itself is optimizing with preference, and the preference is expressed through the ideal reference point with the minimum values for each objective in the achievable space as the aspiration levels. Thus the preferred solutions are the whole PF. What if we use some arbitrary unachievable reference point? Let's say the diamond point in Fig. 5(a). Then the desired solutions should be the points on the PF but within the aspiration levels of the reference point, as shown in Fig. 5(a). The reason is that although such points are dominated by the unachievable reference point, their objective values are more similar with the reference point than the PF points out of the aspiration levels, which means more similar performance in engineering problems. For achievable reference point, the points on the PF within the aspiration levels will all dominate the reference point, as shown in Fig. 5(b). And these points will certainly be the desired points with respect to the achievable reference point. Therefore, reference point can be used to provide the DM's preference information which can be used by MOEA/D.



(a) (b)

Fig. 5 The desired solutions for different reference points. (a) unachievable reference point; (b) achievable reference point

In order to guarantee the obtained solutions are all Pareto optimal, the following achievement scalarizing function is used in rpMOEA/D to decompose the MOP:

$$\min g(x|w,z^*) = \max_{i=1,\ldots,m} w_i e_i + \rho \sum_{i=1}^{m} e_i \qquad (10)$$

where $e_i = f_i(x) - z_i^{ref}$ and z_i^{ref} is the ith objective value of the reference point z^{ref}.

4.2 Fuzzy preference based MOEA/D for MOP with preference (fpMOEA/D)

In this section, the fuzzy preference relation is used to determine the weight vector. In fuzzy preference relations, the preference are expressed by a preference matrix R whose elements $r_{ij} \in [0,1]$ are the preferences of objective f_i over f_j that satisfy the following conditions:

$$r_{ij} + r_{ji} = 1, r_{ii} = 0.5 \qquad (11)$$

Based on this preference relation matrix, the weight for each objective can be obtained by

$$w_i = \frac{S(f_i,R)}{\sum_{j=1}^{m} S(f_j,R)} \qquad (12)$$
$$S(f_i,R) = \sum_{j=1,\cdots,m,j\neq i}^{m} r_{ij}$$

If we specify a proper interval for r_{ij}, then we can obtain a set of weight vectors. Such weight vectors can reflect the preference of the DM, and MOEA/D is then used to optimize the problem using such weight vectors. Finally, we can obtain the preferred solutions.

4.3 Test function cases

The 2-objective functions ZDT1-3 and 6 are used to test the algorithm performance. The population size is set to 20, maximum iteration 300, crossover rate 0.5, and scaling factor 0.5. Fig. 6 gives the test result for rpMOEA/D. We can see that the objective values of the obtained solution are all around to the reference point.

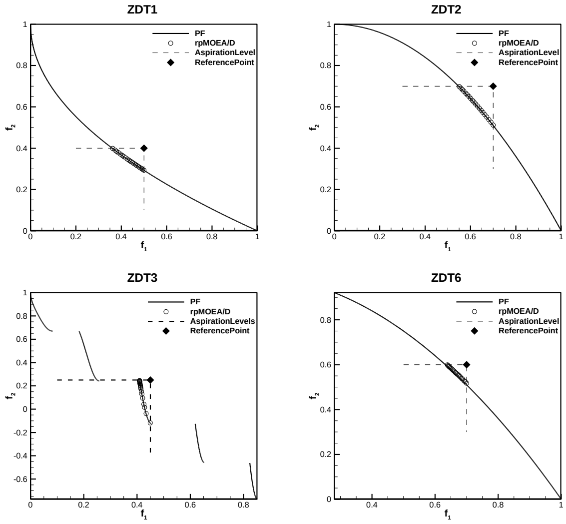


Fig. 6 Test function results for rpMOEA/D.

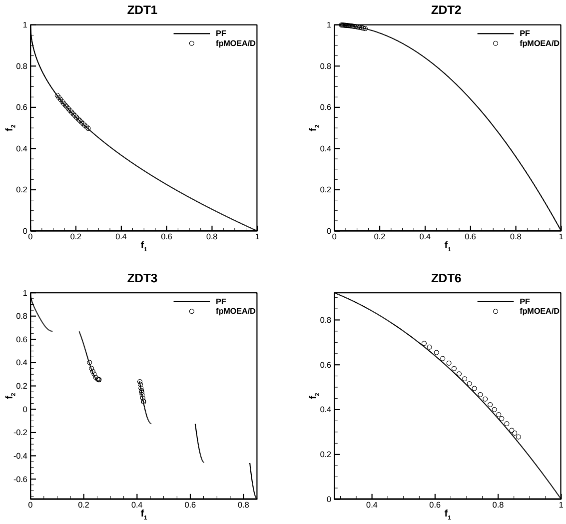


Fig. 7 Test function results for fpMOEA/D.

Secondly, we will present the test result of fpMOEA/D. For the 2-objective case, the method to determine the weight vectors is simple. We assume the two objectives are equally important and the weight vector varies between (0.3, 0.7) to (0.7, 0.3). The population size is set to 20, maximum iteration 300, crossover rate 0.5, and scaling factor 0.5. Fig. 7 shows the test function results for fpMOEA/D. We can see that the results matched with the fuzzy preference except for ZDT2. The reason may be that the decomposition method used in MOEA/D does don't suit will with ZDT2. Other decomposition methods should be studied.

4.4 Airfoil optimization case

In order to validate the methods in the previous sections, multi-objective aerodynamic airfoil shape optimization problem with 2 objectives adopted from reference [24] is used. The goal of this problem is to optimize the shape of a standard-class glider, aiming at obtaining optimum performance for a sailplane at different flight conditions.

PARSEC [24] airfoil representation is adopted to parameterize the airfoil. Fig. 8 shows the concept of PARSEC. There are two leading edge radius in the method used in this paper, the leading edge radius for upper and lower surface, and that is the only difference between the method used in this paper and Fig. 6. Hence, a total of 12 geometric parameters are used to determine the shape of an airfoil. The allowed ranges for the 12 parameters for the two optimization cases are shown in Table 1 according to [24].

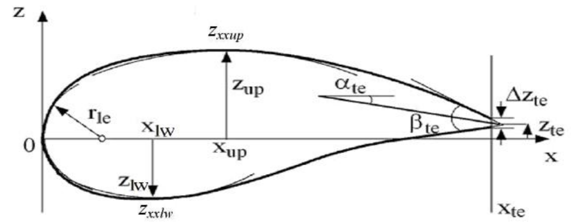


Fig. 8 PARSEC airfoil parameterization method.

Table 1. Ranges for the 12 geometric parameters.

		r_{leup}	r_{lelo}	α_{te}	β_{te}	Z_{te}	ΔZ_{te}
A720	upper	0.0126	0.004	10	14	-0.003	0.005
	lower	0.0085	0.002	7	10	-0.006	0.0025
		X_{up}	Z_{up}	Z_{xxup}	X_{lw}	Z_{lw}	Z_{xxlw}
A720	upper	0.46	0.13	-0.7	0.26	-0.015	0.2
	lower	0.41	0.11	-0.9	0.2	-0.023	0.05

This case is based on the A720 airfoil. Two conflicting objective functions are defined according to a sailplane average weight and operating conditions

$$\min f_1 = \frac{C_d}{C_l}, f_2 = C_d/C_l^{3/2} \qquad (13)$$

for f_1 the constrain is $C_l = 0.63, Re = 2.04 \times 10^6, M = 0.12$, and for f_1 the constrain is $C_l = 1.05, Re = 1.29 \times 10^6, M = 0.08$. The first objecttive represents the inverse of the glider's gliding ratio, whereas the second represents the sink rate. Both objectives are important performance measures for the glider. The computational fluid dynamics solver used is XFOIL [25]. The objective function values of the airfoil A720 is (0.007610, 0.005236). We will choose (0.007, 0.005) as the reference point which is smaller than the values of A720 for the first method. For the second method, we will test three types of fuzzy preference: the first objective is more important than, less important than and equally important as the second one.

Firstly we will show the experiment results of rpMOEA/D. The parameter setting of the optimization is the same with [24] where the population size is 60 and the maximum iteration is 80. The optimization engine is MOEA/D-DE with crossover rate CR=0.5 and the scaling factor F=0.5. The result is shown in Fig. 7. We can see that the solutions obtained by rpMOEA/D are all within the aspiration levels of the reference point. Both the result of rpMOEA/D and the final set of [24] are better than the solution obtained with no preference. Moreover, rpMOEA/D obtained better solutions than the final set of [24].

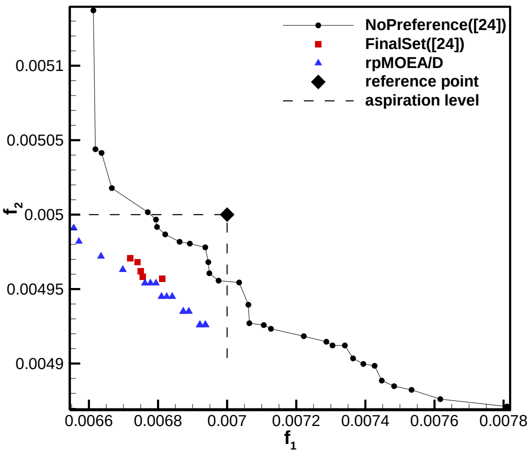


Fig. 9 Result comparison of rpMOEA/D and reference [24].

Secondly, we will show the results of the fpMOEA/D. Since this is a 2-objective case, obtaining the weight vectors will be simple. We conducted three cases: 1) objective 1 is more important than objective 2; 2) objective 1 is equally important than objective 2; 3) objective 1 is less important than objective 2. The results are shown in Fig. 10. We can see that when different fuzzy preference is given, different Pareto optimal solutions are obtained. Comparing with the no preference result in [24], better solutions are obtained using fpMOEA/D.

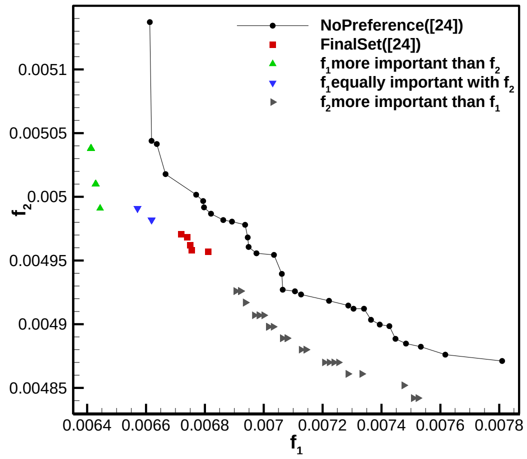


Fig. 10 Result comparison of fpMOEA/D and reference [24].

5 Conclusion

In this paper, we studied the application of MOEA/D to the aerodynamic multi-objective optimization design. Firstly a hybrid operator of CMA-ES and DE is proposed to improve the convergence performance of MOEA/D, and the experimental results show that the new operator can speed up the convergence. Secondly, two methods of optimization with user preference are discussed, and the results show that the reference point method performances better than the fuzzy preference method. The results of this paper suggest that MOEA/D seems a promising algorithm to solve aerodynamic multi-objective optimization problem.

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