

# ANALYSIS OF THE INFLUENCE OF THE EXTERNAL CONFIGURATION ON THE WAVE DRAG OF SUPERSONIC BUSINESS JETS

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## Abstract

*This article presents a volumetric wave drag analysis for the conceptual design of a supersonic business jet airplane (SSBJ), by analyzing the influence of different external configurations (such as wing geometry, engine locations, and intake locations) on the volumetric wave drag. The analysis was done by employing Linearized Supersonic Area Rule, properly implemented on a VBA routine, coupled with a cone-based area extraction routine, implemented on a three-dimensional CAD environment.*

## 1 Introduction

Supersonic business jet aircraft (SSBJ) concepts are nowadays under intense studies, not only because of academic and technological interests, but also because of their potential strategic niche in the business jet market; this is mainly due to their remarkable flexibility, availability, and speed, which results in their potential to be an extremely time-saving means of transportation [1]. Furthermore, SSBJs might be much more viable in the mid-run than other possibly larger supersonic aircraft, because of the mission requirements in speed, range, and number of passengers tend to be more easily achievable for the former; these characteristics

coupled which a possibility of lower sonic boom intensity contribute to enhancing the economic and environmental viabilities of SSBJs [2].

The potential of SSBJs, thus, makes essential that their design is competitive enough, in order to prove their viabilities and technological feasibility [3]; hence, these characteristics must be pursued as early as the conceptual design starts, in order to aim for more efficiency.

One of the most critical areas for a supersonic airplane concept to be successfully efficient is its aerodynamics, not only due to issues with sonic boom intensity but also because supersonic flight implies reduced aerodynamic efficiencies [4]; this is so because of the sharp increase in drag experienced when entering transonic regime, mainly due to the growth of wave drag (both volumetric wave drag and wave drag due to lift). Moreover, although sonic boom intensity also represents an important issue for SSBJ design, because of the current strict regulations about supersonic flight over land [5], it was shown [5] to be possible to reduce supersonic boom intensity by properly modifying the aircraft operation, tailoring the flight path, for example; therefore, since solutions for the sonic boom intensity can be sought from operational criteria, the main design concern, thus, becomes drag reduction.

In comparison, the parasitic drag ( $C_{D0}$ ) experiences greater increase than the induced drag [4] (including wave drag due to lift), when in the transonic and supersonic ranges; also, because roughly 20% of the total supersonic drag in cruise comes from volumetric wave drag [6], the latter was the main focus of the conceptual approach on drag reduction developed within this study.

The methodology employed for evaluation of the volumetric wave drag was based on the Linearized Supersonic Area Rule [7], whose quick and readily available results lend themselves well to optimization routines, for example, which is highly desirable in the conceptual stage of design. The method was implemented on a VBA routine, coupled with an algorithm in a CAD environment for extraction of geometric properties. The computational routine was, thus, named Supersonic Aerodynamic Optimization for Design, version 0.1 (SAOD\_V0.1).

Lastly, in order to conduct a comparative analysis, several external configurations of a typical SSBJ concept, with different wing and tail geometries, and different intake positions were separated to be evaluated with the cited methodology; these changes in configurations were chosen because fuselage–wing and fuselage–propulsion integrations greatly influence the aircraft equivalent area distributions [5], being, thus, of exceptional importance in supersonic design.

## 2 Methodology

### 2.1 Drag Evaluation

The volumetric wave drag was evaluated through the Linearized Supersonic Area Rule [7], with some modifications. This method calculates the drag based on the aircraft equivalent area distributions, i.e., areas are obtained from the models drawn in CAD environment by intersecting the aircraft geometry with planes inclined at the flight Mach angle (Mach planes), at various equally spaced stations along its length.

The original methodology proposes repeating the procedure with Mach planes

rotated with respect to the longitudinal axis of the aircraft, giving, thus, area distributions for each rotation angle; afterwards, the drag values for each angle are calculated, with its effective value being the average of the previous values. In order to reduce the number of CAD iterations, this work proposes obtaining the equivalent area distributions by intersecting the geometry with two half–Mach cones (half cones whose aperture is double the Mach angle), which return two average equivalent area distributions, which are added to yield the final distribution, from which the drag is calculated.

Once the final equivalent area distribution is obtained, its derivatives are calculated with a finite difference scheme, eq. (1), where  $S_i$  represents the area for each station  $i$ ,  $S'_i$  the area derivative, and  $x$  the position of each station, starting from the aircraft nose.

$$S'_i = \frac{S_{i+1} - S_{i-1}}{x_{i+1} - x_{i-1}} \quad (1)$$

The derivative distribution is, then, interpolated with a Fourier series of sines, eq. (2) and eq. (3), where  $N$  represents the number of stations along the airplane length,  $\Delta x$  the spacing between them, and  $A_j$  the Fourier coefficient relative to each harmonic  $j$  ( $j$  going from 1 to  $N$ ).

$$A_j = \frac{2}{N} \sum_{i=1}^N S'_i \sin(j\varphi_i) \quad (2)$$

$$\varphi_i = \frac{\pi x_i}{N\Delta x} \quad (3)$$

Drag computation is achieved through eq. (4), from [7], where  $D$  represents the drag, calculated with local air density ( $\rho$ ) and flight speed ( $V$ ), both coming from the flight Mach number and the flight altitude (considering an International Standard Atmosphere—ISA—model). Lastly, the drag may be made dimensionless, yielding the drag coefficient,  $C_D$ , found from eq. (5), with  $S_{ref}$  being the aircraft planform wing area, also extracted from the CAD drawing.

$$D'_j = \frac{\pi}{8} \rho V^2 \sum_{k=1}^N k A_{kj}^2 \quad (4)$$

$$C_D = \frac{2D}{\rho V^2 S_{ref}} \quad (5)$$

The whole procedure is implemented on a VBA routine, illustrated in the flowchart below, Fig. 1.

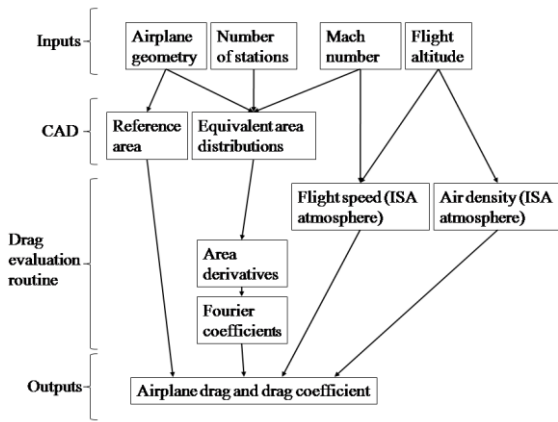


Fig. 1: Drag Evaluation Flow Chart

## 2.2 Validation

In order to validate the implemented procedure, the computed results were compared to those obtained from the analytical methodology present in [8], shown below, in equations (6), (7), (8), and (9):

$$C_{D_{wave}} = \frac{4.5\pi}{S_{ref}} \left( \frac{A_{max}}{L} \right)^2 A \cdot B \quad (6)$$

$$A = E_{WD} (0.74 + 0.37 \cos \Lambda_{LE}) \quad (7)$$

$$B = 1 - 0.3 \sqrt{M - M_{C_{D0}max}} \quad (8)$$

$$M_{C_{D0}max} = \frac{1}{\cos^{0.2} \Lambda_{LE}} \quad (9)$$

Above,  $A_{max}$  represents the maximum cross-sectional area for the aircraft,  $L$  being its length, and  $\Lambda_{LE}$  being its leading edge sweep

angle (for variable sweep,  $\Lambda_{LE}$  was taken as the average of the leading edge sweep angles, weighed by their semi-span); all of these parameters being possible to extract from the CAD environment; in addition, as of [8], the parameter  $E_{WD}$ , empirical wave drag efficiency, was taken to be 2.0, as an average of typical values for usual supersonic aircraft.

## 2.3 Mission Requirements

The analyses in this article were conducted considering flight at two different Mach numbers: 1.6, as cruise Mach number over water, and 1.15, as cruise Mach number over countries which permit supersonic flight over their territory. These values were selected because Mach 1.6 does not present issues of severe aerodynamic heating (Horinouchi, 2005, cited in [3]) and necessity of variable inlet geometry [4], whereas Mach 1.15 is a common approximation [3] for the “cutoff Mach number” (Mach number for the sonic boom not reach the ground).

Flight cruise altitude was taken to be about 11 km (up to 40 000 ft), because it reduces both the environmental impact of the flights and the exposition of the passengers and crew to cosmic radiation [3], in comparison to mid-stratospheric flights. Fuselage length and width were taken as enough to allow a total of nine passengers, which previous researches [1] have elected as a minimum necessary for this aircraft category.

All of the aspects discussed under this section are summarized under Table 1, below.

Mission requirements	
Cruise Mach numbers analyzed	1.6 (over water) and 1.15 (cutoff Mach number for land)
Cruise altitude	40 000 ft
Number of passengers	9

Table 1: Mission Requirements

## 2.4 Analyzed Configurations

Three different aircraft configurations were analyzed and compared, being illustrated by Fig. 2, Fig. 3 and Fig. 4, with their

geometrical properties shown under Table 2, below.

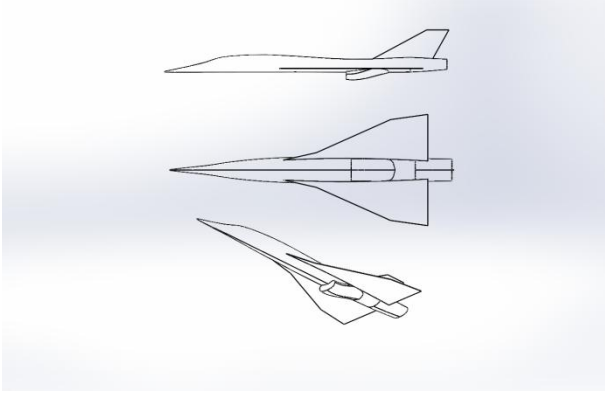


Fig. 2: Model 1 views

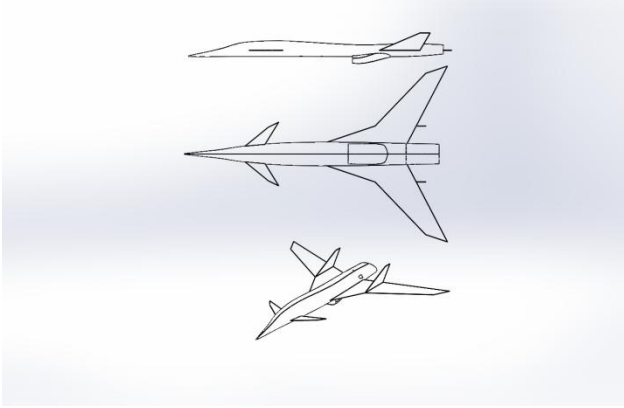


Fig. 3: Model 2 views

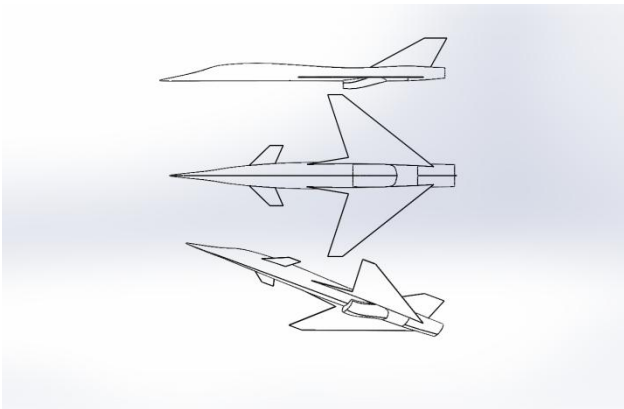


Fig. 4: Model 3 views

All of the analyzed configurations share the same fuselage geometry (shape, length and cross-sections), but with different planform wing shapes and areas, different leading edge

and trailing edge sweep angles (with model 1 not having trailing edge sweep and model 3 having negative leading and trailing edge sweep), and lack of canards for model 1, whose areas are the same, for models 2 and 3.

Aircraft geometrical properties			
Property	Model 1	Model 2	Model 3
Reference area [m <sup>2</sup> ]	74.87	72.91	75.44
Length [m]	29.92		
Wing span [m]	11.82	20.65	16.96
Leading edge sweep angle [deg]	69.62	52.38	-8.15
$M_{C_{D0}max}$	1.235	1.104	1.002
Maximum cross-sectional area [m <sup>2</sup> ]	4.770	5.923	5.041

Table 2: Aircraft geometrical properties

### 3 Results and discussion

The three aircraft models were analyzed using SAOD\_V0.1 (according to section 2.1), dividing each model into 20 stations along its length, and, for comparison and validation, using the analytical methodology from section 2.3; the results are presented below, under Table 3.

Aircraft volumetric wave drag coefficient (in drag counts)				
Methodology	Computational		Analytical	
Mach	1.15	1.6	1.15	1.6
Model 1	95.0	76.5	N/A	68.3
Model 2	91.8	77.8	137.3	115.8
Model 3	94.4	81.4	104.1	90.4

Table 3: Volumetric drag coefficients (in drag counts)

Above, the calculation of volumetric wave drag at Mach 1.15 for Model 1 by the analytical method was not applicable because this value is lesser than the value of  $M_{C_{D0}max}$  for this airplane. In addition, the relative error between methodologies is shown below, under Table 4, being calculated from relation (10).

$$\frac{|C_{D_{computational}} - C_{D_{analytical}}|}{C_{D_{analytical}}} \quad (10)$$



Relative error between computational and analytical methodologies		
Mach	1.15	1.6
Model 1	N/A	12.0%
Model 2	33.1%	32.8%
Model 3	9.33%	10.0%

Table 4: Relative error between methodologies

The area distribution of the three aircraft configurations is performed in RAPID [9][10]. The Fig. 25(a), Fig. 25(b) and Fig. 25(c) shows the area distribution over the length of the aircraft for Mach numbers 1.15 (in blue) and 1.6 (in black).

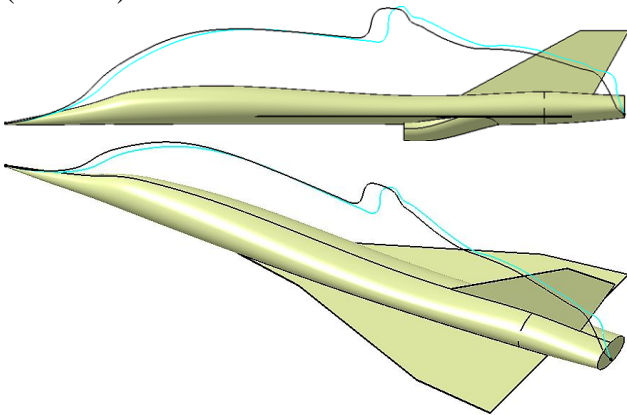


Fig. 5(a): Model 1 area distribution

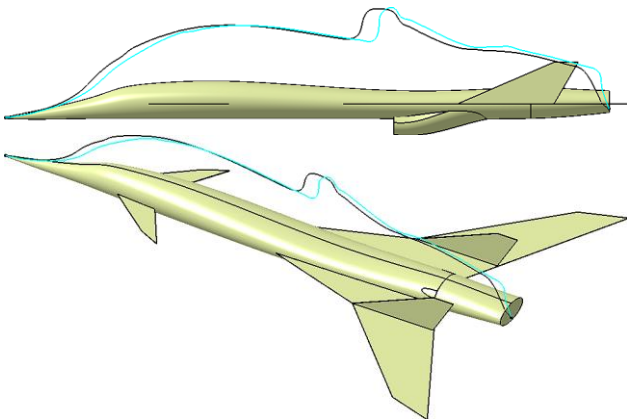


Fig. 5(b): Model 2 area distribution

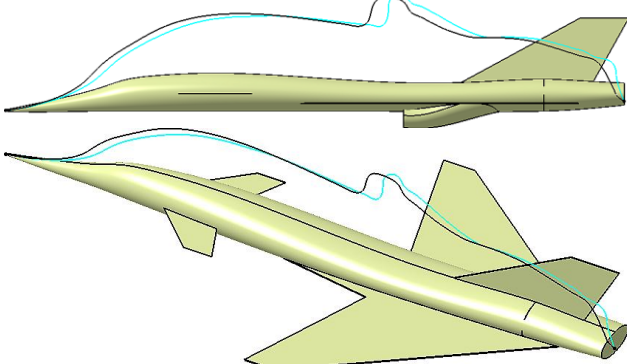


Fig. 5(c): Model 3 area distribution

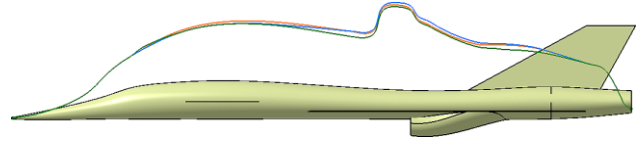


Fig. 5(d): Area distribution of all the three models at Mach 1.6

Fig. 25: Area distribution of three aircraft configurations.

There are two peaks in the area distribution shown in Fig. 25. The first peak above the nose of the fuselage is fairly smooth that is a result of having a progressively increasing diameter of the front section of the fuselage. The second peak is above the air inlet, this is not avoidable as a lot of geometry is suddenly added with the addition of the inlet. The area distribution at second peak smoothens with the increase in speed of the aircraft thus reducing the supersonic drag of the aircraft for cruise speed at Mach 1.6. As seen in Fig. 25(d), the area distribution at Mach 1.6 for Model 1 (in blue), Model 2 (in green), Model 3 (in orange), the Model 2 has the least area distribution among the three aircraft configurations.

From the results obtained, model 2 achieved the smallest wave drag values: this may be explained by the fact that whereas the equivalent cross-sectional area distributions for the three models are close, model 2 has the best distribution, because its wing shape, leading and trailing edge sweep angles, and double vertical tail contribute to smooth the final area distribution downstream of the wing, which, in turn, reduces volumetric wave drag, in comparison to the other models.

## 4 Conclusions

This work presented volumetric wave drag analyses for different proposed supersonic business jet aircraft configurations, done employing code SAOD\_V0.1, developed by the researchers.

The code, designed for effective communication with most 3D CAD environments, was validated by employing an analytical method, from reference [8], and yielded reasonable drag results for the

considered cases. also, further improvements in the code performance and accuracy are possible.

In addition, the code may also be coupled with CAD tools, such as parametric design routines, and with optimization routines, resulting in a desirable resource for aircraft design in conceptual stage; this will be done in future works.

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