MATHEMATICAL MODEL OF COMMERCIAL AIRPLANE DYNAMICS AT STALL

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Abstract

Errors of flight crew during piloting at upset and stall are one of the main reasons of commercial plane crash today. Modern flight augmentation systems provide airplanes with limiters of flight parameters such as angle of attack (AoA), sideslip angle and others. Despite of this fact statistics shows that such accidents continue to occur. Therefore, today it is important to train pilots with correct actions at upset and stall. To achieve this goal, we need in adequate mathematical model of airplane dynamics at high AoA.

This article describes an approach used in TsAGI for simulation commercial airplane dynamics at such regimes. This model includes the dynamic hysteresis in longitudinal aerodynamics characteristics. Moreover, article demonstrate the hysteresis effect on airplane dynamics in comparison with the model without hysteresis. Also the model contains additional forces and moments in three main axes to simulate such phenomena as buffeting and non-symmetric flow separation which occur at high angles of attack.

1 Introduction

Analysis of the behavior of lift force and pitch moment coefficients, given by wind tunnel experiments [1-2], showed that aerodynamic hysteresis at high AoA appears. Identification of characteristics $c_L(\alpha, M)$, $c_m(\alpha, M)$, based on data, given by flight, confirmed that hysteresis in $c_L(\alpha)$ and $c_m(\alpha)$ appears at high AoA for different Mach numbers [3]. Moreover, analysis of hysteresis loops shows that hysteresis takes a place at certain AoA and its behavior depends on AoA derivative. Nowadays, accounting of hysteresis in flight simulators is innovative goal, because it is a critical regime for an aircraft and a pilot, therefore, it is too important to represent the behavior of an aircraft close to real conditions. Accurate representing of critical regimes should lead to more efficient flight crew training. Previously, many authors offer the approaches to a mathematical simulation of aerodynamic hysteresis phenomenon [4-10].

The most prevalent approach is the simulation of aerodynamic hysteresis via first-order filter $\dot{c}_{\text{out}} = \frac{1}{T}(c_{\text{in}} - c_{\text{out}})$. This approach is considered in the paper [7], where parameters T and c_{in} correspond to AoA and its derivative. Another interesting approach to the description of non-steady aerodynamic characteristics is presented in paper [8], where lift force characteristic is separated into linear and non-linear components.

In the range of papers, there are many intricate models of hysteresis. For example, an additional variable, which characterizes the position of separation point on wing chord, is provided by paper [10] for simulation of longitudinal dynamics of an aircraft at high AoA. In this paper, authors offer to use three first-order filters: for calculation of non-linear component of lift force coefficient, for the aerodynamic center of pressure and for representation of flow angularity dynamics. Even more, the extended model that is used six parameters is described in ONERA paper [9]. These models can present more accurate results, than models with first-order filters, but there is lack of recommendations how to choose the parameters for the model in these papers or there are no such recommendations at all. Another and more important disadvantage of these approaches is that they do not provide offering how to use these models for simulation of dynamics of full-size airplane. Consequently, there is no comparison of the models with the data given by real flights.

The model, which we consider in our paper, is based on the model of non-steady aerodynamic, proposed by paper [7], because the authors propose the detailed algorithm of selection of the first-order filter parameters. The paper is organized as follows: in section 2, we present the algorithm of non-steady characteristics calculation that is used in the research of aircraft dynamics and comparison between calculations and experimental data; in section 3, we represent the results of dynamics research of the civil aircraft; section 4 is devoted to such issues as asymmetric flow separation and buffeting.

2 Mathematical Simulation of Hysteresis

This paper presents the modification of approaches of hysteresis simulation in lift force $c_L(\alpha,\dot{\alpha})$ and pitch moment $c_m(\alpha,\dot{\alpha})$ coefficients developed in papers [7,8]. This modification is based on data, given by wind tunnel T-103 TsAGI in the case of harmonic oscillations of AoA at flow velocity 25 m s^{-1}, and data, given by flight [3]. As the result, we developed the algorithm of calculation of lift force and pitch moment non-steady coefficients, which will be described below.

2.1 Mathematical model

First of all, coefficients of lift force and pitch moment, given by wind tunnel, was represented in a composition of two components.

$$\begin{aligned} c_L(\alpha,\dot{\alpha}) &= c_{LD}(\alpha,\dot{\alpha}) + \Delta c_{L\,rd}, \\ c_m(\alpha,\dot{\alpha}) &= c_{mD}(\alpha,\dot{\alpha}) + \Delta c_{m\,rd}, \end{aligned} \tag{1}$$

where $c_{mD}(\alpha,\dot{\alpha})$ and $c_{LD}(\alpha,\dot{\alpha})$ are averaged over all realizations of aerodynamic characteristics $c_m(\alpha,\dot{\alpha})$ and $c_L(\alpha,\dot{\alpha})$, that are called regular components; $\Delta c_{L\,rd}$ and $\Delta c_{m\,rd}$ – random components, which contain the difference between regular components and every realization.

The regular components affect aircraft's dynamic stronger than the random one, therefore, we consider the random components only in the section 4. Further, regular components of lift force and pitch moment calculated via mathematical model will be marked as c_{Lr} and c_{mr} respectively.

For calculation of the regular component of pitch moment coefficient c_{mr} according to paper [7] we use the first-order filter:

$$\dot{c}_{mr} = \frac{1}{T}\left(c_{m\,\mathrm{in}} - c_{mr}\right), \tag{2}$$

where $c_{m\,in}$ – input parameter, T–time constant.

For calculation of lift force regular component c_{Lr} according to paper [8] we use separation of the characteristic into linear and non-linear components. Thus, for lift force coefficient we have the equations below:

$$\begin{aligned} c_{Lr} &= c_{Lm} + c_{Ll}, \\ \dot{c}_{Lm} &= \frac{1}{T}\left(c_{L\,in} - c_{Lm}\right), \\ c_{Ll} &= c_L^{\alpha}\alpha, \end{aligned} \tag{3}$$

where $c_{L\,in}$ – input parameter, T – time constant, c_L^{α} – derivative of the static characteristic at $c_L(\alpha)$ the range of AoA, where $c_L(\alpha)$ have a linear dependence on AoA, c_{Lm} – non-linear component of lift force characteristic.

For calculation of c_{Lr} and c_{mr} we use the stationary aerodynamic characteristics $c_{L\,st}(\alpha)$ and $c_{m\,st}(\alpha)$ and values of AoA α_0, α_1, α_2 and α_3, given by experiment. These values of angles of attack separate all range of AoA into several zones with different types of dynamic characteristics behavior. This separation is represented in fig. 1. Also, the figure shows the behavior of regular components of aerodynamic characteristics of lift force and pitch moment c_{LD} and c_{mD} given by wind tunnel experiments in the case of harmonic oscillations with frequency f = 1 Hz, amplitude A_α = 5° and velocity V=25 m s^{-1}. Paper [7] gives physical interpretation of values α_0, α_1, α_2 and α_3:

α_0 – AoA, where flow ends up rebuilding into detached flow in the case of $\dot{\alpha} < 0$;

α_1 – AoA, where regular components of aerodynamic characteristics start to save tendency that was evolved up to this moment in the case of $\dot{\alpha} > 0$;

α_2 – AoA, where flow ends up rebuilding into separated flow in the case of $\dot{\alpha} > 0$;

α_3 – AoA, where flow starts its rebuilding in the case of $\dot{\alpha} < 0$.

In each zone in fig. 1 the input parameters and the time constants of the first-order filters correspond to different dependencies. Selection of these dependencies generally match with selection method offered by paper [7], but this model has some modifications, therefore, we present dependences of input parameters and time constants for all range of AoA.

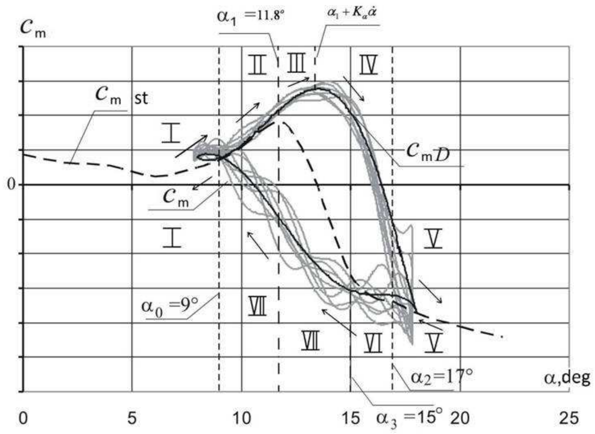


Fig. 1a. Hysteresis in pitch moment coefficient.

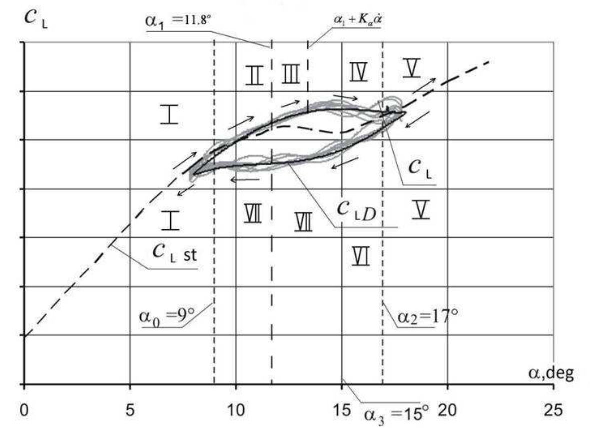


Fig. 1b. Hysteresis in lift force coefficient.

In zones I, II, V and VI the regular components of c_{Lr} and c_{mr} just coincides with the stationary characteristics of $c_{L\,st}$ and $c_{m\,st}$, therefore, parameters of the eq. (2) and (3) are selected as follow. The input parameter in calculation of pitch moment characteristic is equal to the stationary dependence of pitch moment on AoA, in eq. (3) such parameter is equal to non-linear component of stationary lift force characteristic $c_{L\,in}(\alpha) = c^*_{L\,st}(\alpha)$, where:

$$c^*_{L\,st}(\alpha) = c_{L\,st}(\alpha) - c_{Ll}(\alpha) \qquad (4)$$

$c_{Ll}(\alpha)$ is a linear component of stationary characteristic $c_{L\,st}(\alpha)$. The time constant is selected small to provide well agreement of the regular components and the static dependencies. In the paper T=0.025 s in these zones that provide the filter with bandwidth $\omega_d = \frac{1}{T} = 40$ s^{-1}. This value is more than short-motion natural frequency, therefore, output variables will track the input parameters with high accuracy.

In zone III, where dynamic characteristics save the tendency of behavior, which evolved in previous zone (II) the input parameters, are selected as follow:

$$\begin{aligned} c_{m\,in}(\alpha) &= c_{m\,st}(\alpha_1) + \frac{dc_{m\,st}}{d\alpha}(\alpha - \alpha_1), \\ c_{L\,in}(\alpha) &= c^*_{L\,st}(\alpha_1) + \frac{dc^*_{L\,st}}{d\alpha}(\alpha - \alpha_1), \end{aligned} \qquad (5)$$

where the derivatives mean in first approximation as

$$\begin{aligned} \frac{dc_{m\,st}}{d\alpha} &= \frac{c_{m\,st}(\alpha_1) - c_{m\,st}(\alpha_1 - \Delta\alpha)}{\Delta\alpha}, \\ \frac{dc^*_{L\,st}}{d\alpha} &= \frac{c^*_{L\,st}(\alpha_1) - c^*_{L\,st}(\alpha_1 - \Delta\alpha)}{\Delta\alpha}, \\ \Delta\alpha &= 1^\circ \div 2^\circ \end{aligned} \qquad (6)$$

Similarly to previous zones in zone III output variables have to accurately track the input parameters (5), therefore, the time constant is selected as T=0.025 sec. Width of zone III $\Delta\alpha_1$ linearly depends on AoA derivative: $\Delta\alpha_1 = K_\alpha \dot{\alpha}$.

In zones IV and VII flow is rapidly separating ($\dot{\alpha} > 0$) and recovering ($\dot{\alpha} < 0$) respectively and the regular components in these zones tend to their static dependences, therefore, the input parameters are equal to the stationary characteristics $c_{m\,in}(\alpha)$=$c_{m\,st}(\alpha)$, $c_{L\,in}(\alpha) = c^*_{L\,st}(\alpha)$. To take into account the dynamics of flow separation and recovering the time constant is selected as follows:

$$T = \begin{cases} K_n \dot{\alpha}_{n\ \lim} & \text{if} \quad |K_n \dot{\alpha}_n K_{1,2}| < |K_n \dot{\alpha}_{n\ \lim}| \\ K_n \dot{\alpha}_n K_{1,2} & \text{if} \quad |K_n \dot{\alpha}_n K_{1,2}| > |K_n \dot{\alpha}_{n\ \lim}| \end{cases} \qquad (7)$$

Parameter K_n is the coefficient of linear dependence of width of hysteresis loop on AoA derivative.

Parameter $\dot{\alpha}_{n\,\lim}$ is a boundary of AoA derivative, where we start to consider hysteresis phenomenon. If value of $\dot{\alpha}$ is too small the regular components are coincide with the stationary dependences.

For working with experimental data given by wind tunnel, we use a reduced value of AoA derivative. The experimental model was made at scale $\frac{L}{L_m} = 25$, therefore, parameter $\dot{\alpha}_n$ can be selected as $\dot{\alpha}_n = \frac{\dot{\alpha}}{25}$.

Parameters K_1 and K_2 for zones IV and VII respectively are introduced for an accounting the fact of equality between the regular components of aerodynamic characteristics and the static dependencies of corresponding characteristics – within the area where flow is not rebuilding. These parameters are calculated as follows:

$$K_1 = \frac{(\alpha_2 - \alpha)(\alpha - \alpha_1)}{(\alpha_{av1} - \alpha_1)(\alpha_2 - \alpha_{av1})}, \quad K_2 = \frac{(\alpha_3 - \alpha)(\alpha - \alpha_0)}{(\alpha_{av2} - \alpha_0)(\alpha_3 - \alpha_{av2})}, \qquad (8)$$

where $\alpha_{av1} = \frac{\alpha_1 + \alpha_2}{2}$, $\alpha_{av2} = \frac{\alpha_0 + \alpha_3}{2}$.

As the result, we have the algorithm for calculation the regular components of pitch moment $c_{mr}(\alpha, \dot{\alpha})$ and lift force $c_{Lr}(\alpha, \dot{\alpha})$ coefficients.

2.2 Experiment

The example of calculation c_{Lr} and c_{mr} in the case of harmonic oscillations with amplitude $A_\alpha = 5°$, frequency $f = 1$ Hz and initial AoA $\alpha_{init} = 10°$ is represented in fig. 2. In addition, there are the regular components of lift force c_{LD} and pitch moment c_{mD} coefficients from wind tunnel experimental data in the fig. 2. Moreover, fig. 3 demonstrate the results of calculation of aerodynamic characteristics via mathematical model, adapted to real flight conditions, in comparison to results of characteristics identification from real flight.

Fig. 2 and 3 clearly show that the mathematical model, described above, is in good agreement with the wind tunnel experiment and the flight identification. Therefore, we can conclude, that the developed model can be used for an accurate simulation of the hysteresis in lift force and pitch moment coefficients. For the simulation, we have to know the stationary dependencies of corresponding coefficients and the range of parameters determined by dynamic experiments in a wind tunnel.

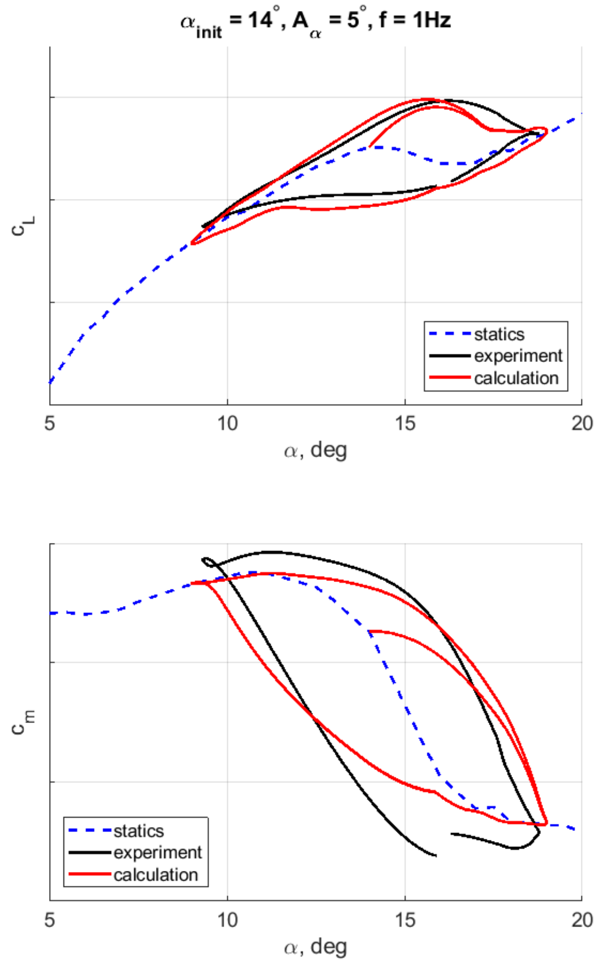


Fig. 2. Comparison model calculation with wind tunnel experiment

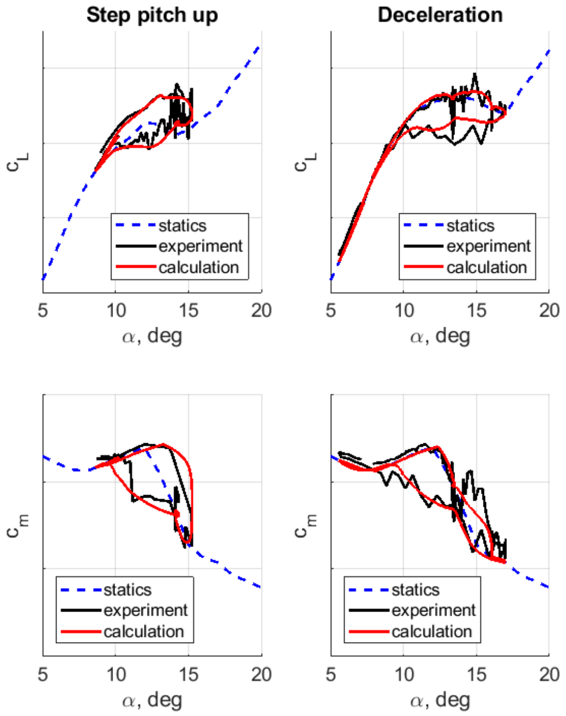


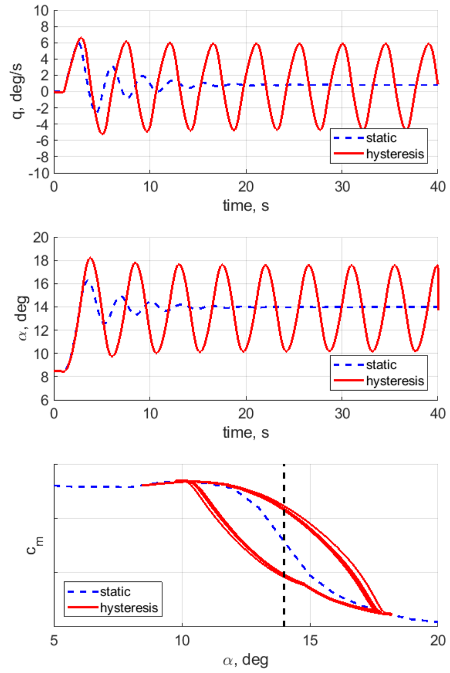
Fig. 3. Comparison model calculation with flight identification. Left – pilot's step pitch up, right – deceleration

3 Research of Dynamics in the Case of Aerodynamics Hysteresis

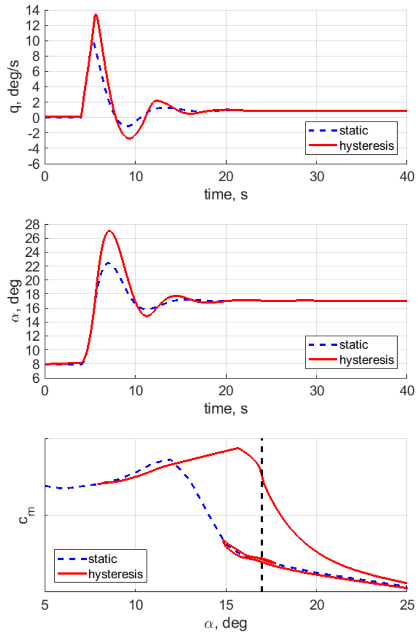
After the variable parameters have been tuned, the model was used for simulation of longitudinal dynamics of civil aircraft.

First, we consider the dynamics of free aircraft (without flight control augmentation system). Pitch rate and AoA transients to step-control deflection in the attempt to reach 14 and 17 degree of AoA are represented in fig. 4. Graphs show the behaviors for two types of aircraft model: model with static aerodynamic characteristics and model with aerodynamic with the hysteresis accounting. The figures demonstrate some following features of short-motion at Mach number 0.3:

- Aerodynamic hysteresis in lift force and pitch moment characteristics has strong effect on longitudinal dynamic of the aircraft without flight control augmentation system.
- The hysteresis effect decrease if we attempt to reach AoA that lay to the right from zone of flow rebuilding.
- Attempts to reach AoA that lay in the middle area of flow rebuilding zone (for example α=14°) lead to self-oscillation regime of the aircraft.



a) α=14°



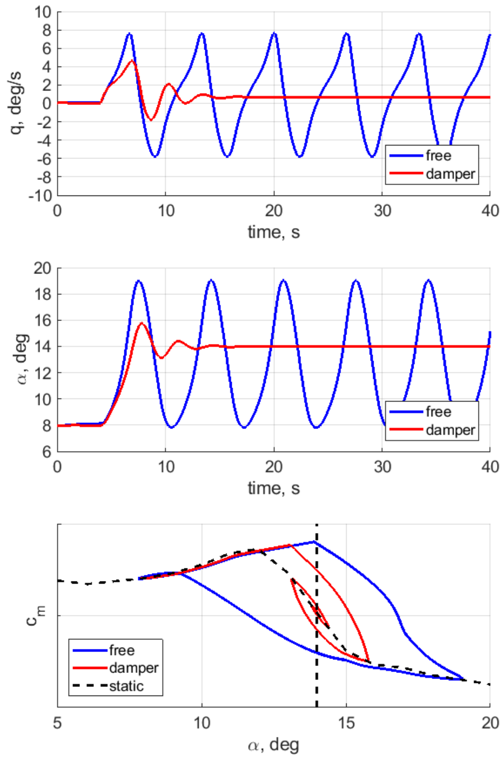
b) α=17°

Fig. 4. Comparison short-motion dynamics of aircraft without control system with static aerodynamic characteristics and hysteresis in attempt to reach different AoA

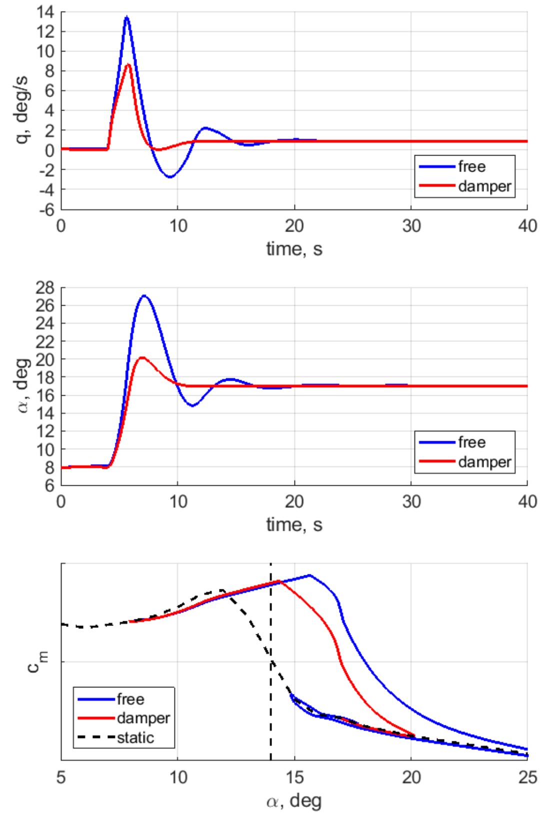
In addition, the figures include pitch moment dependences on AoA for these processes. The graphs demonstrate growth and damping of self-oscillations. Moreover, the graphs represent that the reason of these oscillations is the hysteresis phenomenon.

Now it is important to consider how we can improve stability characteristics via a pitch damper. Fig. 5 shows the responses of the aircraft with the pitch damper to step deflections of elevator in attempts to reach the same AoA (α=14°,17°).

The graphs show that the pitch damper significantly decrease the influence of hysteresis of lift force and pitch moment characteristics. Another advantage of the damper is that it damps the self-oscillations, caused by the hysteresis in the attempt to reach AoA that lays in the middle of the flow-rebuilding zone. On the other hand, the pitch damper cannot reduce the hysteresis effect at all. For example, fig. 5 shows that in the case of aircraft with the pitch damper, the hysteresis leads to more than 20% overshoot increase.



a) α=14°



b) α=17°

Fig. 5. Comparison between short-motion behaviors of aircraft with pitch damper in the case of aerodynamic hysteresis and static characteristics in attempt to reach different AoA

4 Simulation of Civil Airplane Dynamics at Stall AoA

Hysteresis of aerodynamic characteristics is one of the main features of airplane dynamic at high AoA. In addition, airplane has a series of others aerodynamics phenomena, which affect its dynamics. One of these phenomena is buffeting, which occur at high AoA. Identification of flight parameters such as AoA and normal g-load during “decelerations” is illustrated by Fig. 6.

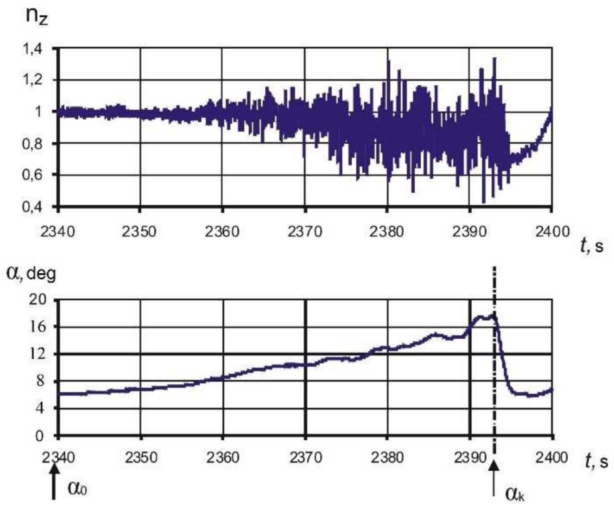


Fig.6a. Identification of “deceleration” flight parameters, cruise configuration

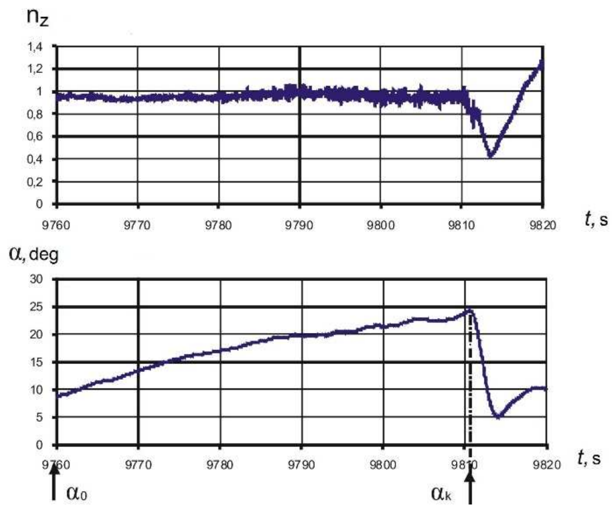


Fig.6b. Identification of “deceleration” flight parameters, flaps extended configuration.

Identification results have two main features: firstly - an appearance and growing of buffeting due to AoA is clearly evident, secondly - extraction of high-lift-devices leads to reduction of buffeting intensity. Further, let’s consider spectral densities, which is presented in Fig. 7. Graphs demonstrate that there are about 4 peaks highlighted in red. Another important feature is that buffeting amplitude grow as a polynomial function of AoA in the range from start of flow rebuilding to the middle of flow rebuilding zone. This dependency is shown as red plot at Fig 7. The following model of buffeting was developed. Summary aerodynamic shaking consists of two components: random component (filtered white noise) and periodical component. Second component is described by pilots as “kicks”.

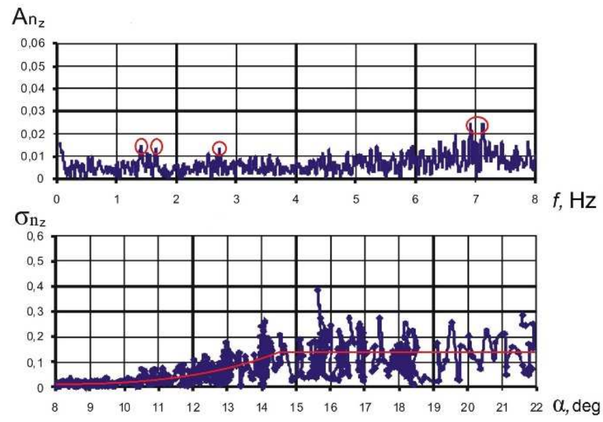


Fig.7a. Spectral densities of normal g-load and standard deviation due to AoA for cruise config.

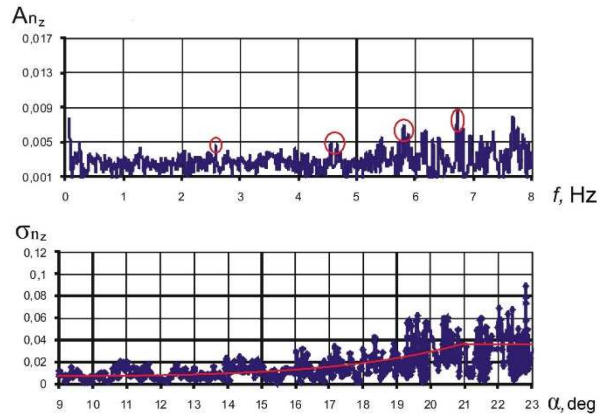


Fig.7b. Spectral densities of normal g-load and standard deviation due to AoA for FE config.

Except shaking in normal direction pilots feel lateral shaking. Spectral density of lateral g-load is analogous to normal g-load. But there are no specific picks in lateral g-load spectral density, that means absence of “kicks” in lateral channel, only random component.

Now let’s consider lateral movement of an airplane. There are two main features in this axes: an asymmetrical flow separation and wing stall. Nose-down stall was not considered in this article because in there is no dangerous features in aircraft dynamics in this case. Typical dependency of roll rate on time at high AoA in the case of a “deceleration” maneuver is represented in Fig. 8. Graph demonstrates common behavior of plane and especially moment of wing stall. Two specific features were extracted from such flight data. First - increase of periodic oscillations of roll rate due to AoA. Such behavior was not induced by pilot. Second – aperiodic intensive increase of roll moment that forced pilot to parry it by side deflection of control lever and reducing AoA by pushing control lever. Such two types of disturbances were included to mathematical model. Magnitude of periodical component value is linear proportional to AoA, aperiodic component of roll moment is increasing up to AoA of flow rebuilding finish, then, it smoothly decreases to zero.

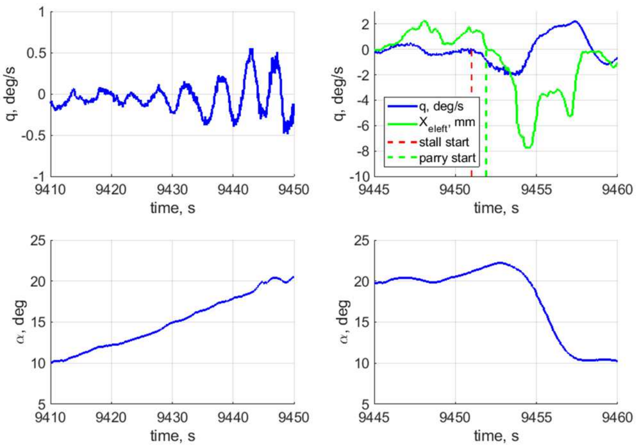


Fig. 8. Behavior of airplane dynamics in lateral channel at high AoA.

Analogous aperiodic component is considered for yaw moment too, to aggravate wing stall effect. Such behavior could be explained be increasing of drag at wing with flow separation.

Total above described model was integrated in common dynamic model of civil airplane. By means of experienced test pilots model parameters were corrected an all model was estimated as adequate.

5 Conclusion

To conclude we should underline some main points. First of all, we've developed the model of non-steady aerodynamic characteristics of lift force and pitch moment coefficients. The model gives the results that are close to the data given by wind tunnel experiment. Moreover, the model can be adapted to real flight conditions and then it gives the results that are in good agreement with data given by flight identification. Secondly, the researches of longitudinal dynamics of the civil aircraft in the case of aerodynamic hysteresis show that we have to consider this phenomenon to represent the correct behavior of the aircraft at high angles of attack. This accounting is valuable because the hysteresis leads to features of aircraft's dynamic that we cannot get in the case of aircraft with static aerodynamic. Thirdly, we can reproduce buffeting and asymmetrical flow separation that leads to wind stall. Designed model allows to calculate full motion of an airplane at high AoA. In addition, it is important to notice, that the model was highly appreciated by test pilots.

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