LINEAR MODELING BASED ON INSTRUMENTAL MODEL ESTIMATION AND PID PARAMETERS TUNING IN FREQUENCY-DOMAIN OF AIRCRAFT ENGINE

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Abstract

The linear model of aircraft engine was established by using instrumental model estimation. The validation results indicate the high accuracy of the static and the dynamic performance of the linear model. PID tuning method based on partial model matching on frequency-domain is introduced, and applied this method to design the PID controller of the aircraft engine high pressure rotor speed in flight envelope, and the control effects were evaluated by the nonlinear model. Simulation results show, under the effect of the PID designed controller, the system had quick dynamic response with zero overshoot and zero steady-state error.

1 Introduction

Although aircraft engine is an extreme complex thermodynamic system with quite strong nonlinearity, linear control theories are widely employed. Linear model of aircraft engine is important to engine research, especially engine control system. Obtaining high accuracy linear model is always the hot research point. The linear model can be obtained by using small perturbations [1] and curve fitting [2], but the results of these methods are not robust to the noise. In this paper, instrumental model estimation is employed to establish the linear model of aircraft engine. This method is an improved algorithm of least square estimation, which is an unbiased estimation about color noise [3]. In this method, it uses the measurable information to construct an instrumental model of the system, and the instrumental model is updated in real-time, in order to obtain the consistent estimation of the unknown parameters of the system.

As mentioned previously, linear control theories are used on aircraft engines. Among linear control methods, PID control has been applied widely to industrial process control system, so as aircraft engine control system. In this paper, a frequency domain PID design method is introduced, which is called partial model matching method. It is a linear model-based PID tuning methods from the frequency characteristics, and it has been applied to some control practice [4, 5]. In this method, PID parameters are calculated to minimize the frequency characteristic errors between the system controlled system with PID and a chosen reference model at a limit set of frequency points

In this paper, based on instrumental model method, linear aircraft engine models in flight envelope are established from nonlinear model. Then for these linear models, partial model matching method on frequency-domain was employed and applied to PID parameters tuning. The PID controllers at different operation points in flight envelope for aircraft engine were obtained and the well effects were validated by dynamic simulation comparison between linear and nonlinear model.

2 Linear Model of Aircraft Engine

For aircraft engine, the fuel flow is used to control the high pressure rotor speed in this paper, and the transfer function can be written as:

$$G_{NH}(s) = \frac{\Delta N_H}{\Delta W_f} \quad (1)$$

where, ΔN_H is the normalized increment speed of high pressure rotor, ΔW_f is the normalized increment of fuel low.

Consider the error output model with instrumental model of aircraft engine in Fig.1.

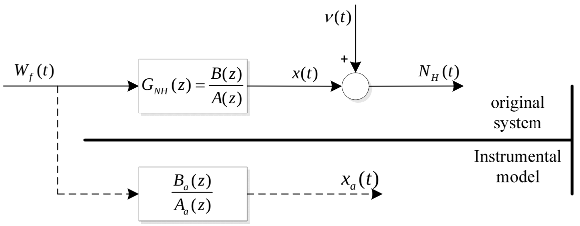


Fig.1 Error Output Model with Instrumental Model of Aircraft Engine

where, $G_{NH}(z)=\dfrac{B(z)}{A(z)}$, $A(z)=1+a_1z^{-1}+a_2z^{-2}+\cdots+a_{n_a}z^{-n_a}$, $B(z)=b_1z^{-1}+b_2z^{-2}+\cdots+b_{n_b}z^{-n_b}$, it is discrete-time transfer function of $G_{NH}(s)$, $W_f(t)$ is the fuel flow input, $x(t)$ is the true value of engine output, but it is unmeasured in fact, $v(t)$ is random noise, $N_H(t)$ is the engine output with noise. According to the experience of [6], the high pressure rotor system can be described as a transfer function with one zero and two poles. After discretized, $n_a=2$, $n_b=2$.

Define the parameter vector $\theta=[a_1,a_2,b_1,b_2]^T$ is to be estimated and system information vector $\varphi(t)=[-x(t-1),-x(t-2),-W_f(t-1),-W_f(t-2)]^T$.

Then, the engine output with noise is $N_H(t)=x(t)+v(t)=\varphi^T(t)\theta+v(t)$.

Because the true value of engine output included in $\varphi(t)$ is unable to gotten, it cannot be used to estimate the unknown parameters directly. Considering an instrumental model $\dfrac{B_a(z)}{A_a(z)}$ which has the same structure with $\dfrac{B(z)}{A(z)}$ in Fig.1, the output instead of the true value of engine output is used to obtain the information vector $\hat{\varphi}(t)$. And $\dfrac{B_a(z)}{A_a(z)}$ can be obtained by $\hat{\theta}$ which is the estimation of θ. The minimization function can be established:

$$J(\theta)=\sum_{j=1}^{t}\left[N_H(j)-\varphi_a^T(j)\hat{\theta}\right] \qquad (2)$$

The recursive estimation algorithm of instrumental model can be established, and the asymptotic consistency has been proved [7].

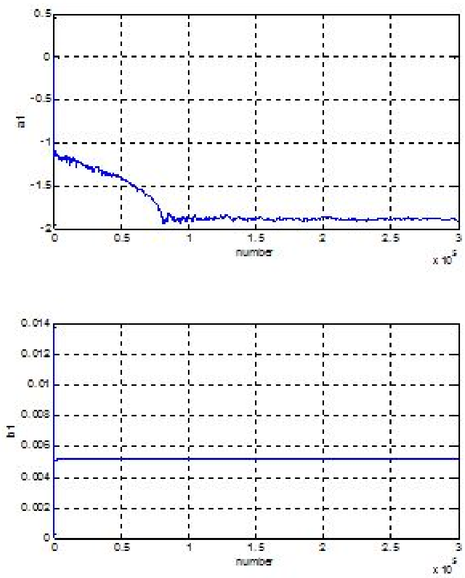
$$\begin{cases}\hat{\theta}(t)=\hat{\theta}(t-1)+L(t)[N_H(t)-\hat{\varphi}(t)\hat{\theta}(t-1)]\\ L(t)=P(t)\hat{\varphi}(t)=\dfrac{P(t-1)\hat{\varphi}(t)}{1+\hat{\varphi}^T(t)P(t-1)\hat{\varphi}(t)}\\ P(t)=P(t-1)-\dfrac{P(t-1)\hat{\varphi}(t)\hat{\varphi}^T(t)P(t-1)}{1+\hat{\varphi}^T(t)P(t-1)\hat{\varphi}(t)}\\ \quad =[I-L(t)\hat{\varphi}^T(t)]P(t-1) \quad P(0)=p_0I\\ \hat{\varphi}(t)=[-x_a(t-1),-x_a(t-2),u(t-1),u(t-2)]^T\end{cases} \qquad (3)$$

After the aircraft engine runs up to a certain steady-state operation point, the uncorrelated and zero-mean random signal is superimposed on the fuel flow, and the maximum amplitude is ±5%. According to the fuel input W_f and the response of high pressure rotor, the θ can be estimated by instrumental model method.

Fig.2 shows the parameter variation curves at height *H*=0km, Mach number *Ma*=0.5. At this operation point, the estimation result is $\theta=[1.901578,0.904577,0.005140,-0.004913]^T$.

The continuous-time transfer function can be obtained with 0.02s sample time of engine system:

$$G_{NH}(s)=\frac{0.2642s+0.5966}{s^2+5.014s+7.882} \qquad (4)$$



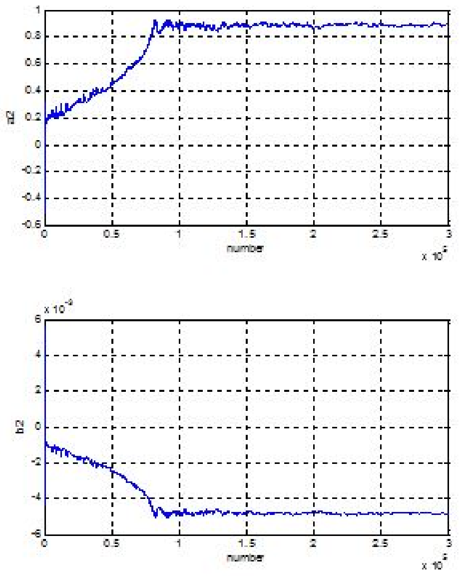


Fig.2 Parameter Variation Curves at *H*=0km, *Ma*=0.5

Fig.3 gives the step response of linear model $G_{NH}(s)$ and nonlinear model. It obviously validates the accuracy of linear model established by instrumental model method. The accuracy is high enough to as the substitute for nonlinear model.

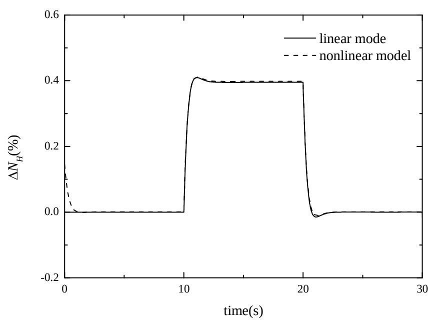


Fig.3 Step Response of Linear Model and Nonlinear Model for W_f 2% step at *H*=0km, *Ma*=0.5

Table 2 shows the high pressure rotor speed identification results of partial operation points in flight envelope.

Table.2 Linear Model in Flight Envelope

H(km)	*Ma*	Identification results
0	0	$G_{NH}(s)=\dfrac{0.2888s+0.7717}{s^2+5.456s+10.07}$
5	0.5	$G_{NH}(s)=\dfrac{0.2642s+0.5966}{s^2+5.014s+7.882}$
10	1.2	$G_{NH}(s)=\dfrac{0.2814s+0.4607}{s^2+3.24s+4.264}$
20	1.5	$G_{NH}(s)=\dfrac{0.4193s+0.9118}{s^2+11.48s+22.68}$

2 PID Parameters Tuning Using the Partial Model Matching of Frequency-domain

The structure of PID control loop is shown in Fig 4.

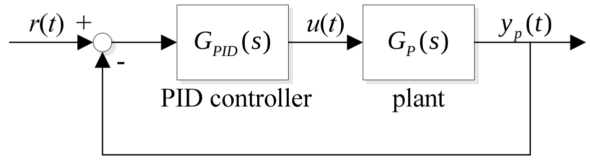


Fig.4 Block Diagram of PID Control System

where, $G_P(s)$ is plant, $C_{PID}(s)$ is PID controller, $C_{PID}(s)=k_p+\dfrac{k_i}{s}+\dfrac{k_d s}{1+\gamma s}$, considering the realizability, the derivative part is filtered by a first-order filter with small time constant γ. Here, let $\gamma=0.1$. $r(t)$ is reference input, $u(t)$ is control input, $y_P(t)$ is output.

Reference model is given as Fig.5 and its dynamic quality satisfies the system requirement.

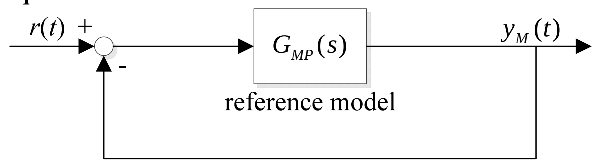


Fig.5 Block Diagram of Reference Model System

where, $G_{MP}(s)$ is reference model, $y_M(t)$ is output of reference model.

The original algorithm of model matching on frequency-domain is limited to a finite selected frequency to found the PID parameters (k_p , k_i , k_d), which minimize the least square error of open loop frequency characteristics between $G_P(s)C_{PID}(s)$ and $G_{MP}(s)$.

Define the relative errors function between $G_P(s)C_{PID}(s)$ and $G_{MP}(s)$ at the concerned band $[\omega_a,\omega_b]$ as follows

$$\varepsilon(j\omega)=\frac{G_{MP}(j\omega)-G_P(j\omega)C_{PID}(j\omega)}{G_{MP}(j\omega)},\quad \omega\in[\omega_a,\omega_b] \tag{5}$$

Further, it can be represented as

$$\varepsilon(j\omega_k)=1-\phi^T(j\omega_k)\theta \tag{6}$$

where, $\theta=[k_p \quad k_i \quad k_d]^T$ is the PID parameter vector, $\phi^T(j\omega)=[A(j\omega) \quad \frac{A(j\omega)}{j\omega} \quad \frac{A(j\omega)j\omega}{1+j\omega\gamma}]$, $A(j\omega)=\frac{G_P(j\omega)}{G_{MP}(j\omega)}$.

Define the following loss function at a set of frequency points belong to the band. $[\omega_a,\omega_b]$.

$$J(\theta)=\sum_{k=1}^{N}|\varepsilon(j\omega_k)|^2+\sum_{k=1}^{N}|\varepsilon(-j\omega_k)|^2 \\ =(I-\Phi\theta)^H(I-\Phi\theta) \tag{7}$$

where, $I=[1,\cdots,1]^T\in R^{2N}$, $\Phi=[\phi(j\omega_1),\cdots,\phi(j\omega_N),\phi(-j\omega_1),\cdots,\phi(-j\omega_N)]^T$.

The least square solution of Eq.7 is given as

$$\hat{\theta}=(\Phi^*\Phi)^{-1}\Phi^*I \tag{8}$$

This result does not take in to account any constraints of the system. However, closed loop stability is crucial to the aircraft engine control system. In order to guarantee the system stability, Hurwitz stability criterion is added as the constraint, an improved PID tuning method of partial model matching on frequency-domain with the loss function as the object is proposed.

$$\min\{J(\theta)\} \\ s.t.\begin{cases}\Delta_1=a_1>0 \\ \Delta_2=\begin{vmatrix}a_1 & a_3 \\ a_0 & a_4\end{vmatrix}>0 \\ \vdots \\ \Delta_n>0\end{cases} \tag{9}$$

where, $a_0,a_1,\cdots,a_n$ are the coefficients of characteristic polynomial in Fig.4, $a_0s^n+a_1s^{n+1}+\cdots+a_{n-1}s+a_n(a_0>0)$.

3 PID Controller Design of Aircraft Engine

At H=0km, Ma=0 of design point, the closed loop performance index is the setting time less than 1 second, no overshoot and no steady-state error. To meet the requirements, the reference model can be chosen as

$$G_{MP}(s)=\frac{0.75s+5}{s(0.05s+1)} \tag{10}$$

The closed loop of rotor speed bandwidth is commonly 0-10rad/s [8], the frequencies to be matched should be in this range. Here, select the frequency set Ω={0.03 0.1 0.8 1.4 2.8 4.7} (rad/s). Establish the optimization problem with the loss function and stability, and PID parameters can be solved as $k_p=18.36$, $k_i=64.69$, $k_d=2.25$.

The bode diagram of $C_{PID}(s)G_P(s)$ and reference model $G_{MP}(s)$ are given in Fig.6, ‘X’ and ‘○’ denote the frequency of $C_{PID}(s)G_P(s)$ and $G_{MP}(s)$ in Ω separately. It shows that the frequency characteristic of the two loops overlap in the range of bandwidth. PID controller can match the aircraft engine and reference model with a high degree of accuracy.

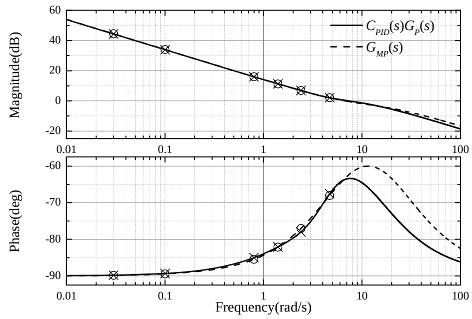


Fig.6 Bode Diagram of $C_{PID}(s)G_P(s)$ and $G_{MP}(s)$

The system of nonlinear model with the PID controller is simulated to verify the control effect. Fig.7 shows the step response of N_H for 2% step command. The step response of the PID control system using Ziegler-Nichols tuning method is also presented for comparison. There is no steady-state error with both methods. But there is overshoot with Ziegler-Nichols method, and no overshoot with improved method.

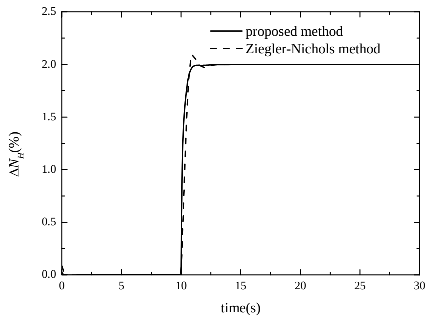


Fig.7 Step Response of Nonlinear Model for N_H 2% Step at H=0km, Ma=0

The non-design point performance at H=5km, Ma=0.5 is shown in Fig.8, with the similar results at other non-design points. As the same as the design point, there is overshoot with Ziegler-Nichols, and idea response with proposed method.

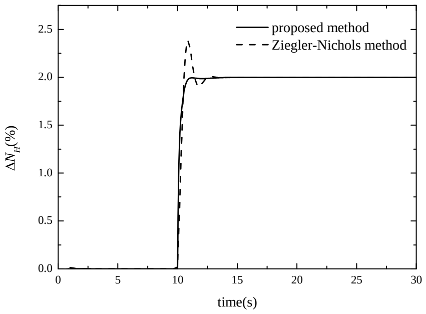


Fig.8 Step response of nonlinear model for N_H 2% step at *H*=5km, *Ma*=0.5

Table.3 lists the partial PID results in flight envelope. When height and Mach number change from low to high, k_p is increased first and then decreased, k_i is increased monotonously, k_d is decreased first and then increased.

Table.3 PID parameters in flight envelope

H(km)	Ma	k_p	k_i	k_d
0	0	18.36	64.69	2.25
5	0.5	20.60	65.37	2.39
10	1.2	15.92	44.42	2.78
20	1.5	20.30	123.49	1.23

4 Conclusion

In flight envelope, high accuracy aircraft engine linear model was established by instrumental model method. Based on the PID tuning method of partial model matching on frequency-domain, an improved algorithm was introduced to ensure the stability of the system. High pressure rotor PID controller in flight envelope was designed using the algorithm. Simulations show that the designed controller acquired the system an idea dynamic performance which achieved the design aim.

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