ICAS 2016
30th Congress of the International Council of the Aeronautical Sciences
DCC, Daejeon, Korea ; September 25-30, 2016

UNCERTAINTY QUANTIFICATION OF AIRFOIL DESIGN FOR ADVANCED PROPELLER MODELS

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Keywords: *physics-based modeling, multifidelity, probabilistic analysis, ensemble modeling*

Abstract

There has been a recent push by industry and government to make commercial air travel more environmentally friendly by striving to meet aggressive fuel burn, noise, and emissions goals simultaneously. This has forced engineers to move away from traditional turbofan engines mounted on tube-and-wing aircraft, and look toward advanced concepts. The development of advanced concepts requires engineers to utilize parametric design space exploration techniques to facilitate their understanding of the concept. The research presented in this paper focuses on formulating a stochastic method to enable parametric design space exploration for advanced concepts by creating an accurate representation of the high fidelity design space. As an initial step toward propeller design, the method is implemented on NACA 4-series airfoils as a proof of concept. High fidelity data is used to bias lower fidelity data using a distance based formulation. The combined data set is then sampled and fit with a Gaussian process regression to form the overall combined model. The results indicate that using a distance based method results in an improved high fidelity design space representation.

Nomenclature

$\underline{x}$	vector of input variables
$\underline{X}$	matrix of input vectors
μ	mean
$k(\cdot,\cdot)$	covariance function
$K(\cdot,\cdot)$	covariance matrix
λ	hyperparameter
D	number of design variables
l	design variable index
n	number of training points
n^*	number of test points
p,q	indices for the covariance function inputs
σ_f^2	signal variance
$\mathbf{f}^*$	Gaussian process predictions at test inputs
$\mathbf{f}$	vector of training outputs

1 Introduction

The aviation industry has become, and will continue to be a vital part of the world economy. In a report published in 2016, the FAA predicts the available seat miles to increase by approximately 3.5% annually for the next 20 years[11]. Boeing and Airbus project a need for a approximately 33,000 new aircraft over the next 20 years to replace aging aircraft and to increase the global fleet which is expected to nearly double[6, 4]. While these projections are encouraging for the aviation industry and the global economy, the environmental impact, namely noise and air pollution, is becoming a major source of concern for local and global communities at large. Coupled with these rapidly emerging environmental issues is the ever-rising cost of fuel. In order to address these economic and environmental concerns, industry and governments from around the world are striving to meet aggressive goals to reduce the fuel burn, noise, and emissions of commercial aircraft. Existing technologies are capable reaching these goals individually, but meeting them simultaneously creates additional challenges. This has forced aircraft manufacturers to move away from traditional technology and instead research advanced airframe and propulsion concepts, such as the blended wing body or the open rotor engine.

There are many challenges associated with conducting research on advanced concepts, most importantly that these goals often cause conflicting requirements. Due to these conflicting requirements, engineers need the ability to parametrically explore the design space to conduct trade-off studies. Representing a high fidelity design space is challenging with advanced concepts. Historical data do not exist so experiments must be planned and performed to generate data. However, experimental testing is expensive and must be done in the most economical way possible. This results in a sparse amount of high fidelity tests. Low fidelity tests can be conducted at numerous places throughout the design space, but they are considered to be much less accurate due to missing physics. Additional high fidelity computer simulations can also be conducted in a limited manner. With this in mind, the bottom line is that the data for advanced concepts is sparse at the high fidelity levels and inaccurate at the lower fidelity levels.

The ultimate use of experimental data is to help technology development decision makers determine which technologies to continue investing in. However, the lack of high fidelity data and the uncertainty associated with the existing data from lower fidelity tests can provide a misleading depiction of the expected performance the technology or concept will provide. Therefore, decision makers need to have an understanding of the uncertainty associated with a performance model that is created from experimental data. A quantitative definition of uncertainty will assist decision makers when they are deciding which technologies to pursue and which experiments should be conducted in the future. Given this information, the overall objective of this research is to formulate a transparent and traceable methodology that enables the performance prediction of an immature technology throughout a parametric design space in the presence of sparse, uncertain data.

A multifidelity approach is outlined to more efficiently use the limited amount of available data. Multifidelity methods use a limited amount of high fidelity data to augment the results of the low fidelity simulations[14, 17]. Furthermore, surrogate modeling techniques will be used to further reduce the necessary amount of resources required for high fidelity performance prediction[12, 15, 23]. The methodology will utilize the uncertainty of each data set to create one combined set and then use Gaussian process regression models to create a probabilistic parametric performance prediction space. This research concludes with a demonstration of the methodology on NACA 4-series airfoils where three design variables, maximum camber, maximum camber location, and thickness ratio, and the angle of attack will be used to predict the coefficient of lift. A demonstration using airfoils was chosen because the design space is not overly complicated, yet it is a meaningful demonstration step in the overall goal of advanced propeller design since airfoil design is a major portion of propeller design.

The following section will provide an outline of the technical approach. Relevant background information for the development of the technical approach will be presented where appropriate in the section. Section 3 provides an explanation of the use case utilized to demonstrate the methodology as well as the results. The paper concludes with the conclusions presented in Section 4.

2 Technical Approach

The approach formulated for this research has four main steps: defining the uncertainty for the different datasets, combining the datasets, fitting the performance model, and representation of the model uncertainty. The approach is depicted in Figure 1, and the process starts at the top left of the figure. The process is divided into two tracks, one for the low fidelity (LF) data and one for the high fidelity (HF) data so the reader can visually determine where the data is combined and the steps that happen before/after the combination. In reality several datasets may be used, but only two are shown in the flowchart.

Furthermore it is important to note that this methodology as a whole is not globally applica-

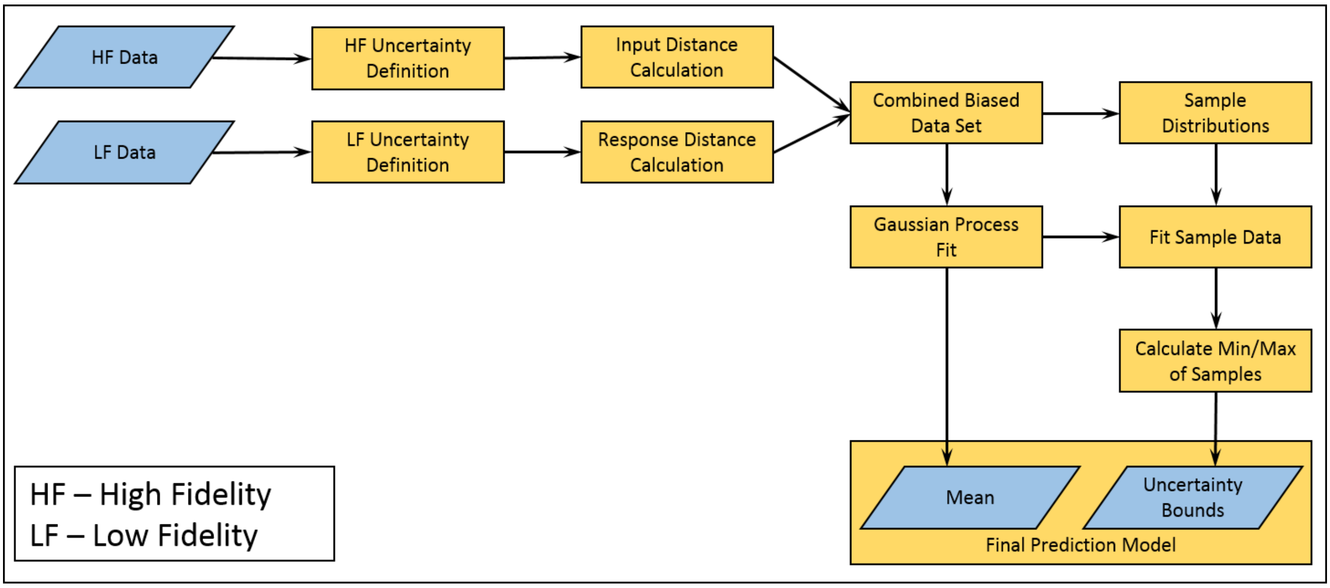

Fig. 1 Methodology flow chart

ble and two key assumptions are made with regard to the type of problem for which this method can be applied. The assumptions are as follows:

1. There is a limited amount of experimental data from prior feasibility studies.

2. The data is smooth, meaning the responses from two inputs located a short distance from each other are similar.

The first assumption reiterates the motivation behind the problem this research is attempting to tackle. In situations where a large amount of high fidelity data is available, other existing methods may provide a more efficient model. The assumption regarding the 'smoothness' of the data is important because the methodology cannot handle large or discrete changes in function values.

The remainder of this section will focus on providing details and background research relevant to each step of the process, as well as important assumptions that were made during the formulation.

2.1 Defining the Uncertainty

The first step of the process is to define the uncertainty associated with each data set that will be utilized. It is important that the uncertainties are established prior to combining or fitting any of the data because it will impact the final results. In general one would only be dealing with two different types of data: data from physical experiments and computational data.

For the experimental data, Oberkampf and Roy state that the uncertainty surrounding experimental data should be represented by a Gaussian distribution[19]. This is a sentiment that is echoed in the literature by many other experts and one that is utilized in practice throughout many technology development programs. Therefore, the experimental data utilized will be represented with a Gaussian distribution.

Next, the uncertainty for the computational data was determined. In the literature there are differing opinions regarding how the uncertainty for computational data should be represented. Kennedy and O'Hagan believe data from a computational simulation should be represented by a Gaussian distribution similar to experimental data[16]. However, Oberkampf and Roy believe that computational simulations should be represented by an interval[19].

For this research it is important that all of the distributions have a peak, or a most expected value for the random variables, which aligns most with Kennedy and O'Hagan. Any number of distributions will likely work. The authors used triangular distributions on a previouse research project with industry. Thus, the authors elected to utilize triangular distributions to represent the uncertainty for computational data. The triangular distributions will be set by defining the peak of the distribution by the results of the computational simulations. The upper and lower bounds will be defined through comparison to the highest fidelity data.

There are a myriad of other methods for defining the uncertainty of a data set, but defining the best definition of uncertainty for this application is beyond the scope of this research. The intent of this research is to formulate a method for accurately representing disparate uncertain data sources. The reader in encouraged to research the most appropriate uncertainty representation for their application.

2.2 Data Set Combination

The next step in the process is to combine the data sets. It was observed in the literature that methods that use likelihood ratio tests such as Bayesian model averaging or Bayesian model combination are commonly used to combine data sets[24, 13]. The problem with these methods is that they use some or all of the existing high fidelity data for the model combination process. This can cause problems when the data set is sparse because there is not enough data left for the final model after the combination process. The authors came across this problem during previous research where they fit separate regressions to each data set and then used the likelihood ratio to combine the separate regressions into one[22].

This current research combines the data sets into one set prior to fitting any sort of regression. However, the low fidelity data cannot simply be appended to the high fidelity data. The regression method therefore needs to be biased toward the high fidelity data. The low fidelity data is biased toward the high fidelity data depending on the distance between the high and low fidelity data points. The squared exponential distance function, Equation 1, is used to define the magnitude of the bias based on distance.

$$k(x,x') = e^{(-\frac{1}{2}(x-x')*(x-x')^T)} \tag{1}$$

Gaussian processes and other kernel methods use similar distance based techniques, but what is different about this research is that it uses the distances between the inputs as well as the responses to bias the data. The squared exponential distance is calculated between every low and high fidelity point in the input and response spaces. If the two points are close in the input space, then the high fidelity point will have a strong influence on the low fidelity point. If the responses of the two points are far away from each other, then the high fidelity point will again have a strong influence on the low fidelity point. The results from the distance calculations in both the input and response spaces are combined to give an overall influence on the lower fidelity point. This overall influence is used to bias the mean of the lower fidelity uncertainty distribution. So a low fidelity point that is close to a high fidelity point in the input space, but far away in the response space will be heavily biased by the high fidelity point. This results in a new data set where the presence of higher fidelity data points has been used to bias the lower fidelity data points. Recall that triangular distributions are used to represent the uncertainty for the low fidelity data points. The data combination process described results in a shifting of the peak of each low fidelity data point toward a high fidelity data points.

2.3 Gaussian Process Regression

The next step of the process is to fit the combined data set with a regression. A Gaussian process regression method was chosen for this research because of its versatility and it has been observed to work well with smooth data sets. In machine learning, Gaussian processes fall under the category of kernel methods which are a class of algorithms used for pattern analysis. See *Pattern Recognition and Machine Learning* by Bishop for a detailed explanation of kernel methods[5]. The squared exponential kernel will be used for this research. Another reason for using a Gaussian process is that future work on this topic will likely involve experimenting with different kernels. If the reader is familiar with Bayesian linear regression, then it may be helpful to know that Gaussian process regression with the squared exponential kernel is the same as Bayesian linear regression with an infinite number of radial basis functions[20].

A Gaussian process is defined as a collection of random variables where any finite number of which have a joint Gaussian distribution. It is a class of stochastic processes that, given a set of data, uses the multivariate Gaussian distribution to make predictions at unobserved locations in the input space. It is a statistically rigorous approach for constructing surrogate models of deterministic computer codes[20]. A Gaussian process is commonly derived from a Bayesian perspective, similar to kriging[9, 18]. Gaussian processes are also commonly referred to as kriging (given a specific kernel) or Design and Analysis of Computer Experiments (DACE)[15, 8]. The DACE approach was popularized by Sacks et al. and Cressie[21, 7]. Different distance based kernels can be chosen based on the user's preference.

The following brief explanation given on Gaussian processes comes from chapter 2 of *Gaussian Processes for Machine Learning* by Rasmussen and Williams[20]. The reader is encourage to read this chapter for a more detailed explanation.

A Gaussian process is specified by its mean function $m(\underline{x})$ and covariance function $k(\underline{x},\underline{x}')$ which are defined as

$$m(\underline{x}) = \mathbb{E}[f(\underline{x})] \tag{2}$$

$$k(\underline{x},\underline{x}') = \mathbb{E}[(f(\underline{x}) - m(\underline{x}))(f(\underline{x}') - m(\underline{x}'))]. \tag{3}$$

The mean function is typically set to zero in practice which makes the derivation easier to follow. The covariance function specifies the covariance between pairs of inputs. The reader can think of the covariance function as a representation of the similarity between variables. For this research the squared exponential coveriance function is used. When no additional noise is added, the function is a follows.

$$cov(f(\underline{x}_p), f(\underline{x}_q)) = k(\underline{x}_p, \underline{x}_q) = \sigma_f^2 exp\left(-\frac{1}{2\lambda^l}(\underline{x}_p^l - \underline{x}_q^l)^2\right) \tag{4}$$

where $\lambda^1, \lambda^2, ..., \lambda^D$ are the hyperparameters for a D dimensional space, and σ_f is the signal variance. The hyperparameters are referred to as the characteristic length scale for each dimension. They show the degree of nonlinearity in the dimension they are associated with. Therefore, they can be used as a sensitivity study. For instance, the dimension with the largest hyperparameter is responsible for the largest amount of variability in the response for the given ranges.

Next, the user chooses a set of test inputs $\underline{X}^*$ and then uses the covariance function to create a $n^* \times n^*$ covariance matrix $K(\underline{X}^*, \underline{X}^*)$. The covariance matrix can then be used to create random multivariate Gaussian prior.

$$\mathbf{f}^* \sim \mathcal{N}(0, K(\underline{X}^*, \underline{X}^*)) \tag{5}$$

The usefulness of a Gaussian process comes when knowledge gained through experimentation is incorporated to train the function. The joint distribution of the training outputs, $\mathbf{f}$, and the test outputs $\mathbf{f}^*$, based on the prior is

$$\begin{bmatrix} \mathbf{f} \\ \mathbf{f}^* \end{bmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} K(\underline{X},\underline{X}) & K(\underline{X},\underline{X}^*) \\ K(\underline{X}^*,\underline{X}) & K(\underline{X}^*,\underline{X}^*) \end{bmatrix}\right), \tag{6}$$

where the $*$ represents the test data. Thus, $K(\underline{X},\underline{X}^*)$ represents the $n \times n^*$ covariance matrix of n training points and n^* test points. The posterior distribution is found by conditioning the joint Gaussian prior distribution on the training data. Think about it as if you are restricting the joint prior distribution to contain only those functions that agree with the observed data points. The predictive equations for the Gaussian process regression are

$$\mathbf{f}^* | \underline{X}, \mathbf{f}, \underline{X}^* \sim \mathcal{N}(\bar{\mathbf{f}}^*, cov(\mathbf{f}^*)) \tag{7}$$

where

$$\bar{\mathbf{f}}^* = K(\underline{X}^*, \underline{X}) K(\underline{X}, \underline{X})^{-1} \mathbf{f} \tag{8}$$

$$cov(\mathbf{f}^*) = K(\underline{X}^*,\underline{X}^*) - K(\underline{X}^*,\underline{X}) K(\underline{X},\underline{X})^{-1} K(\underline{X},\underline{X}^*) \quad (9)$$

The MATLAB code that accompanies the text by Rasmussen and Williams is used to generate the Gaussian process regressions.

2.4 Overall Uncertainty Representation

The final step of the process is to assess the combined uncertainty of the newly generated model. The previous section regarding data set combination describes how the mean of the lower fidelity model is biased in this multifidelity model.

The combined uncertainty is determined by sampling each distribution and fitting the resulting data with a regression model. The maximum upper and lower bounds of all the regression fits represent the overall bounds of the uncertainty for the combined model. There is no need to create a new fit for every sample set because the fit should be similar to the original combined data set. Creating new Gaussian process models for every sample set would be unnecessarily expensive. Therefore, the same model is fit to every sample set.

3 Methodology Demonstration

The methodology outlined in Section 2 was implemented on a simplified problem to demonstrate its capabilities. The design of an airfoil was chosen as a preliminary test case to demonstrate the process because it is an important aspect of propeller design, the performance can be represented simply through four input variables, and a variety of data was readily available to the authors. The NACA 4-series airfoils were utilized for this demonstration. Three different design variables (maximum camber, maximum camber location, and thickness ratio) were used in conjunction with the angle of attack to develop a performance model that predicts the coefficient of lift.

NACA 4-series experimental data from the *Theory of Wing Sections* by Abbott and Von Doenhoff was used as the high fidelity data[2]. The experimental results given by Abbott and Von Doenhoff are input as the mean and the standard deviation as defined by the measurement error documented in the experimental report. The standard deviation of the distribution is defined such that the total error defined in the experimental report represents the 95% confidence interval. The airfoil experiments were conducted in the Langley two-dimensional low-turbulence model pressure tunnel. Unfortunately the documentation describing the measurement error for these tests says that the error is negligible[3]. Therefore, the authors have decided to use the measurement error from a modern wind tunnel apparatus which is 0.5% [1].

The different airfoil geometries and the angles of attack used as part of this research are listed in Table 1. A total of 45 data points make up the full high fidelity data set.

Table 1 NACA 4-series airfoil geometries from Abbott and Von Doenhoff[2]

Airfoil	AoA Range	Number of Points
0006	0 – 14	6
0009	0 – 16	5
0012	0 – 16	6
1408	0 – 16	6
1410	0 – 16	5
1412	0 – 16	6
2408	0 – 14	6
2410	0 – 16	5
2412	0 – 16	6
2415	0 – 16	6

Table 2 NACA 4-series airfoil ranges for the data generated by XFOIL.

	Lower Bound	Upper Bound	Step
Max Camber	0	4	1
Max Camber Location	0	4	1
Thickness Ratio	6%	24%	2%
Angle of Attack	0	16	4

The lower fidelity data for the NACA 4-series airfoils was generated using XFOIL, which is a potential flow solver that can optionally include an interactive boundary layer formulation and stability model[10]. The coefficient of lift was predicted for sweeps of all four input variables. The overall ranges for each variable are shown in Table 2 resulting in a total of 630 data points.

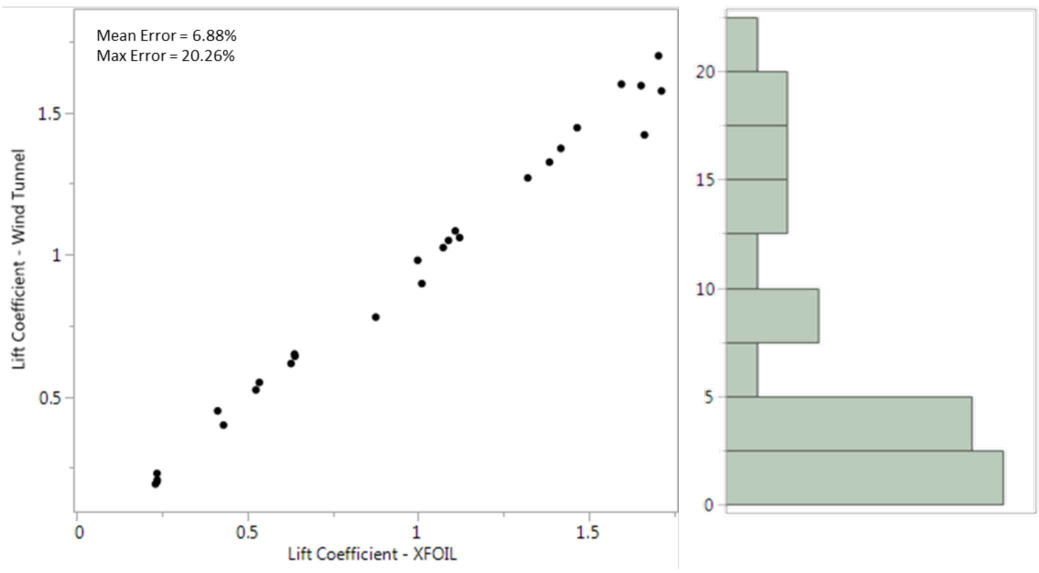

Fig. 2 Plot of the actual lift coefficient from the wind tunnel data versus the predicted lift coefficient from XFOIL. A histogram of the error is shown to the right of the plot.

The uncertainty for the XFOIL data points was determined by comparing the XFOIL predicted lift coefficient values to the experimental data. A subset of the high fidelity, experimental data points were selected and the lift coefficient values were compared to the predicted lift coefficient values from XFOIL for the same points in the design space. The maximum error between the points was utilized as the overall uncertainty for the XFOIL data points. An actual versus predicted plot is shown in Figure 2 where the actual lift coefficient from the wind tunnel data is compared against the predicted lift coefficient from XFOIL. Note that a line starting from the origin with a positive slope of one would indicate a perfect fit. The airfoil data follows the ideal trend well, but there is some error, especially toward the higher values for lift coefficient. A histogram of the error has also been included on the right side of the figure as well. The mean of the error is 6.88% and the maximum error is 20.26%. Recall, it was decided that triangular distributions would be used for the computer simulations. The peak values of the distributions were set to equal the XFOIL prediction values and the upper and lower bounds were set to equal $+/-20\%$ of the peak value.

The next step is to combine the data so that it can be used to create a performance prediction model. As a test case for this methodology, a model was created to predict the performance of the NACA 2410 airfoil. For this prediction model, the wind tunnel data for the NACA 2410 airfoil was removed from the high fidelity data set. This subset of the high fidelity data was combined using the method described in Section 2.2. A Gaussian process regression was then fit to the combined data set. This initial regression fit represents the mean of the model. Next, 100 sample data sets were generated by sampling the uncerainty distributions for each data point. Each new data set was individually fit with the same Gaussian process model that was used to create the overall mean of the model. The upper and lower bounds from these sample regression fits is used to represent the overall unceraint of the model. Figure 3 show the results of the model prediction of the lift curve for a NACA 2410 airfoil. The black dashed line represents the mean of the prediction and the dashed red lines represent the uncertainty bounds. The blue circles are the actual wind tunnel results that were left out of the high fidelity data set. Notice how the model predicts the wind tunnel data well, and that the wind tunnel data falls within the uncertainty bounds.

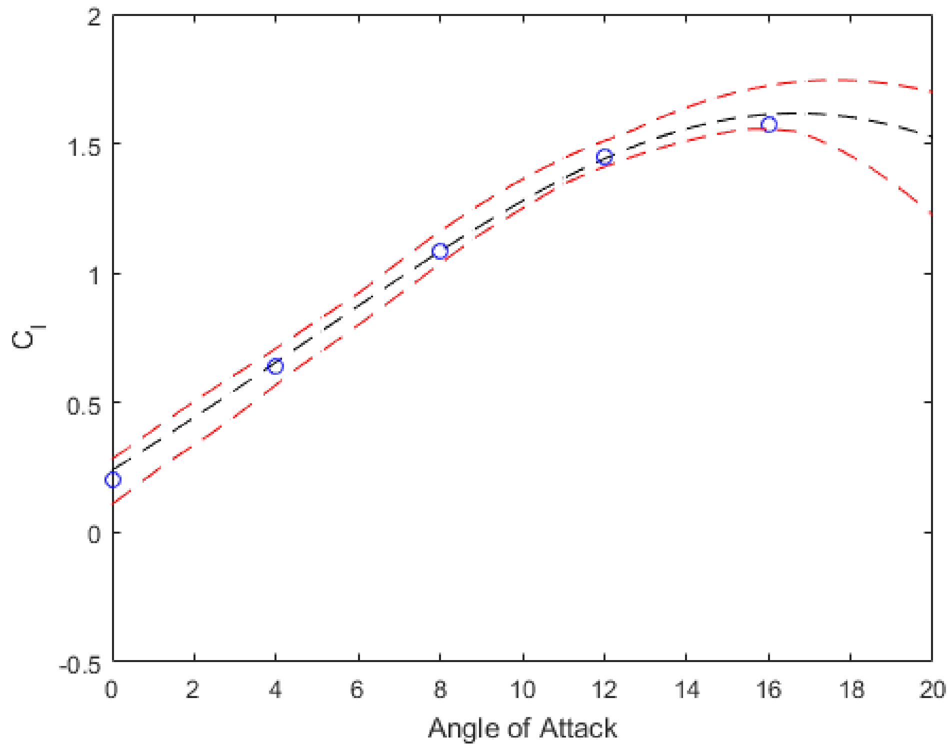

Fig. 3 Lift coefficient predictions of a NACA 2410 airfoil as a function of angle of attack.

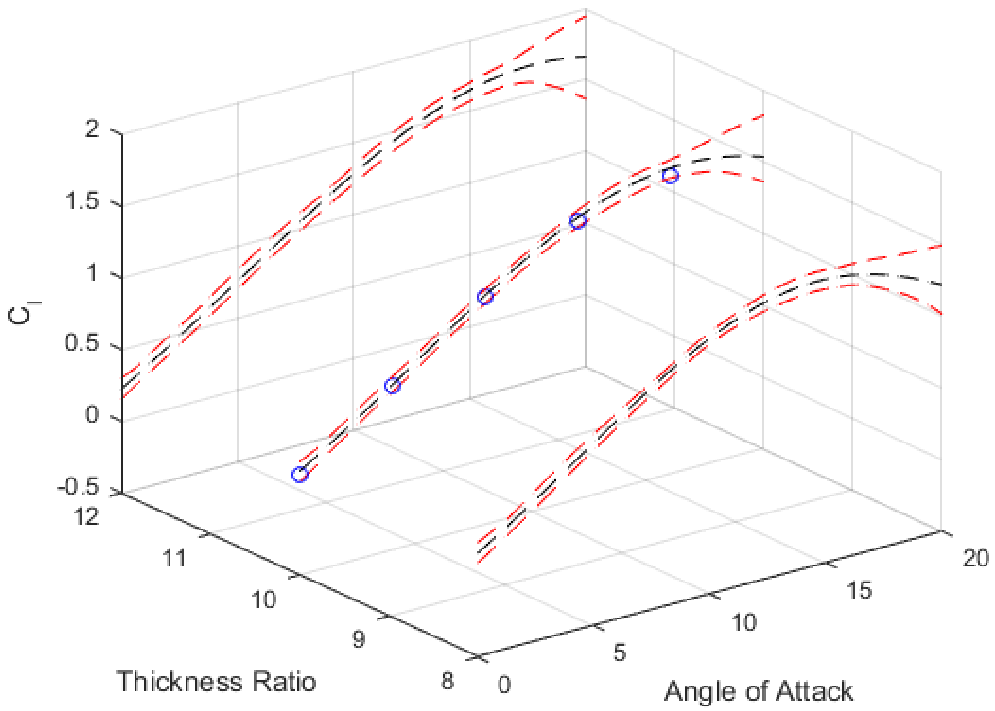

Fig. 4 Lift coefficient predictions of a NACA 24*XX* airfoils as a function of angle of attack and thickness.

The next set of results show how this method can be used to create surfaces in multiple dimensions. The same data set, excluding the 2410 again, was used in this prediction. Figure 4 shows the prediction of the lift curve as a function of both angle of attack and thickness. Predictions were made for the 2408, 2410, and 2412 airfoils to create a surface. Again the black lines represent the mean of the prediction and the red lines represent the uncertainty bounds. The blue circles represent the wind tunnel data for the 2410 airfoil that was left out of the data set. Notice how the method predicts the actual wind tunnel results well.

4 Conclusions

The aviation industry forecasts the need for a large fleet of next generation aircraft that can provide a high level of performance while reducing the environmental impact of the industry as a whole. The design of these advanced concepts can be aided by enabling parametric design space exploration. It was observed that traditional methods for creating these parametric models are not compatible in situations where sparse data sets exist. This research helps engineers overcome this issue by providing a methodology where uncertainty surrounding the data can be used to bias the data sets to allow the combination of the data and formulation of a model in a more efficient and informative manner than traditional Bayesian techniques.

As mentioned in Section 1, advanced propeller design is an area that could benefit from this kind of methodology. Due to the importance of airfoil design when it comes to designing advanced propellers, an airfoil use case was seen as a usefull demonstration step for this methodology. Therefore, this multifidelity approach was applied to a NACA 4-series airfoil use case and it was demonstrated that the combined data sets can provide more insight into the performance, and uncertainty surrounding the performance, for a given design space. The next step for this research will be to apply the methodology to advanced propeller concepts. Additionally, future work will focus on investigating the different options within the Gaussian process formulation, such as the selection of the appropriate kernel and optimization of the hyper-parameters, as well as different distance calculation methods for biasing the mean of the lower fidelity data. Furthermore, the methodology could be used in the future to investigate areas of the design space where high levels of uncertainty exist and future experimentation should be focused.

References

[1] *Modern Machine & Tool CO., INC.* www.mmtool.com/balances.html.

[2] Abbott, I. H. and Von Doenhoff, A. E., *Theory Of Wing Sections*. New York: Dover Publications, 1959.

[3] Abbott, I. H., Von Doenhoff, A. E., and Stivers Jr., L. S., "Summary of Airfoil Data," *NACA Report No. 824*, 1945.

[4] Airbus, "Mapping Demand: Global Market Forecast 2016-2035," tech. rep., Airbus, 2016.

[5] Bishop, C. M., *Pattern Recognition and Machine Learning*. New York: Springer Science, 2006.

[6] Boeing, "Current Market Outlook 2015-2034," tech. rep., Boeing Commercial Airplanes, 2015.

[7] Cressie, N., "The Origins of Kriging," *Mathematical Geology*, vol. 22, no. 3, pp. 239–252, 1990.

[8] Cressie, N. A. C., *Statistics for Spatial Data*. New York: John-Wiley and Sons, 1993.

[9] Currin, C., Mitchell, T., Morris, M., and Ylvisaker, D., "Bayesian Prediction of Deterministic Functions, With Applications to the Design and Analysis of Computer Experiments," *Journal of the American Statistical Association*, vol. 86, no. 416, pp. 953–963, 1991.

[10] Drela, M. and Youngren, H., "Xfoil 6.9 User Primer," 2001.

[11] Federal Aviation Administration, "FAA Aerospace Forecast: Fiscal Years 2016-2036," tech. rep., 2016.

[12] Forrester, A. I. J., Sóbester, A., and Keane, A. J., *Engineering Design via Surrogate Modelling: A Practical Guide*. John-Wiley and Sons, 2008.

[13] Hoeting, J. A., Madigan, D., Raftery, A. E., and Volinsky, C. T., "Bayesian Model

Averaging : A Tutorial," *Statistical Science*, vol. 14, no. 4, pp. 382–417, 1999.

[14] KEANE, A. J., "Cokriging for Robust Design Optimization," *AIAA Journal*, vol. 50, pp. 2351–2364, nov 2012.

[15] KEANE, A. J. and NAIR, P. B., *Computational Approaches for Aerospace Design: The Pursuit of Excellence*. West Sussex: John-Wiley and Sons, 2005.

[16] KENNEDY, M. C. and O'HAGAN, A., "Predicting the output from a complex computer code when fast approximations are available," *Biometrika*, vol. 87, pp. 1–13, 2000.

[17] MARCH, A. and WILLCOX, K., "Multifidelity Approaches for Parallel Multidisciplinary Optimization," in *12th AIAA Aviation Technology, Integration, and Operations (ATIO) Conference*, no. September, (Indianapolis), pp. 1–23, 2012.

[18] NEAL, R. M., "Regression and Classification Using Gaussian Process Priors," in *Bayesian Statistics 6* (BERNARDO, J. M., BERGER, J. O., DAWID, A. P., and SMITH, A. F. M., eds.), pp. 475–501, Oxford University Press, 1998.

[19] OBERKAMPF, W. L. and ROY, C. J., *Verification and Validation in Scientific Computing*. New York: Cambridge University Press, 2010.

[20] RASMUSSEN, C. E. and WILLIAMS, C. K. I., *Gaussian Processes for Machine Learning*. MIT Press, 2006.

[21] SACKS, J., WELCH, W. J., MITCHELL, T. J., and WYNN, H. P., "Design and Analysis of Computer Experiments," *Statistical Science*, vol. 4, no. 4, pp. 409–435, 1989.

[22] SCHWARTZ, H. D. and MAVRIS, D. N., "Model Form Uncertainty Representation to Enable Multifidelity Design of Advanced Concepts," in *29th Congress of the International Council of the Aeronautical Sciences*, (St. Petersburg), pp. 1–11, 2014.

[23] SIMPSON, T. W., LIN, D. K. J., and CHEN, W., "Sampling Strategies for Computer Experiments: Design and Analysis," *International Journal of Reliability and Applications*, vol. 2, no. 3, pp. 209–240, 2001.

[24] WASSERMAN, L., "Bayesian Model Selection and Model Averaging.," *Journal of mathematical psychology*, vol. 44, pp. 92–107, mar 2000.

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