REDUCED ORDER MODELING OF AVIATION ENVIRONMENTAL DESIGN TOOL WITH PROPER ORTHOGONAL DECOMPOSITION AND KRIGING

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Abstract

A high fidelity simulation is preferred for its remarkable accuracy for engineering problems. However, it requires long computational time, which leads to significant overhead and eventually hinders its application to design study. To overcome such impediments, a reduced-order method is utilized. Reduced-order method can effectively represent a simulation output as a linear combination of a basis and weighting coefficients on a low dimensional space. To construct a reduced-order model, this research adopts proper orthogonal decomposition to collect an empirical orthogonal basis and ordinary Kriging interpolation to predict weighting coefficients. Afterwards, this research applies reduced-order methodology to an aviation environmental design tool for the rapid prediction of arrival and departure noise.

1 Introduction

The U.S. federal aviation administration (FAA)'s office of environment and energy, in collaboration with the national aeronautics and space administration (NASA), has developed a comprehensive software system that models aircraft performances in space and time to estimate fuel burn, emissions, the numerical propulsion system and noise. This program is called aviation environmental design tool (AEDT), which is designed to model individual studies ranging from a single flight at an airport to scenarios at regional, national, and global levels [1]. The AEDT carries out a high fidelity simulation with superior accuracy, but it often requires considerable time and resources. For this reason, repetitive numerical calculation and real-time computing becomes intractable. In that case, a reduced-order method (ROM) can be conducive for high fidelity model since it improves computational speed.

Conceptually, reduced-order method consists of a basis and weighting coefficients. However, a commercial program, such as the AEDT, generally is difficult to evaluate the basis of software outputs and to determine weighting coefficients. To surmount this hurdle, Ref. [2] applied proper orthogonal decomposition based ROM to the numerical propulsion system simulation (NPSS) in aircraft design. With the help of expectation-maximization algorithm for probabilistic principal component analysis (EM-PCA) and neural networks, this research achieved orthogonal bases from NPSS engine deck responses and evaluated weighting coefficients. Ref. [3] showed uncertainty quantification (UQ) methodology of the AEDT. The authors addressed aircraft performance, aircraft noise, and aircraft emissions, executing a combination of surrogate modeling and grouped sensitivity analysis.

Accordingly, this research capitalizes on proper orthogonal decomposition (POD) to obtain an empirical orthogonal basis from AEDT noise outputs and ordinary Kriging interpolation to predict weighting coefficients. With the help POD and Kriging, this research constructs a reduced-order AEDT for noise prediction at both arrival and departure.

2 Formulations

2.1 Reduced-order method

In general, a high fidelity model can be expressed as

$$y = f(x; \theta), \tag{1}$$

where $y \in \mathbb{R}^d$ is an output of interest, $x \in \mathbb{R}^m$ is a spatial coordinate and $\theta \in \mathbb{R}^p$ is a parameter of interest. For instance, when we examine a pressure distribution across the wing of an aircraft by finite volume method, y corresponds to a pressure distribution, x is spatial coordinates and θ could be an angle of attack or a Mach number.

For high fidelity analysis in form of Equation (1), a reduced-order method deals with the parameter θ within a particular range. Given parameter $\theta \in \mathcal{D} = [\theta_\ell \times \theta_u] \subset \mathbb{R}^p$, y can be approximated as

$$y = f(x; \theta) \approx \sum_{i=1}^{r} a_i(\theta) v_i(x) \quad \forall_\theta \in \Theta, \tag{2}$$

where $a_i \in \mathbb{R}$ is weighting coefficients and $V = \{v_i\}_{i=1}{}^r \in \mathbb{R}^{d\times r}$ is a basis that spans y.

This research utilizes proper orthogonal decomposition for the evaluation of basis $v_i(x)$ and ordinary Kriging interpolation for the prediction of basis coefficients $a_i(\theta)$.

2.2 Proper orthogonal decomposition

Proper orthogonal decomposition extracts an empirical basis from given observations. For each snapshot $y \in \mathbb{R}^d$, a sample covariance matrix $S \in \mathbb{R}^{d\times d}$ is formulated with a snapshot ensemble $Y = \{y_i\}_{i=1}{}^n \in \mathbb{R}^{d\times n}$ as follows:

$$S = \frac{1}{n-1} Y Y^{\mathrm{T}}.$$

An orthogonal basis $V = \{v_i\}_{i=1}{}^r \in \mathbb{R}^{d\times r}$ can be extracted by the singular value decomposition (SVD) of matrix S. If the number of snapshot d is larger than the number of observations n, covariance matrix S becomes enormous, leading to heavy computation. For efficient basis extraction, a method of snapshot evaluates a sample covariance matrix $R \in \mathbb{R}^{n\times n}$ such that

$$R = \frac{1}{d-1} Y^{\mathrm{T}} Y.$$

We can obtain an orthogonal basis by SVD of sample covariance matrix R with row to column space transformation.

2.3 Ordinary Kriging interpolation

Ordinary Kriging [4] is an interpolation method based on a Gaussian process. Kriging treats deterministic response y as a realization of a random variable $Y(x)$. According to this stochastic viewpoint, a random variable $Y(x)$ is expressed as a combination of a trend function $m(x)$ and a random variable $Z(x)$ shown as below.

$$Y(x) = m(x) + Z(x)$$

In case of ordinary Kriging, $m(x)$ is assumed to be constant such that $m(x) = \mu$. Random variable $Z(x)$ is presumed to be a Gaussian random variable, thereby $Y(x)$ also becomes a Gaussian random variable. As a result of the aforementioned assumptions, a likelihood can be formulated with observations $y \in \mathbb{R}^n$ as

$$\mathcal{L}(\mathrm{y}; \mu, \sigma^2, \theta) = \frac{1}{(2\pi\sigma^2)^{n/2}|\Psi|^{1/2}} \exp\left[-\frac{(\mathrm{y} - \mu\mathbf{1})^{\mathrm{T}}\Psi^{-1}(\mathrm{y} - \mu\mathbf{1})}{2\sigma^2}\right],$$

where σ^2 is a process variance, and $\mathbf{\Psi} \in \mathbb{R}^{n\times n}$ is a correlation matrix.

The parameters of Kriging, such as μ, σ^2 and θ can be estimated by the method of maximum likelihood. A prediction of an output $y_o \in \mathbb{R}$ at a new input $x_o \in \mathcal{D} \subset \mathbb{R}^p$ can be obtained as

$$\hat{y}_o = \hat{\mu} + \psi^{\mathrm{T}} \Psi^{-1} (\mathrm{y} - \hat{\mu}\mathbf{1}),$$

where $\psi \in \mathbb{R}^n$ is a correlation between x and x_o.

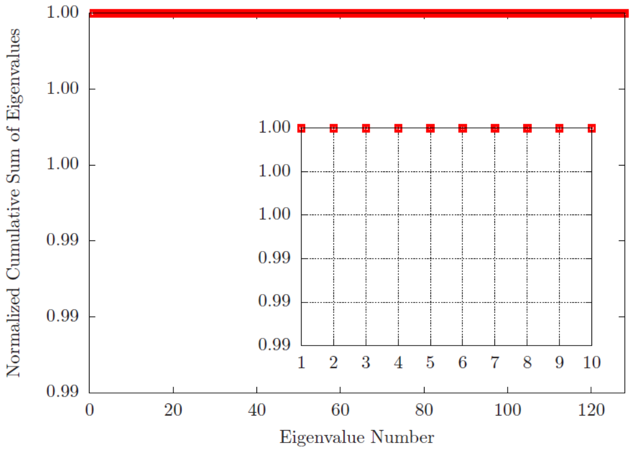
3 Reduced order modeling

This research constructs a reduced-order AEDT for departure and the approach noises as shown in Equation (2). To generate a reduced-order model, we adopted three parameters of noise conditions and indicated the ranges of the parameters in Table 1. A total of 128 training samples was populated by a maximum-entropy experiment design and was used for model construction. For the verification of the reduced-order AEDT model, 128 test samples were populated by a random design.

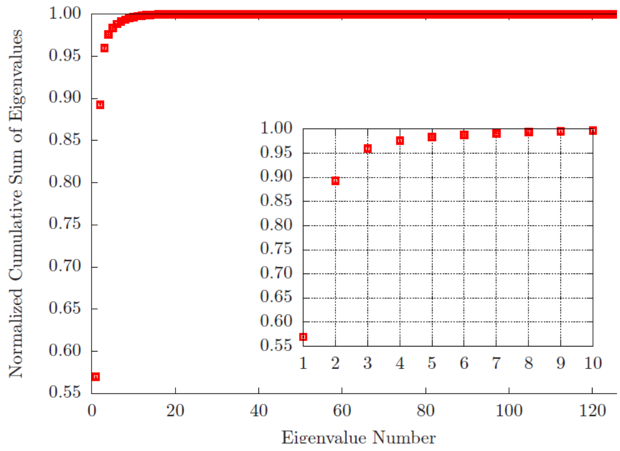
First, the basis is obtained by POD and the relative importance of the basis is depicted in Fig. 1. Fig. 1(a) shows that only 7 basis functions are enough to delineate the 99% of variation in noise and surprisingly only 1 basis suffices to estimate entire approach noise as shown in Fig.1(b). Table. 2 demonstrates that the original dimensionality *d* can be reduced effectively to *q* on a low dimensional space.

Table. 1. Ranges of parameters for noise evaluation

Inputs	Temperature	Pressure	Humidity
Lower bounds	29	22.4	0
Upper bounds	89	32.58	100



(a) Departure noise



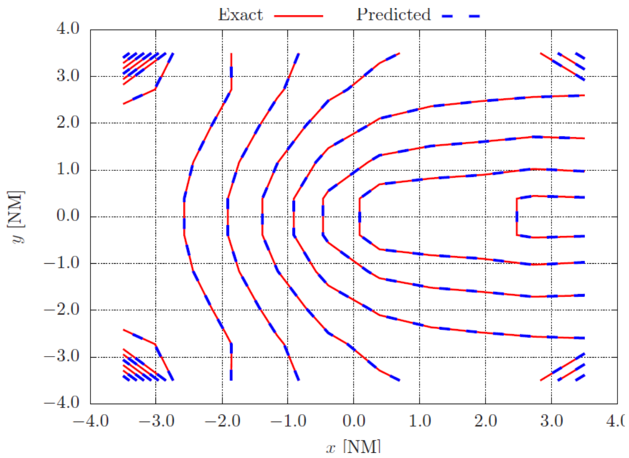
(b) Approach noise

Fig. 1. Normalized cumulative sum of eigenvalues

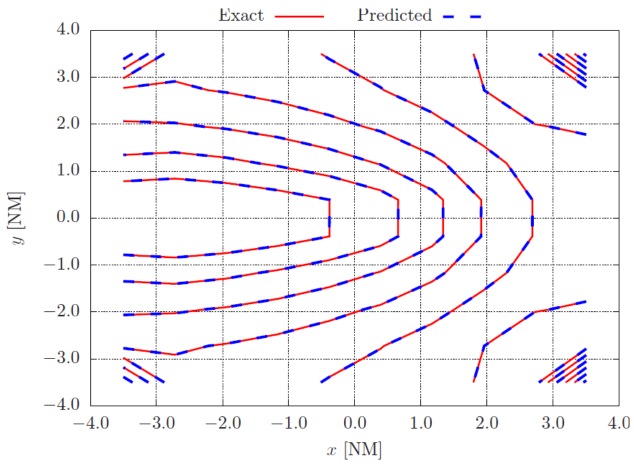
Table. 2. Dimensionality reduction

Response	d	q	$\sum \lambda^q_{i=1} / \sum \lambda^n_{i=1}$	d/q
Departure	5041	7	0.9912	720
Approach	5041	1	1.0000	5041

To verify the goodness-of-fit of the reduced-order AEDT model based on POD and ordinary Kriging, this research employs a coefficient of determination (R^2) that indicates the overall fitness of the estimated values to the exact values. The authors investigated the worst cases that correspond to the lowest values of the R^2 in the training and test samples. Fig. 2 shows the noise contours of the worst cases for the training samples, and Fig. 3 shows the noise contours of the worst cases for the test samples. Even for the results of the worst case, the prediction of the generated reduced-order model is almost the same as the exact counterparts.

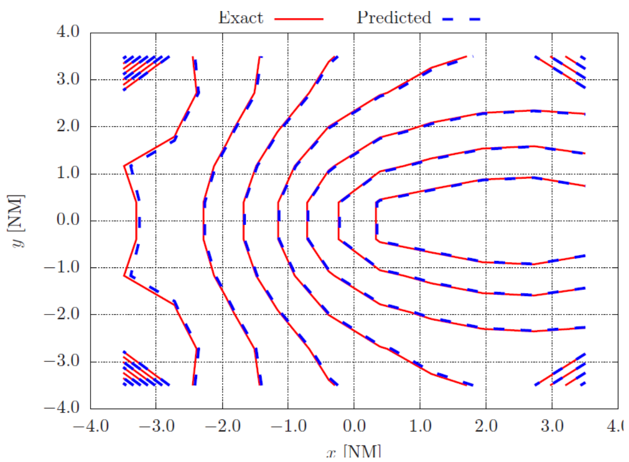


(a) 100th snapshot: departure

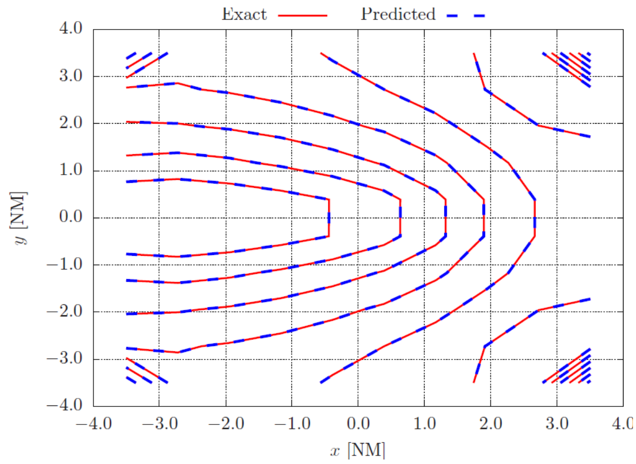


(b) 62th snapshot: approach

Fig. 2. Noise contours of worst cases for training samples



(a) 176th snapshot: departure



(b) 181th snapshot: approach

Fig. 3. Noise contours of worst cases for test samples

4 Conclusion and future works

This research proposed the use of POD and ordinary Kriging interpolation to attain the model order reduction of commercial program. The prediction investigations with both the training and test data sets substantiate that the reduced-order AEDT model is accurate and reliable. With further research, an another surrogate model can be estimated for the prediction of weighting coefficients of an aviation environmental design tool.

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