

# LOW REYNOLDS PROPELLER META-MODEL FOR MULTIDISCIPLINARY OPTIMIZATION

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## Abstract

*This study presents a model developed for low Reynolds propeller to be used in a multidisciplinary optimization. The proposed model covers the aerodynamics and structural performance of a propeller. The aerodynamics of the airfoils is a metamodel that results from X-Foil simulations and experimental results from several references. The metamodel considers the influences of Reynolds, Mach numbers and also the maximum camber and thickness of the airfoil. The propeller aerodynamics is the Blade Element Theory with induction velocities obtained from a free-stream Lifting Line model of the propeller wake. The structural model is a unidirectional finite element, with the deformations calculated with Castigliano Theorem for each element. This model is resumed in a software that obtains the aerodynamics and structural performance of a propeller and generates a CAD script for visualization of the propeller geometry, the flexibly deformed geometry, and the free-wake geometry. To validate the results of software, it was simulated for several propellers and comparing with experimental results. The optimization was simulated for some configurations with simplex method. The software advantage is a fast and stable simulation that makes viable to optimize propellers and obtain the main characteristics, as chord, incidence, airfoil thickness, camber and sweep distributions along the blade radius.*

## 1 Introduction

The model proposed in this article was developed based a bibliographic research of several references. In the early studies of

propellers theory of Rankine [1] e Froude, [2] on the momentum propeller theory, Drzewiecki, [3] on the Blade Element Theory, Betz and Prandtl [4] on the Blade Element Momentum Theory and further developments of Bothezat [5] and Theodorsen [6]. Also the aerodynamics theories for potential flow over a lifting body, as in Prandtl [7] for the Lifting-Line Theory and Garner [8] for the Lifting Surface Theory. Also analyzing the studies in propeller optimizations of Burger [9]. In the airfoil theory was analyzed the studies of Drela [10] for Mach corrections, Ostowari and Naik [11], Tangler [12] and Spera [13] for airfoil post-stall model, Smith [14] for the closed algebraic model for Mach, and airfoil thickness and Totah [15] for the model with Mach and Reynolds considerations. In the optimization field, the studies analyzed were Taheri and Mazaheri [16] and Hsin, Chen, et al., [17] for gradient based methods, Holland [18] for the Genetic Algorithm Optimization, and Nelder [19] for Simplex method of optimization.

The model proposed is divided into two disciplines, aerodynamics, and structures. The aerodynamics model composed of the 2D airfoil meta-model and a 3D propeller lifting-line model. The 2D airfoil model that calculates de aerodynamic coefficients of an airfoil based on its maximum camber and thickness and Reynolds and Mach numbers. And the 3D propeller lifting-line model calculates the induced velocities on the airfoil and the geometry of the free-wake roll-up of the propeller stream. The structures model is composed of a one-dimensional Finite element method to obtain Strain and Stress of the propeller. This work is a contribution to the low Reynolds propeller design introducing a new procedure of low cost computational for MDO.

## 2 Methodology

The logic of the methodology was based on premises for the development and they are:

- An optimum propeller properties have to maximize the aerodynamic efficiency and constrained the structural stress to a limit.
- The propeller performance is dependent on Reynolds and Mach numbers.
- The angle of attack in a propeller are large, reaching the post-stall conditions.
- Simplified methods of inductions velocities heave poor quality in high loadings or low advanced ratios.
- A reliable and robust method is necessary for a good optimization process.

### 2.2 Aerodynamic model

Atmospheric Model:

The Atmospheric model is based on the ISA 76, a study of the atmospheric proprieties with the dependence of the altitude. With this model the density, viscosity and sound speed are defined.

Airfoil Aerodynamics Model:

The aerodynamic 2D model was obtained with X-Foil simulations for the Clark-Y airfoil modified in several cambers and thickness. The simulations were realized in the low Reynolds range, from 30.000 to 1.000.000, and in Mach from 0 to 0.8, for angles of attack from  $-15^\circ$  to  $20^\circ$ .

The camber simulated was from 0.5% to 6.5%, and the thickness simulated was from 6% to 21%, covering 95% of the studied low Reynolds airfoils database.

The model is a set of equations that are exponentially dependent on the Reynolds number, and their coefficients are dependent on camber and thickness of the airfoil, with further mach corrections. The model predicts the influence of laminar bubble formation on the aerodynamics characteristics. The Fig. 1 and Fig. 2 show the comparison of the model and the simulations for the base airfoil.

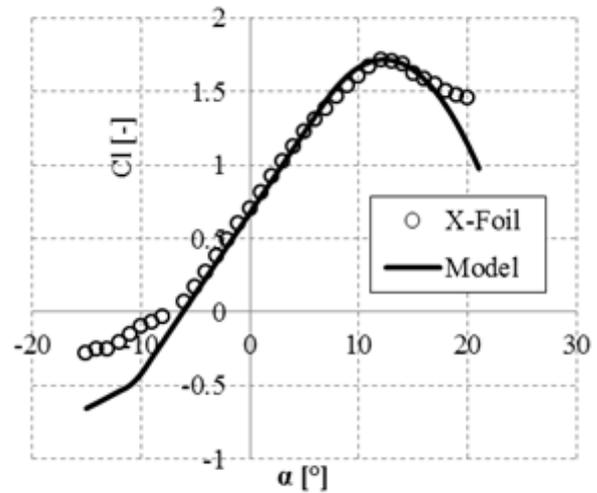


Fig. 1 Meta-model and XFoil comparison for  $CL \times \alpha$ .

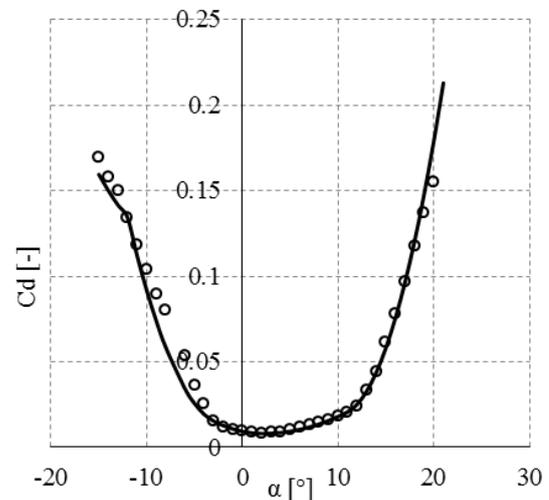


Fig. 2 Meta-model and XFoil comparison for  $CD \times \alpha$ .

Also the Fig. 3 and Fig. 4 shows the comparisons of the airfoil efficiency with the angle of attack in the full range of Reynolds defined. The mean deviation in the lift to drag ratio is 3.6.

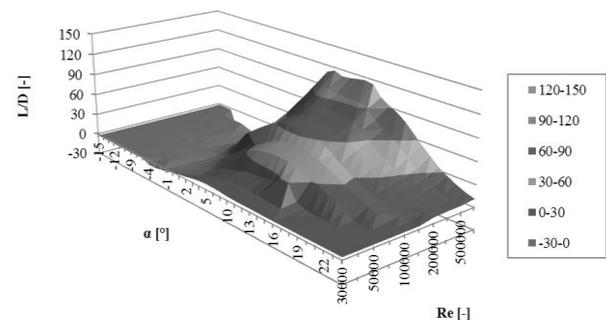


Fig. 3 L/D in X-Foil Simulations.

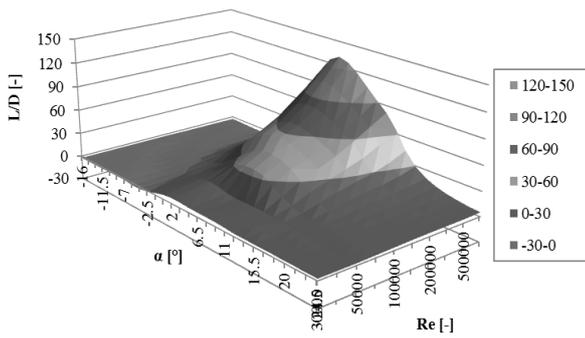


Fig. 4 L/D of the meta-model.

Propeller Aerodynamics Model: The propeller Aerodynamics model uses the Lifting Line Theory to predict the induced velocities on the airfoil sections. By numerically integrating the Biot-Savart formulation for aerodynamics, of the discrete shed vortex of each section of the blade. The same equation is used to obtain the induced velocities on the stream and predict the free-stream roll-up, as observed in Fig. 5.

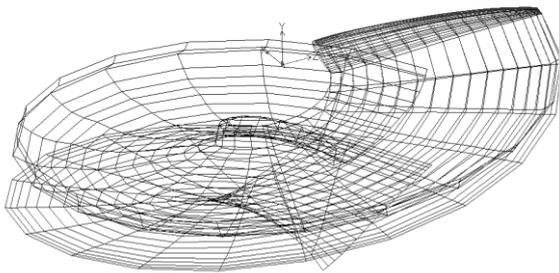


Fig. 5 – Blade Free-stream wake roll-up.

### 2.3 Structural Model:

The structural model uses the same discrete sections of the aerodynamic model, considering the blade as a beam fixed on the root. At the airfoil sectors, between two sections, the moments in X and Y axis are considered, and the forces in the 3 axes as well. The centrifugal inertial loads are also considered in affecting the moments in the blade. This consideration has the most influence in propellers with a large sweep in the blade tip. The flexibility model is a numerical integration energy method of the Castigliano equation. The Fig. 6 shows the flexible propeller geometry in black and the rigid in gray.

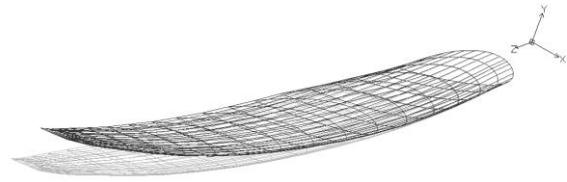


Fig. 6 flexible propeller (scale factor =20).

## 3 Results

Simulations in the developed program of a 2 blade propeller, with 0.168 meters of diameter were carried for several RPM's exploring the full range of Mach and Reynolds Numbers of the model. As observed in Fig. 7, the efficiency of the propeller varies with the RPM, for very low RPM the efficiency reduces due to low Reynolds number effect on the airfoils sections of the blade. Also for high RPM, above 14800, the increase of drag due to high Mach numbers of the tip sections of the blade. These influences are affected by the airfoil characteristics as camber and thickness and heave a large role in the optimizations simulations.

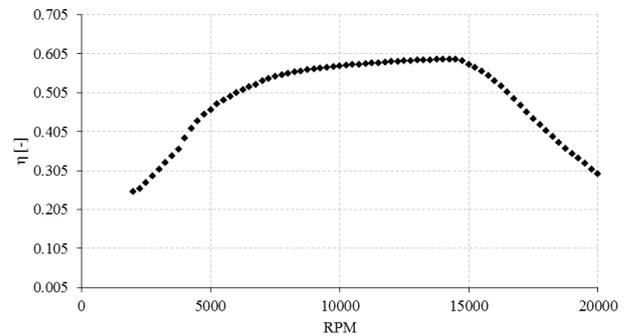


Fig. 7 – RPM x efficiency simulation results.

Also, simulations for varying the number of blades were carried, and as observed in Fig. 8 the maximum efficiency reduces 12% from a single blade propeller to a 7 blade propeller.

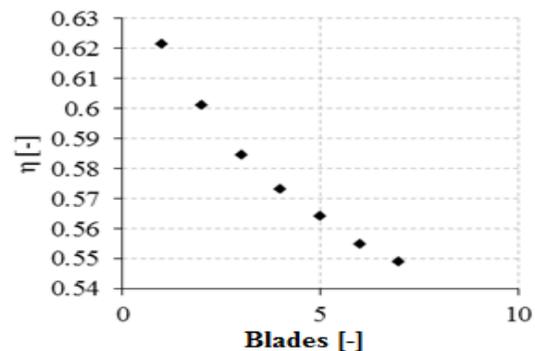


Fig. 8 Number of Blades x Efficiency

Also, the experimental results presented in Selig [20] were compared with simulations in the software developed. The comparisons in Fig. 9 and Fig. 10 shows that the model is representative of the experimental results. The discrepancies are most associated with the camber and thickness distributions that are not considered in the meta-model, thus the maximum camber and maximum thickness that are considered in the meta-model can represent equivalent results, and therefore gives support to the optimization process. Also, the turbulence level can affect the laminar properties of the airfoil, altering the airfoil drag and therefore the power coefficient  $C_p$ .

The main constraint is the propeller torque, that was considered a value of 0.6 N.m, equivalent of a 10 cubic centimeters two-stroke engine, very common in air models. The objective of the optimization is to increase the propeller thrust at a design speed. The Fig. 11 and Fig. 12 shows the result of the optimum propeller, the propeller diameter is 11.57 inches. The characteristics most pronounced is the propeller sweep at the tip, to increase the high Mach performance of the blade tip. Also to compensate de moments in Y axis at the blade root the medium sections of the blade projects forward in X axis. The monotonic reduction of the blade incidence and thickness is verified,

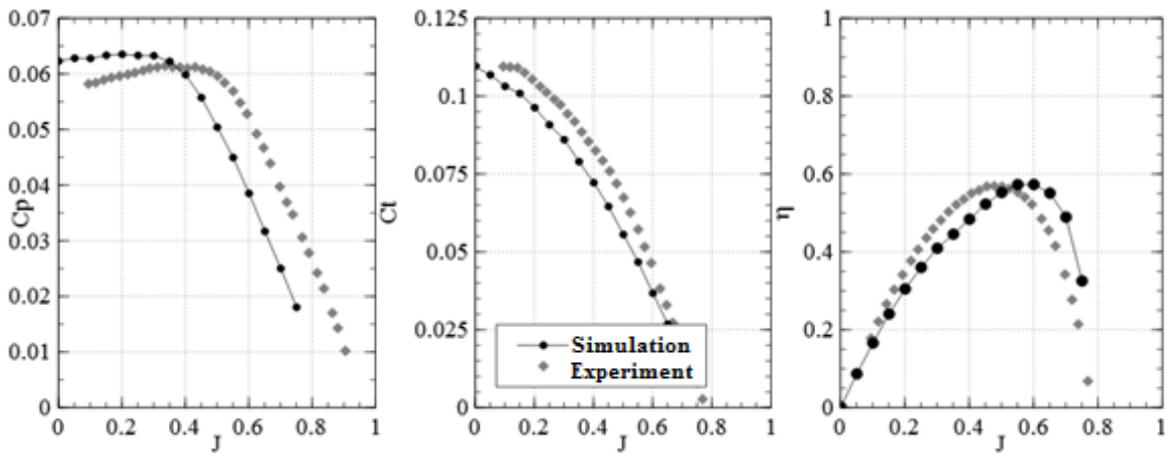


Fig. 9 – Simulation and experiment of SP 9x7 propeller at 6693 RPM

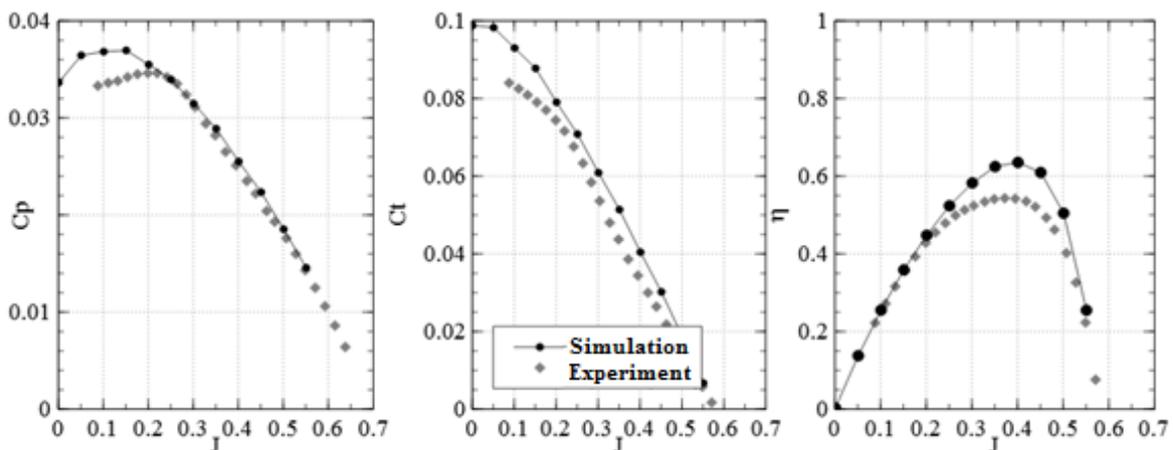


Fig. 10 – Simulation and experiment of E 11x5.5 propeller at 6001 RPM

### 3.1 Optimization

An optimization of a two blade propeller using the Simplex Method was realized in order to explore the developed program capability.

these characteristics are very common in propellers to meet the aerodynamic and structural requirements. The camber also reduces to the tip, these characteristics influences the stream roll-up and therefore the induced velocities in the propeller blade

sections. To optimize a propeller with multiple blades was considered a torque of 50.0 Nm and design speeds of 25m / s, 45m / s and 60m / s. The propeller characteristics obtained are similar in many respects to the propeller with two blades optimized for a small diameter; its thickness distributions and incidence are monotonic, the sweep occurs at the tip of the blade, the camber is reduced at the root and tip of the blade and there is reduction of the chord at the tip of the blade. However, the intensity of these features is different: the chord decreases more strongly towards the tip, especially because the Reynolds number is greater for this propeller and reduces the chord with a higher intensity. The incidence is higher throughout the blade, since the advance design ratios are higher, thus requiring a more blade pitch. The reduction of the camber is increased, mostly because the increased amount of blades increases the intensity of the induced velocities. The thickness is larger at the tip as the Reynolds is also higher and, under these conditions, the airfoil increases aerodynamic efficiency. The sweep is most acute at the tip characteristic associated with increased number of blades. Figures 13 and 14 shows the optimized multi-bladed propeller.

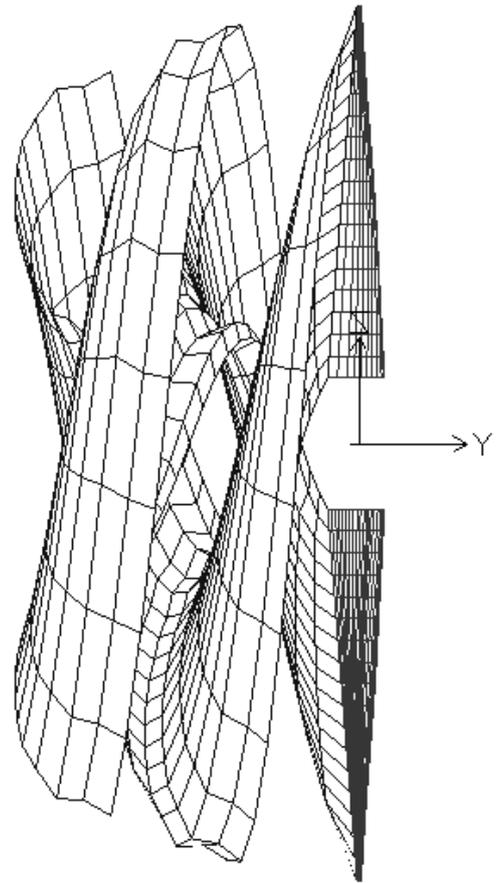


Fig. 9 Two-blade propeller lateral view

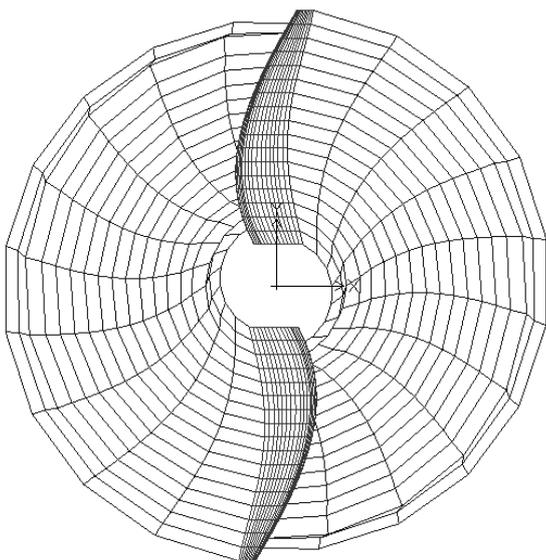


Fig. 11 Two-blade propeller frontal view

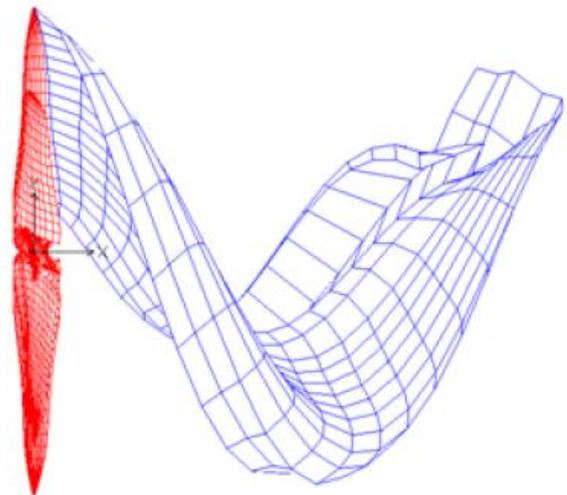


Fig. 13 Multi-bladed propeller lateral view.

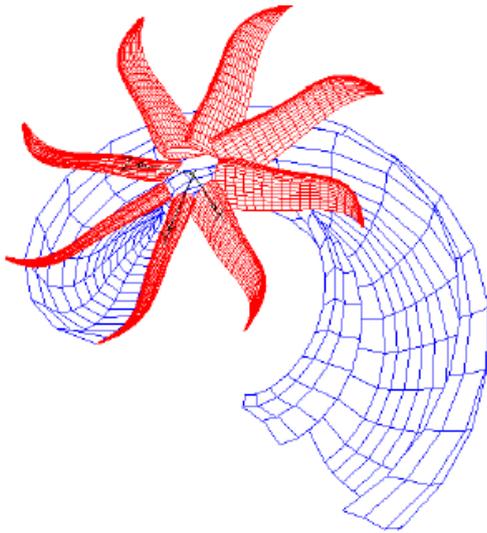


Fig. 10 Multi-bladed propeller 3D view

## 4 Conclusions

With the proposed model was possible to develop an optimization process for propellers with low Reynolds numbers. The results show a representative model, with significant stability e reliability that accelerates the achievement of the optimum propeller. The trade-off between structural and aerodynamic characteristics considered shows that optimum propellers are driven to a result that is not in the extremes of the limits of the geometry inputs. Therefore the model proposed can cover large variations in terms of the propeller geometry and consider their influence on the performance of the propeller.

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