OPTIMIZATION FOR FLIGHT CONTROL SYSTEM WITH CONSTRAINTS SUPPLEMENTED HANDLING QUALITIES

Young-Sam Kim*, Min-Jea Tahk**
***Hanwha Corporation, **KAIST**

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Abstract

In this study, the direct method including evaluation criteria of handling qualities as optimization method's constraints are carried out to complement problems of the indirect method that assesses whether a flight control system designed by existing optimization techniques satisfies handling qualities or not. For the direct method, co-evolutionary augmented Lagrangian method is used. F-16 longitudinal flight control system is performed the optimization with the performance index and constraints.

Moreover, present optimization technique can optimize flight condition one by one. In case of performing optimization for several flight conditions, it should be executed as many times as flight conditions. Although reliability of the optimization result can be high in that case, it is required considerable time for the optimization. To solve this problem, an optimization method is constructed with gain scheduling. Then we study techniques that have lower reliability about performances than one of the former but computation time is able to abridge.

1 Introduction

Although the classical PID controller has been widely used in flight control of various kinds of vehicles, its gain adjust tends to depend on designer's experience or trial and error. An optimization based PID controller design is a very attractive issue in that it provides a systematic design procedure and guarantees consistent controller performance especially in gain scheduling. If the controller's structure cannot be changed, an optimization of a PID controller is to find the best gain set, which conforms to designer's intention.

An optimization method based on the line search, like Sequential Quadratic Programming (SQP), has fast convergence but it has a weakness that a cost function must be continuously differentiable, In order to overcome this weakness, researchers have applied various kinds of stochastic optimization techniques to design controller. Park has applied the Co-Evolutionary Augmented Lagrangian Method (CEALM) for an autopilot design and the Particle Swarm Optimization (PSO) on an airship control[1, 2]. Krishinakumar done a control system optimization using the Genetic Algorithm (GA)[3]. Chen has studied an optimal PID control employing the GA[4].

In order to apply a real aircraft designed flight control system, stability of the system must be verified. Also, modifications of the control loop are required to improve stability of an unstable flight control system. In this manner verified standards for control loop correction and performance evaluation are required to improve a flight control system's stability. Even though a variety of studies and standard about performance evaluations and improvements of aircrafts are suggested, it is difficult to select controller design and evaluation standard suitable for each aircraft. MIL-STD-1797A made by U.S. Department of Defense states flying qualities to perform appropriate mission and stability grades for high-fidelity aircrafts at each flight condition. It also provides suitable evaluation standards divided into longitudinal and lateral direction[5].

Many researchers have carried out studies to evaluate the system's performance after design of aircraft's flight control system using aforementioned various kinds of optimization methods. The flight control system designed using this way, however, cannot guarantee to satisfy performance evaluation standards every time. So, the necessity of manners to complement these problems is raised

This paper is composed of five sections as follows. Section 2 explains the definition of flying and handling qualities (FQ and HQ) and standards about longitudinal direction specified by MIL-STD-1797A and MIL-F-8785C. Optimization techniques are described briefly in Section 3. Next, the simulation model is constructed in Section 4. Then simulation results are given. The objective of Section 5 is to conclude the paper.

2 FQ and HQ of aircraft

FQ and HQ of an aircraft are a measure that indicates how an aircraft's response is easy, predictable, and effective about pilot commands for performing a mission[7].

2.1 Classification for evaluation of FQ and HQ

FQ and HQ specified by MIL-F-8785C are introduced in this section[6]. Airplane's class is divided into four classes by weight and size like Table 1 in MIL-F-8785C. It also classifies levels of FQ and flight phase in Table 2 and 3.

Table 1. Classification of airplanes[6]

Class I	Small, light airplanes - Light utility, Primary trainer, Light observation
Class II	Medium weight, low-to-medium maneuverability airplanes - Heavy utility/search and rescue - Light or medium transport/cargo/tanker - Early warning/electronic countermeasures/airborne command, control, or communications relay - Antisubmarine, Assault transport, Reconnaissance, Tactical bomber, Heavy attack, Trainer for Class II
Class III	Large, heavy, low-to-medium maneuverability airplanes - Heavy transport/cargo/tanker, Heavy bomber - Patrol/early warning/electronic countermeasures/airborne command, control, or communications relay - Trainer for Class III
Class IV	High maneuverability airplane - Fighter/interceptor, Attack, Tactical reconnaissance, Observation, Trainer for Class IV

Table 2. Levels of flying qualities[6]

Level 1 (Satisfactory)	Flying qualities clearly adequate for the mission Flight Phase
Level 2 (Acceptable)	Flying qualities adequate to accomplish the mission Flight Phase, but some increase in pilot workload or degradation in mission effectiveness, or both, exists
Level 3 (Controllable)	Flying qualities such that the airplane can be controlled safely, but pilot workload is excessive or mission effectiveness is inadequate, or both. Category A Flight Phases can be terminated safely, and Category B and C Flight Phases can be completed

Table 3. Flight phase categories[6]

Nonterminal Flight Phase	**Category A**	Require rapid maneuvering, precision tracking, or precise flight path control - Air-to-air combat(CO) - Ground attack(GA) - Weapon delivery/launch(WD) - Aerial recovery(AR) - Reconnaissance(RC) - In-flight refueling(receiver)(RR) - Terrain following(TF) - Antisubmarine search(AS) - Close formation flying(FF)
	Category B	Normally accomplished using gradual maneuvers and without precision tracking, although accurate flight path control may be required - Climb(CL) - Cruise(CR) - Loiter(LO) - In-flight refueling(tanker)(RT) - Descent(D) - Emergency descent(ED) - Emergency deceleration(DE) - Aerial delivery(AD)
Terminal Flight Phase	**Category C**	Normally accomplished using gradual maneuvers and usually require accurate flight path control - Takeoff(TO) - Catapult takeoff(CT) - Approach(PA) - Wave-off/go-around(WO) - Landing(L)

2.2 Longitudinal HQ

They are explained referred to [6] and [8].

2.2.1 Phugoid stability

The phugoid is a long-period (low frequency) mode in which forward speed (kinetic energy) and altitude (potential energy) are interchanged. It has the frequency under 1 rad/sec generally. The requirements for the phugoid are expressed in terms of damping ratio for stable phugoid and time-to-double-amplitude, T_2, for unstable phugoid. It is expressed as Eq. (1). No distinction is made between classes of airplane.

$$T_2 = -\frac{0.693}{\zeta_p \omega_p} \tag{1}$$

If the phugoid is unstable, $\zeta_p < 0$.

Unconventional response-types, such as attitude-command, usually do not exhibit phugoid characteristics at all. For conventional or angle of attack command response-type, equivalent systems have been shown to work quite well when phugoid-type dynamics do appear in the augmented response. The frequency range of match should be expanded to include the phugoid characteristics. Eq. (2) is shown equivalent system that includes phugoid and short-period mode.

$$\frac{\theta}{\delta} \approx K_\theta \frac{\left(s+\frac{1}{T_{\theta_1}}\right)\left(s+\frac{1}{T_{\theta_2}}\right)}{\left(s^2+2\zeta_p\omega_{n_p}s+\omega_{n_p}^2\right)\left(s^2+2\zeta_{sp}\omega_{n_{sp}}s+\omega_{n_{sp}}^2\right)} \quad (2)$$

The phugoid alone can be matched say in the range 0.01 to 1 rad/sec using Eq. (3).

$$\frac{\theta}{\delta} \approx K_{\theta_p} \frac{\left(s+\frac{1}{T_{\theta_2}}\right)}{\left(s^2+2\zeta_{sp}\omega_{n_{sp}}s+\omega_{n_{sp}}^2\right)} \quad (3)$$

Natural frequency requirements of the phugoid mode do not exist, but it is recommended that they separate with short-period mode natural frequency. Typically phugoid mode should be satisfied following condition to stabilize the velocity of airplane about perturbations.

Level 1: ζ_p at least 0.04

Level 2: ζ_p at least 0.0

Level 3: T_2 at least 55 seconds

2.2.2 Short-period flying qualities

The short-period is a relatively rapid mode that governs the transient changes in angle of attack, pitch, flight path and normal load factor that occur following rapid control or gust inputs. Forward speed remains relatively constant during short-period oscillations. The mode is usually a stable under-damped second order oscillation.

Requirements of the short-period response are short-period frequency, acceleration sensitivity, and short-period damping. The equivalent short-period damping ratio, ζ_{sp}, shall be within the limits of Table 5.

Table 5. Short-period damping ratio limits[6]

Level	Category A and C flight phases		Category B flight phases	
	Min	Max	Min	Max
1	0.35	1.30	0.30	2.00
2	0.25	2.00	0.20	2.00
3	0.15		0.15	

2.2.3 Control Anticipation Parameter

Control Anticipation Parameter (CAP) is defined the ratio of the instantaneous angular acceleration in pitch to the steady-state change in load factor when the pilot applies a step input to the longitudinal control. It is given as Eq. (4).

$$CAP = \frac{\dot{q}_0}{\Delta n_{ss}} = \left(\frac{L_\alpha}{W\omega_{n_{sp}}^2}\right)^{-1} = \frac{\omega_{n_{sp}}^2}{n/\alpha} \quad (4)$$

This theory is based on the fact that in order to make precise adjustments to the flight-path, pilot must infer from the initial attitude response of the vehicle, the ultimate response of the flight-path. If CAP is too small, pilot tends to over-control and rates the pitch response as sluggish. If CAP is too large, pilot tends to understand his desired flight-path corrections, and rate the response as fast, abrupt, and too sensitive.

2.2.4 Bandwidth criteria

From the frequency response of the pitch attitude to longitudinal controller the bandwidth frequency is the frequency where the phase margin is 45°, or where the gain margin is 6 dB. The bandwidth hypothesis is that the pilot can adequately follow input commands with frequencies up to the bandwidth. If he tries aggressively to follow higher-frequency commands, he will approach instability. The phase roll-off, or slope at high frequency, that causes this characteristic is essentially the same as equivalent time delay and is measured using a parameter called τ_p. Thus bandwidth criterion does contain the physical elements that underlie the pilot-induced oscillations or the flying qualities cliffs. Time delay of bandwidth criteria does not fix a constant value. So, it is determined by transfer function of each aircraft. Time delay τ_p is expressed in Eq. (5) where ω_{180} denotes the frequency at phase of -180° and

$\phi_{2\omega_{180}}$ is the phase angle at frequency of double of ω_{180}. If time delay is large, airplane has bad closed-loop characteristics.

$$\tau_p = -\frac{\phi_{2\omega_{180}} + 180^\circ}{57.3 \times 2\omega_{180}} \tag{5}$$

2.2.5 Neal-Smith criterion

Neal-Smith criterion evaluates flight performances based on frequency response analysis. Pilot compensator is designed to improve flight performances and vibration characteristics in low frequency ranges. Flight performance analysis is based on analysis of open-loop characteristics with respect to the pitch command. Compensation is defined as the phase angle of the pilot's compensation measured at the specified bandwidth frequency and the resonance is presented in dB. Fig. 1 shows this criterion, and the bandwidth, minimum droop and maximum resonance corresponded with flight phases and levels of flying qualities are given in Table 6 and 7.

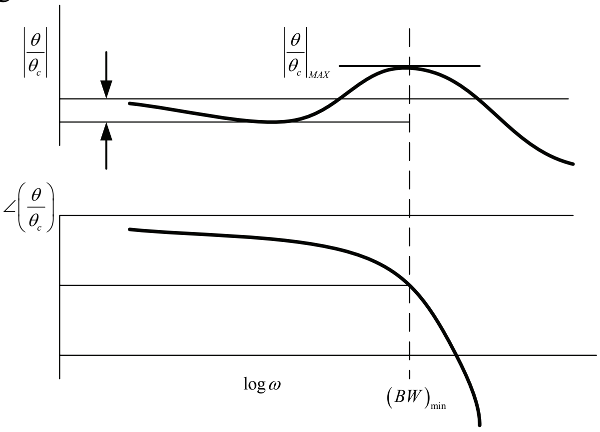

Fig. 1. Neal-Smith criterion[8]

Table 6. Bandwidth of the Neal-Smith criterion[8]

Flight phase	Bandwidth (rad/sec)
Category A	3.5
Category B	1.5
Category C	1.5
Landing	2.5

Table 7. Minimum droop and maximum resonance of the Neal-Smith criterion[8]

	Minimum droop(dB)	Maximum resonance(dB)
Level 1	-3	3
Level 2	-3	9

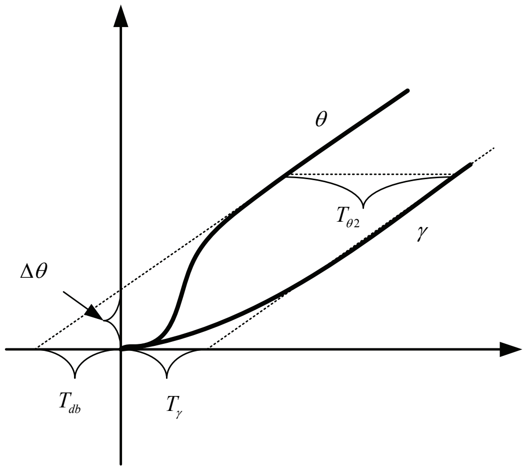

Fig. 2. Dropback time requirement[8]

2.2.6 Gibson's dropback criterion

Excessive lead in the Neal-Smith criterion results in excessive lag requirements for the pilot but the bandwidth and equivalent systems criteria do not adequate cover the effects of excessive phase lead. The dropback criterion had been added to the bandwidth criterion to catch the excessive lead cases.

Dropback is characteristics of the pitch attitude response and is positive if the attitude decrease to a lower state value after the removal of the control input. The opposite case is known as negative dropback or overshoot. The pitch rate's overshoot ratio defines the level of dropback. Dropback time is illustrated like Fig. 2 and expressed as follows:

$$\begin{aligned} T_{db} &= \frac{\Delta\theta}{q_{ss}} \\ \frac{\gamma}{\theta} &\approx \frac{1}{T_{\theta 2}s + 1} \\ T_{\theta 2} &= T_{db} + T_\gamma \approx T_{db} + \frac{2\zeta_{sp}}{\omega_{n_{sp}}} \end{aligned} \tag{6}$$

3 Optimization method

A general constrained minimization problem is formulated as Eq. (8) where $x = (x_1, x_2, \cdots, x_n)$ is a vector of variables, $f(x)$ is a cost function, $h(x) = [h_1(x), h_2(x), \cdots, h_m(x)]^T$ is a set of m equality constraints, and

$g(x)=\left[g_1(x),g_2(x),\cdots,g_k(x)\right]^T$ is a set of k inequality constraints. All $f(x)$, $h(x)$ and $g(x)$ can be either linear or nonlinear, continuous or discontinuous[11].

$$\begin{aligned}&\min f(x)\\&\text{subject to } h(x)=0\\&\qquad\qquad g(x)\le 0\end{aligned} \qquad (7)$$

Methods for solving constrained optimization problems exist several ways as following[9].

- Methods based on preserving feasibility of solutions.
- Methods based on penalty functions.
- Methods based on a search for feasible solutions.
- Hybrid methods: combinations of stochastic evolutionary methods and deterministic optimization procedures.
- Other methods: co-evolution approach, multi-objective optimization.

Using these optimization methods, robustness of levels that are able to obtain from modern control theory can be guaranteed. Besides design procedure of control systems is systematic and automatic. It can be designed optimal control system for minimizing given performance index. In this study, we design flight control system that satisfies FQ, HQ and other constraints using CEALM which is the best performance about convergence of solutions among above methods.

3.1 CEALM

The CEALM is based on evolution strategies for selection, recombination and mutation of two populations with opposite objectives to solve saddle-point problems[10]. Using the augmented Lagrangian formulation, a constrained problem can be regarded as a mini-max game between the parameter vector and the multiplier vector by duality.

A mini-max game with payoff function $F=F(u,v)$ is to be minimized by u and maximized by v. It is represented as follows:

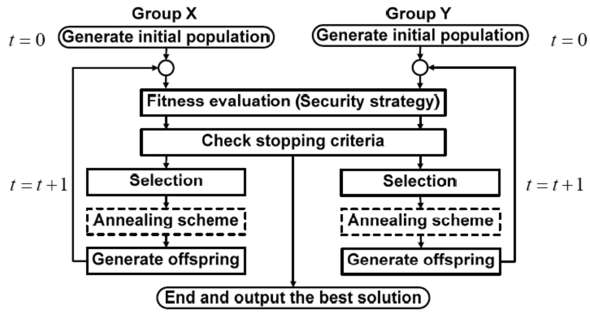

Fig. 3. Flowchart of co-evolution for saddle-point problems

$$\min_u F(u,v^*)=\max_v F(u^*,v)=F(u^*,v^*)=F^* \qquad (8)$$

The saddle-point solution satisfies the pair of following saddle-point inequalities:

$$F(u^*,v)\le F(u^*,v^*)\le F(u,v^*) \qquad (9)$$

The mini-max game can be interpreted in the context of co-evolution as

$$\begin{aligned}&\text{fitness of } x_i=\max_j F(x_i,y_j)\\&\text{fitness of } y_j=-\max_i F(x_i,y_j)\end{aligned} \qquad (10)$$

For convex problems, there exists a saddle-point solution, which corresponds to the optimal value of the parameter vector and the Lagrangian multiplier. For non-convex problems, the augmented Lagrangian method is used to obtain convexity. Fig. 3 shows a flowchart of co-evolution for saddle-point problems[11].

4 Simulation modeling and result

F-16 model using in this paper is designed a control system, which assigns all gains to optimization parameters consisted of inner loop's Stability and Control Augmentation Systems (SCAS) and outer loop's autopilot, to minimize given performance index and to satisfy given constraints. In general design process SCAS is designed first and then gain of outer loop is determined. In case of using optimization techniques, however, all gains are determined at once through parameter optimization. Optimization is executed after that linearization of the nonlinear model at trim point is performed and then appropriate perfomance index and constraint are determined considering SCAS design, autopilot's structure determination, system's dynamic characteristic and objective of control system[12].

4.1 Longitudinal linear flight control system

The outer loop control system is designed to track the guidance command by considering the designed SCAS as the inner loop. Assume that the guidance command of longitudinal flight control system is the velocity and altitude. Generally, thrust controls the velocity and pitch angle change by elevator deflection controls the altitude. Applying longitudinal control concepts the velocity control system is designed PID controller to reduce errors between reference velocity and aircraft's velocity and the altitude control system is using PI controller. The pitch attitude control system consists of PID controller to track the pitch attitude angle provided for tracking altitude command. Schematic diagram is shown in Fig. 4.

A and B matrices of $\dot{x} = Ax + Bu$ at $h = 10000\,ft$ and $V_t = 700\,ft/\sec$ are obtained by performing linearization with longitudinal nonlinear model. They are given as following. $u = \begin{bmatrix}\delta_{thr} & \delta_{ele}\end{bmatrix}^T$ and $x = \begin{bmatrix}V_t & \alpha & \theta & q & h & pow\end{bmatrix}^T$ denote control and state variables.

$$A = \begin{bmatrix} -0.0179 & 25.7107 & -32.1700 & -0.0637 & 0 & 0.3432 \\ -0.0001 & -1.0502 & 0 & 0.9310 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 & 0 \\ 0 & -3.5854 & 0 & -1.4219 & 0 & 0 \\ 0 & -700.0000 & 700.0000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.0000 \end{bmatrix} \qquad (11)$$

$$B = \begin{bmatrix} 0 & 0.2832 \\ 0 & -0.0022 \\ 0 & 0 \\ 0 & -0.2628 \\ 0 & 0 \\ 64.9400 & 0 \end{bmatrix} \qquad (12)$$

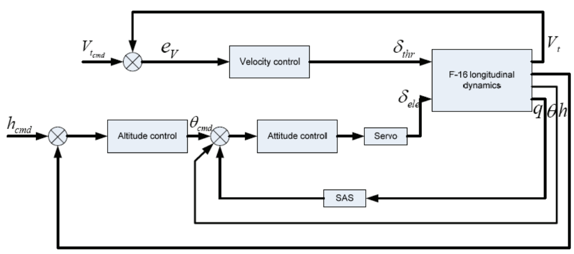

Fig. 4. Schematic diagram of F-16 longitudinal control system

4.2 Performance index and constraint

Performance index of F-16 longitudinal flight control system using CEALM is as follow:

$$\min_u J = \int_{t_i}^{t_f} \left(w_1 \hat{x}_1^2 + w_2 \hat{x}_2^2 \right) dt \qquad (13)$$

where w_1 and w_2 denote weighting. $\hat{x}_1$ and $\hat{x}_2$ are altitude errors and velocity errors.

To optimize the control system, we can consider many constraints. Used constraints for the optimization are given in Eq. 14 ~ 18.

$$\max\left[\mathrm{Re}\left(P_i\right)\right]_j \le 0 \qquad (14)$$

$$\left[GM_d - \left|GM\right|\right]_j \le 0 \qquad (15)$$

$$\left[PM_d - \left|PM\right|\right]_j \le 0 \qquad (16)$$

$$\left[\%OS_d - \%OS\right]_j \ge 0 \qquad (17)$$

$$\left[T_{s_d} - T_s\right]_j \ge 0 \qquad (18)$$

where $j = 1, 2$. They denote the velocity control system and altitude control system. P_i is i th pole of the designed control system. GM_d, PM_d, $\%OS_d$, and T_{s_d} is desired gain margin, phase margin, %overshoot, and settling time.

Moreover, criteria related with longitudinal HQ should be supplemented as constraints. F-16 fighter should satisfy HQ criteria corresponding to Class IV, Level 1, and Category B. They are described detail in Section 2. Eq. (19) ~ (24) summarize them.

$$\text{Phugoid stability} \quad \zeta_p \ge 0.04 \qquad (19)$$

$$\text{Short period} \quad 0.3 \le \zeta_{sp} \le 2.0 \qquad (20)$$

$$0.28 \le CAP \le 3.6 \qquad (21)$$

$$\begin{aligned} &\text{Bandwidth criteria } 0 \le \tau_p \le 0.1 \Rightarrow \omega_{BW} \ge 1.4\,rad/\sec \\ &0.1 \le \tau_p \Rightarrow \omega_{BW} \ge 4\tau_p + 1\,rad/\sec \end{aligned} \qquad (22)$$

$$\text{Dropback time} \quad T_{db} \le 0.25\sec \qquad (23)$$

$$\text{Neal-Smith criterion} \quad BW \geq 1.5 rad/\sec, \quad droop \geq -3dB, \quad resonance \leq 3dB \tag{24}$$

where constraint in Eq. (23) is more rigorous value according to Category A for precise tracking. So, we do not contain dropback time as a constraint of the optimization and only evaluate it for HQ after the optimization.

4.3 Simulation results

4.3.1 Optimization without considering HQ

To compare with results of this study, we execute optimization of F-16 longitudinal flight control system at a trim condition ($h = 10000 ft$ and $V_t = 700 ft/\sec$) without considering HQ as constraints. Reference altitude and velocity command are 5 ft and 0 ft/sec. Desired values and obtained gains are tabulated in Table 8 and 9.

Using these obtained gains, we check robustness and HQ. They are shown in Table 10. As you can see it, all values related with robustness satisfy design requirements. For HQ, phugoid stability, short-period FQ, CAP and bandwidth criteria satisfy design requirements. However, dropback time and Neal-Smith criterion do not satisfy them. It is possible that dropback time is not satisfied because its requirement is not adequate for Category B. Simulation results are given in Fig. 5 ~ 8. Responses of the aircraft follow reference commands quite well. Fig. 6 and 7 denote responses of short-period and phugoid low order equivalent systems (LOES).

4.3.2 Optimization considering HQ

Neal-Smith criterion is adequate for pilot-in-the-loop concept but our model does not include pilot model. By reason of it, Eq. (24) also does not consider as a constraint of the optimization and evaluate it for HQ after the optimization.

All design requirements of robustness are same with Table 8. HQ requirements are Eq. (19) ~ (23). We only change trim conditions and simulate 9 cases. Due to limitation of paper, results of same trim condition with Section 4.3.1 are only illustrated. As you can see in Table 12, design requirements related with robustness are better than results of Section 4.3.1. In this case, Neal-Smith criterion and dropback time also do not satisfy. Mismatches of short-period mode and droop of Neal-Smith criterion, however, are reduced than ones of Section 4.3.1. They are shown in Fig. 9 ~ 12.

Table 8. Desired values

	Desired Value
Gain Margin	6 dB
Phase Margin	45 deg
%Overshoot	20 %
Settling Time	20 sec

Table 9. Obtained gains without considering HQ

Altitude Controller	Kp_h	5.7534469
	Ki_h	0.0000000
Attitude Controller	Kp_t	17.2014973
	Ki_t	1.5600291
	Kd_t	18.1880209
SAS	K_a	-149.6398982
	K_q	-79.7839880
Velocity Controller	Kp_v	0.8534295
	Ki_v	0.0000000
	Kd_v	1.0000000

Table 10. Analysis of robustness and HQ

Robustness		
Gain Margin(dB)	h	16.4554
	Vt	Inf
Phase Margin(deg)	h	49.5812
	Vt	78.0700
%Overshoot (%)	h	7.3473
	Vt	4.0462
Settling Time(sec)	h	20
	Vt	6.7200
Handling Qualities		
Phugoid	ζ_p	3.6505
	Mismatch	9.3371
Short-period	ζ_{sp}	0.7712
	CAP	1.2233
	Mismatch	9.6723
Dropback	T_{db} **(sec)**	0.6249
Neal-Smith	BW **(rad/sec)**	-
	$Droop$ **(dB)**	-18.2271
	$Resonance$ **(dB)**	-
Bandwidth	τ_p **(sec)**	-0.0069
	ω_{BW} **(rad/sec)**	2.5873

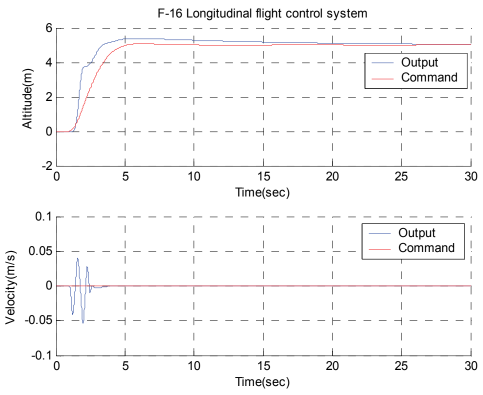

Fig. 5. Time response of the aircraft

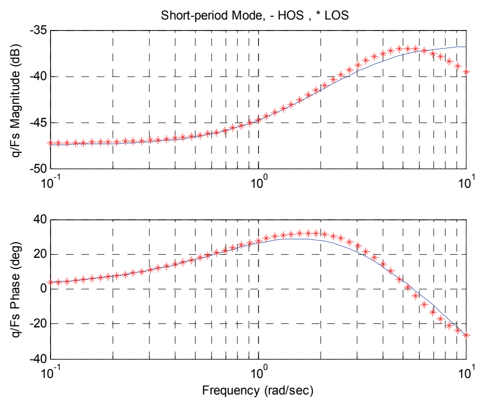

Fig. 6. Response of short-period LOES

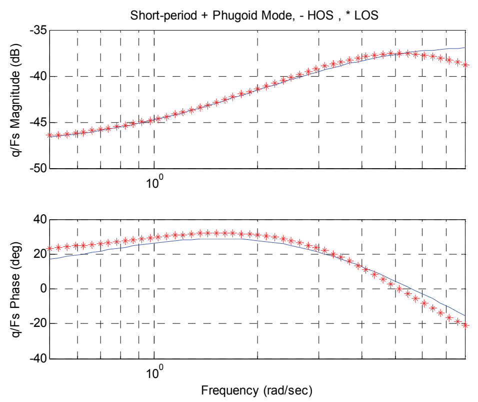

Fig. 7. Response of phugoid LOES

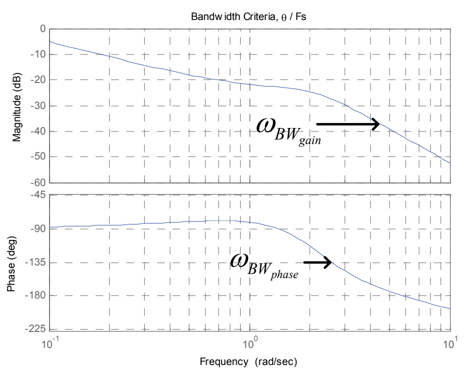

Fig. 8. Bandwidth criteria

Table 11. Obtained gains considering HQ

Altitude Controller	Kp_h	1.0229805
	Ki_h	0.0000484
Attitude Controller	Kp_t	18.0942788
	Ki_t	0.0061321
	Kd_t	28.0351966
SAS	K_a	-149.5342099
	K_q	-26.3404038
Velocity Controller	Kp_v	0.6110085
	Ki_v	0.0433254
	Kd_v	0.9516431

Table 22. Analysis of robustness and HQ

Robustness		
Gain Margin(dB)	h	27.4767
	Vt	Inf
Phase Margin(deg)	h	52.6878
	Vt	79.2470
%Overshoot (%)	h	0.5824
	Vt	0.0842
Settling Time(sec)	h	4.7700
	Vt	5.3800
Handling Qualities		
Phugoid	ζ_p	4.1513
	Mismatch	14.4583
Short-period	ζ_{sp}	0.3192
	CAP	1.6797
	Mismatch	1.9252
Dropback	T_{db} **(sec)**	0.8094
Neal-Smith	BW **(rad/sec)**	-
	$Droop$ **(dB)**	-13.8211
	$Resonance$ **(dB)**	-
Bandwidth	τ_p **(sec)**	-0.0069
	ω_{BW} **(rad/sec)**	2.5873

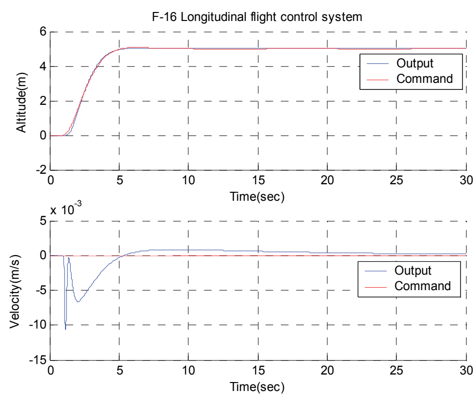

Fig. 9. Time response of the aircraft(HQ)

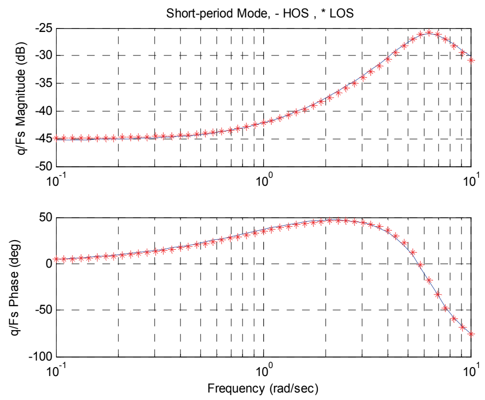

Fig. 10. Response of short-period LOES(HQ)

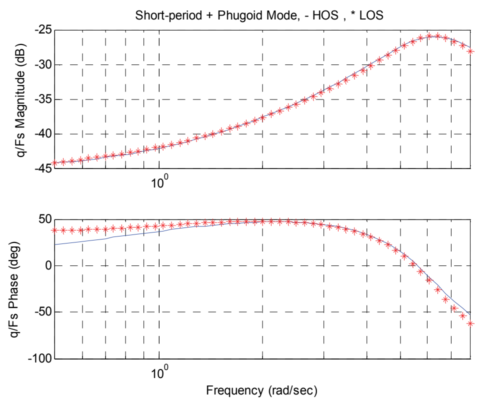

Fig. 11. Response of phugoid LOES(HQ)

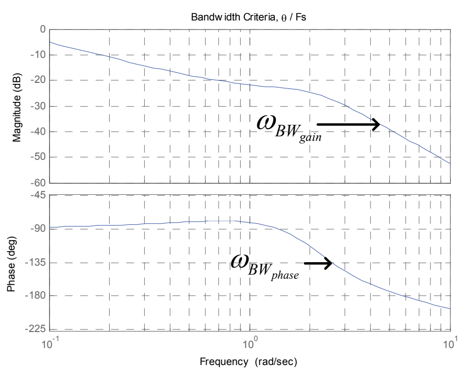

Fig. 12. Bandwidth criteria(HQ)

4.3.3 Gain scheduling

In this section, we derive equation of each gain through least-square curve fitting based on the previously obtained gains. The purpose of this approach is to get the gain satisfying HQ easily at other trim condition without optimization. Basic form of the equation is expressed as following.

$$F(h,Vt) = a_1h^3 + a_2h^2Vt + a_3hVt^2 + a_4Vt^3 + a_5h^2 + a_6hVt + a_7Vt^2 + a_8h + a_9Vt \tag{25}$$

The result of least-square curve fitting is sensitive against to initial guess values. By reason of this, we found adequate initial guess values through trial and errors. Using least-square curve fitting function in MATLAB, we obtained following equations.

$$\begin{aligned} Kp\_v(h,Vt) = &-1.666\times10^{-12}h^3 + 8.2528\times10^{-11}h^2Vt - 2.1415\times10^{-9}hVt^2 \\ &+1.598\times10^{-8}Vt^3 + 1.5784\times10^{-8}h^2 + 9.2232\times10^{-9}hVt \\ &+2.2001\times10^{-8}Vt^2 + 2.1387\times10^{-8}h + 2.88386\times10^{-8}Vt \end{aligned} \tag{26}$$

$$\begin{aligned} Ki\_v(h,Vt) = &\big(-2.0003\times10^{-12}h^3 + 9.9912\times10^{-11}h^2Vt - 2.4042\times10^{-9}hVt^2 \\ &+1.5017\times10^{-8}Vt^3 + 1.5285\times10^{-8}h^2 + 8.4686\times10^{-9}hVt \\ &+2.0568\times10^{-8}Vt^2 + 2.0568\times10^{-8}h + 2.6888\times10^{-8}Vt\big) \\ &\times10^{-3} \end{aligned} \tag{27}$$

$$\begin{aligned} Kd\_v(h,Vt) = &-1.8839\times10^{-12}h^3 + 9.0382\times10^{-11}h^2Vt - 2.1872\times10^{-9}hVt^2 \\ &+1.5902\times10^{-8}Vt^3 + 1.5856\times10^{-8}h^2 + 9.3181\times10^{-9}hVt \\ &+2.2117\times10^{-8}Vt^2 + 2.1603\times10^{-8}h + 2.9143\times10^{-8}Vt \end{aligned} \tag{28}$$

$$\begin{aligned} Kp\_h(h,Vt) = &-1.6537\times10^{-12}h^3 + 1.0768\times10^{-10}h^2Vt - 2.3451\times10^{-9}hVt^2 \\ &+1.5785\times10^{-8}Vt^3 + 1.7076\times10^{-8}h^2 + 1.1371\times10^{-8}hVt \\ &+2.5239\times10^{-8}Vt^2 + 2.4981\times10^{-8}h + 3.4702\times10^{-8}Vt \end{aligned} \tag{29}$$

$$\begin{aligned} Ki\_h(h,Vt) = &\big(-1.3718\times10^{-12}h^3 + 5.947\times10^{-11}h^2Vt - 1.8848\times10^{-9}hVt^2 \\ &+1.6998\times10^{-8}Vt^3 + 1.5823\times10^{-8}h^2 + 9.6391\times10^{-9}hVt \\ &+2.3387\times10^{-8}Vt^2 2.1869\times10^{-8}h + 3.0447\times10^{-8}Vt\big)\times10^{-2} \end{aligned} \tag{30}$$

$$\begin{aligned} Kp\_t(h,Vt) = &\big(1.1884\times10^{-11}h^3 - 6.229\times10^{-10}h^2Vt + 6.4833\times10^{-9}hVt^2 \\ &+4.8562Vt^3 + 2.7214\times10^{-8}h^2 + 2.6985\times10^{-8}hVt \\ &+6.0008\times10^{-8}Vt^2 + 5.0699\times10^{-8}h + 7.5052\times10^{-8}Vt\big)\times3 \end{aligned} \tag{31}$$

$$\begin{aligned} Ki\_t(h,Vt) = &\big(-1.9006\times10^{-12}h^3 + 9.5274\times10^{-11}h^2Vt - 2.3774\times10^{-9}hVt^2 \\ &+1.5126\times10^{-8}Vt^3 + 1.5265\times10^{-8}h^2 + 8.4597\times10^{-9}hVt \\ &+2.0615\times10^{-8}Vt^2 + 2.0073\times10^{-8}h + 2.6884\times10^{-8}Vt\big)\times10^{-2} \end{aligned} \tag{32}$$

$$\begin{aligned} Kd\_t(h,Vt) = &\big(1.1131\times10^{-11}h^3 - 1.1924\times10^{-9}h^2Vt + 1.5339\times10^{-8}hVt^2 \\ &-8.8578\times10^{-9}Vt^3 + 2.6321\times10^{-7}h^2 + 2.6088\times10^{-7}hVt \\ &+2.315\times10^{-7}Vt^2 + 5.8332\times10^{-7}h + 5.6493\times10^{-7}Vt\big)\times2 \end{aligned} \tag{33}$$

$$\begin{aligned} K\_a(h,Vt) = &5.2964\times10^{-11}h^3 - 1.4354\times10^{-9}h^2Vt + 7.4915\times10^{-8}hVt^2 \\ &-5.41\times10^{-7}Vt^3 - 8.1385\times10^{-7}h^2 - 2.5224\times10^{-5}hVt \\ &-6.239\times10^{-5}Vt^2 - 5.082\times10^{-5}h - 0.00012684Vt \end{aligned} \tag{34}$$

$$\begin{aligned} K\_q(h,Vt) = &4.4949\times10^{-11}h^3 - 2.2907\times10^{-9}h^2Vt + 6.4489\times10^{-8}hVt^2 \\ &-5.0088\times10^{-7}Vt^3 - 4.6548\times10^{-7}h^2 - 1.5645\times10^{-6}hVt \\ &-3.4554\times10^{-6}Vt^2 - 2.4443\times10^{-6}h - 5.7034\times10^{-6}Vt \end{aligned} \tag{35}$$

To evaluate the results of least-square curve fitting, we execute them at other trim point, i.e. $h = 16000\,ft$ and $V_t = 560\,ft/\sec$. All constraints related with HQ are satisfied except for Neal-Smith criterion and dropback time. They are given in Table 13. Besides, phase margin of altitude does not satisfy the design requirements, too.

Table 33. Analysis of robustness and HQ(curve fitting)

Robustness		
Gain Margin(dB)	h	6.0248
	Vt	Inf
Phase Margin(deg)	h	31.2982
	Vt	78.7256
%Overshoot (%)	h	0.2831
	Vt	1.4967
Settling Time(sec)	h	6.9700
	Vt	9.4100
Handling Qualities		
Phugoid	ζ_p	2.6479
	Mismatch	10.9402
Short-period	ζ_{sp}	0.5071
	CAP	2.7616
	Mismatch	12.3194
Dropback	T_{db} (sec)	1.2095
Neal-Smith	BW (rad/sec)	-
	$Droop$ (dB)	-13.9679
	$Resonance$ (dB)	-
Bandwidth	τ_p (sec)	-0.0127
	ω_{BW} (rad/sec)	1.8924

5 Conclusions

We studied the optimization for flight control system with constraints supplemented HQ. Robustness of the result considering HQ is improved even if short-period damping ratio is decreased. Besides, HQ are satisfied.

In addition to the optimization, research about gain scheduling was executed. In case of performing optimization for several flight conditions, it is performed as many times as flight conditions. To solve this problem, we carried out the equation of each gain through least-square curve fitting. Although some gains obtained by curve fitting had large discrepancy compared with optimal gains, they satisfied all constraints related with handling qualities. By reason of this, we know that equations got through least-square curve fitting are able to use to calculate gains fast even if its performance is degraded.

A major drawback of the equation of each gain is the existence of difference between scheduled gain and optimal gain at the same trim point referred to obtain the equation. Besides, dissatisfaction of several requirements of robustness is existed. Furthermore, optimization considering handling qualities has a burden that is the computation time.

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