THE SIZE EFFECT OF SIC FIBER STRENGTH ON THE GAUGE LENGTH

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Abstract

It is known that the single-modal Weibull model describes well the size effect of brittle fiber tensile strength. However, some ceramic fibers have been reported that single-modal Weibull model provided biased estimation on the gauge length dependence. A hypothesis on the bias is that the density of critical defects is very small, thus fracture probability of small gauge length samples distributes in discrete manner, which makes the Weibull parameters dependent on the gauge length. The objectives of this study is thus to assess if the Weibull parameters are dependent on the gauge length for the case of small gauge length samples.

Tyranno ZMI Si-Zr-C-O fiber (UBE Industry Co.) has been selected as an example fiber. The tensile tests have been done on several gauge lengths. The derived Weibull parameters have shown a dependence on the gauge length.

1 Introduction

Ceramic Matrix Composites (CMCs) have excellent heat resistance and specific tensile strength. However, it is difficult to design reliable CMC components due to the large distribution in the strength. Mechanical properties of reinforcement fibers have major effect on the strength distribution of CMCs. Thus, ceramic fibers must be evaluated not only in the mean strength but also in the deviation, beforehand the CMCs production [1] [2].

It is known that the strength distribution of brittle material is described well with single-modal Weibull model. Ceramic fibers are brittle, thus the single-modal Weibull model may fit well to the strength. However, some ceramic fibers have been reported that single-modal Weibull model provided biased estimation on the gauge length dependence [3] [4]. A hypothesis on the bias is that the density of critical defects is very small thus fracture probability of small gauge length samples distributes in discrete manner, which makes the Weibull parameters dependent on the gauge length. The objectives of this study is thus to assess if the Weibull parameters are dependent on the gauge length for the case of small gauge length samples.

To analyze the dependence of the Weibull parameters on the gauge length, we make single fiber tensile tests on the gauge lengths of 200mm, 100mm, 50mm, and 20mm on Tyranno ZMI Si-Zr-C-O monofilaments (UBE Industry Co.) as an example ceramic fiber.

2 Experiments

2.1 Sample

Tyranno ZMI fiber has excellent heat resistance and mechanical properties. Table 1 shows the attributes of Tyranno ZMI fiber [5].

Table 1 Attributes of Tyranno ZMI Fiber

Property	Unit		ZMI
Nominal Diameter of Filament	μm		11
Number of Filament	/one bundle		800
Tex	g/1000m		200
Fracture Stress	GPa		3.4
Young's Modulus	GPa		200
Fracture Strain	%		1.7
Density	g/	3	2.48
Composition	wt.%	Si	56
		C	34
		O	9
		Ti	-
		Zr	1
		Al	-
Thermal Expansion Coefficient	10^{-6}/K		4.0 room temperature-1000
Coefficient of Thermal Conductivity	W/mK		2.5

Fig.1   Diameter Distribution of Tyranno ZMI Fiber

2.2 Axial Admeasurements of Sample Fibers' Diameters

Laser Scan Micrometer type LSM-6000 (MITUTOYO, Corp.) has been applied for the diameter measurements of the sample fibers along each 1mm of the 500mm gauge length, as some ceramic fibers have been reported to show variable diameters both along the gauges and between each fibers at a bundle [6] [7] [8] [9]. An example of measurement results is shown in Fig.1

Diameters vary approximately from 10μ m to 14μ m in Fig.1 suggesting that maximum cross section area is more than twice larger than the minimum cross section area. It implies that to calculate fracture stress with a diameter may provide biased data. Thus, to measure diameters in gauge section beforehand the tensile tests is indispensable to derive accurate Weibull parameters.
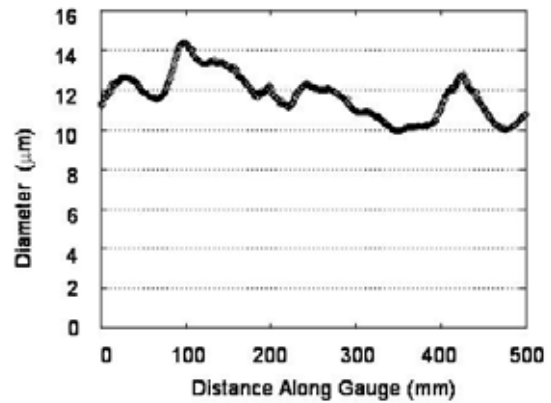
2.3 Tensile Test

1) specimen

For the single fiber tensile tests, Tyranno ZMI fibers were prepared as depicted in Fig.2. Paper holder has a slot in the central part for the gauge lengths are 200mm, 100mm, 50mm, 20mm. However, 5mm was added at both ends of the slot to avoid the error due to the clamp effect.   Thus, the data of the samples that fractured within the 5mm were neglected in the statistical analyses.
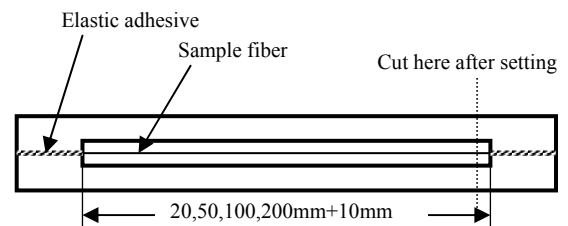

Fig.2   Dimension of specimen

Complete form of specimen is as follows (Fig.3). At tensile test, we filled protection films up with a surfactant. Note that the protection films did not touch the fiber by the paper thickness, thus fiber strength was measured without the friction bias.
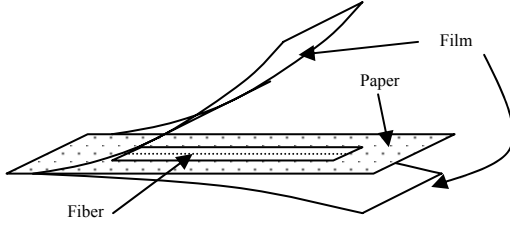
Fig.3   Tensile specimen

2) tensile test

Universal Tensile Test System type 5542 (INSTRON, Corp) with 10N load cell was used with the crosshead speed of 0.1mm/min.   The test setup is shown in Fig.4.
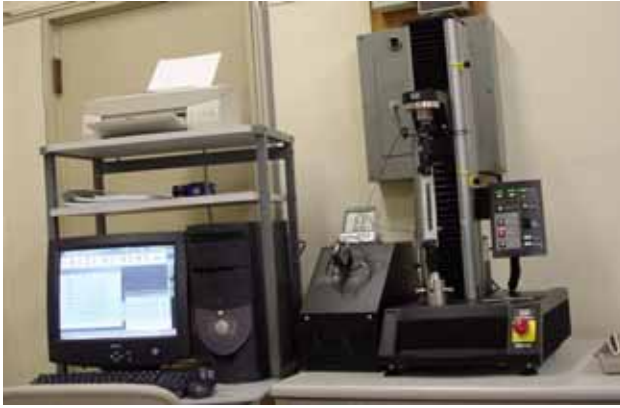

Fig.4 Tensile test setup

3 Statistical analyses

Single-modal Weibull distribution function is given by;

$$F(\sigma) = 1 - \exp\left[-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right] \qquad (1)$$

Where, V_0, V, m, and σ_0 are standard volume, volume of specimen, shape parameter and scale parameter, respectively.   The V/V_0 is proportional to the gauge length l of the fiber if the diameters are uniform along the gauge [10] [11]. Thus, equation (1) may be rewritten as follows.

$$F(\sigma) = 1 - \exp\left[-l\left(\frac{\sigma}{\sigma_0}\right)^m\right] \qquad (2)$$

However, the equation (2) is not suitable for the strength estimation of variable diameter fibers. For long fibers of variable diameters, some modifications have been reported for the exact strength scalings [12] [13]. For the short gauge length fibers, however, a new modification is essential to apply the single-modal Weibull model. Thus, in this study, a gauge section was divided into short length "Δl" sections at which stress is considered to be uniform. Then cumulative fracture probability at each section was calculated.   The fracture probability of whole gauge section was derived by multiplying the fracture probabilities of "Δl" sections.

The whole gauge length had been divided as the cylinders of "Δl=1mm" as schematically shown in Fig.5.   Stress of each cylinder was calculated with the fracture load and the diameter, then the fracture probability of jth cylinder, which is defined as $F_j(\sigma_j)$ was assigned.
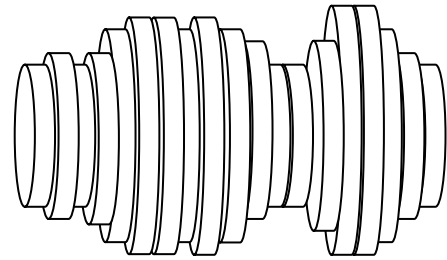

Fig.5 A Model for the Weibull analysis of variable diameter fiber

Next, cumulative fracture probability of whole gauge length $F(\sigma)$ was given as following.

$$F(\sigma) = 1 - \prod_j \left[1 - F_j(\sigma_j)\right] \qquad (3)$$

The cumulative fracture probabilities calculated with this method were with an hypothetical value of m and σ_0. The calculations with the equation (3) provide n cumulative probabilities if the sample number is n. By ordering the results from the lowest probability to the highest probability, the ith result F_i in the set of n samples may be reassigned a cumulative probability of failure $F_{i, rank}$ by the estimators such as $F_{i.rank}=i/(n+1)$. If a set of Weibull parameters m and σ_0 is ideal and the sample number n is large enough, the F_i and $F_{i, rank}$ must be equal. Thus, we set a factor G that is defined by equation (4) and select a set of m and σ_0 when the G is minimum.

$$G = \sum_i \left\{ F_i(\sigma_i) - F_{i,rank} \right\}^2 \qquad (4)$$

In this calculation, V_0 is given as 1.9×10^{-12} m^3, which equals to the cylinder volume of 11μm in the diameter of manufacture's data sheet and 20mm in the length.

For the comparison to the parameters above, Weibull plots were executed assuming each sample diameter was equal to the diameter at fracture point. Equation (5) was used for the Weibull plots.

$$\ln\ln\left(\frac{1}{1-F}\right) - \ln\left(\frac{V}{V_0}\right) = m\ln\sigma - m\ln\sigma_0 \quad (5)$$

4 Results and Discussion

We have measured the axial diameter distributions on 200 pieces of Tyranno ZMI Fiber, whose gauge length was 500mm. The result showed that the diameters vary widely along the gauges. Thus, we must take into account the fact that the tensile stresses are not uniform along the gauge length. Thus, the Weibull plot using a diameter yields excessive error that may cover the size effect of Tyranno ZMI Fiber. An example of diameter data is shown in Fig. 6.
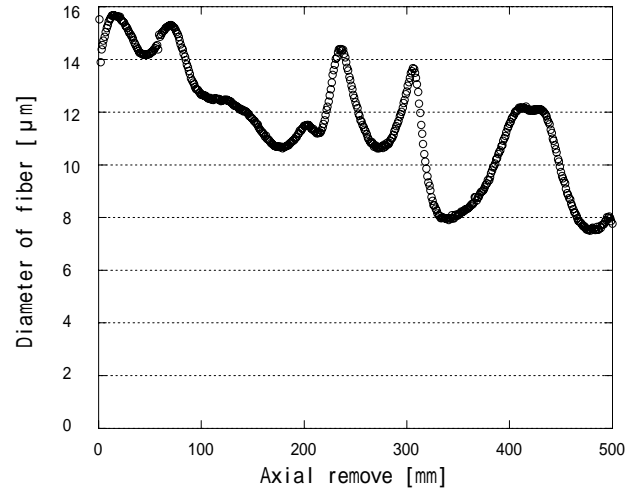

Fig.6   Diameter Distribution of Tyranno ZMI Fiber

Statistical data of tensile tests is given in table 2, which by the Weibull plots assuming each sample diameter was equal to the diameter at fracture point.

Table 2 Statistical data of tensile tests

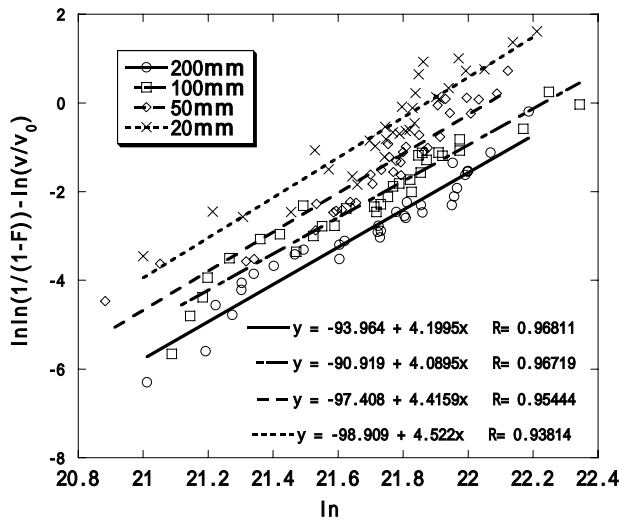| Gauge Length (mm)	200	100	50	20
Mean fracture stress [GPa]	2.70	2.75	2.81	2.88
Variance [× 10^{17}]	5.75	7.43	4.30	4.39
Standerd deviation [× 100MPa]	7.58	8.62	6.56	6.62
Coefficient of variation	0.280	0.314	0.233	0.230

Fig.7 shows the Weibull plots.

Fig.7 Weibull plots for the gauge length of 20mm, 50mm, 100mm and 200mm

Shape parameters and scale parameters derived from above Weibull plots are listed in Table.3.

Table 3 m and σ_0 derived by the Weibull plot

Gauge length[mm]	200	100	50	20
m	4.2	4.1	4.4	4.5
σ_0 [GPa]	5.2	4.5	3.8	3.2

Shape parameter m seems to be less sensitive to the gauge length, however, scale parameter σ_0 is sensitive to the gauge length.

Table 4 shows shape parameter m and scale parameter σ_0 calculated by layered cylindrical model method.

Table 4 m and σ_0 by the layered cylindrical model

Gauge length[mm]	200	100	50	20
m	3.6	4.7	5.5	5.9
σ_0 [GPa]	5.5	4.1	3.5	3.1

Shape parameter m has a tendency to become small with longer gauge length, and scale parameter σ_0 becomes large with longer gauge length (Table 4).

The parameters of single-modal Weibull model are shown to be dependent on the gauge lengths. Thus, single-modal Weibull model may provide biased size effect in scaling the fiber strength.

5 Conclusions

The following conclusions may be drawn from the studies above.

(1) The diameter of Tyranno ZMI fiber varies widely along the gauge.

(2) A diameter such as mean diameter at a bundle may provide biased fracture stress.

(3) The parameters of single-modal Weibull model are dependent on the gauge lengths when the gauge length is small.

(4) Single-modal Weibull model provides a biased size effect of Tyranno ZMI fiber strength.

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