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ANALYSIS OF AIRCRAFT STOCHASTIC MOTION AFTER LOOSING THE CONTROL

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For increasing the size of new aircraft and for increasing the timeablebility (flying in poor wether the modern future aircraft should have a higher flight safety. Therefore, the investigation of the unconventional flights e.g. flights out of the normal flight and load envelopes is a very actual practical and theoretical problem.

The lecture investigates the aircraft real motion after loosing the conventional control.

The free flying model of aircraft is used in the practical investigation. The motion of airplane model with simulation of loosing the control is recorded on video. The film is digitalized and the flight path is reconstructed on the basis of special marks made on the flying model.

The Fokker-Planc equations are used during the description of the real motion of aircraft. The entire process of aircraft motion is analyzed as the realizations of time series parameters and set up a hypothesis on the approximation of diffusion coefficient functions.
So, the statistical data of aircraft motion after loosing the control is obtained by solution of diffusion equations. The results are demonstrated in from transition probability matrix functions describing the changes in flight characteristics of airplane after loosing its conventional control.

The results of investigations can be applied to the aircraft accident investigations and crash analysis.

1. The problem

Building up the mathematical model required for the statistical analysis of the motion parameters after the control (X_{sp};Y;etc.) of an aeroplane, as well as the planning of the experimental process.
The aim of building up three-dimensional 3D geodetic grids is the determination of the grid points 3D (x;y;z) (or their orientation, respectively). In our case, the application of is method provides the possibility of orientation in plane or in space of the corrected geodetic co-ordinates of the cameras used in our experiment.

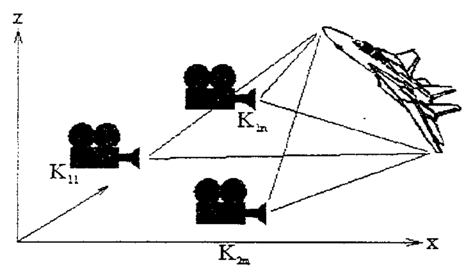

Fig. 1/a.

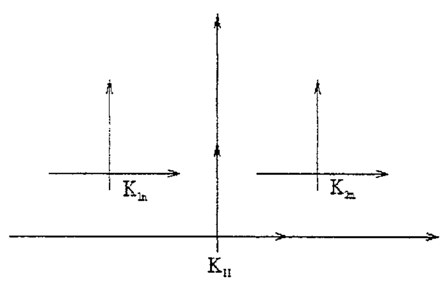

Fig. 1/b.

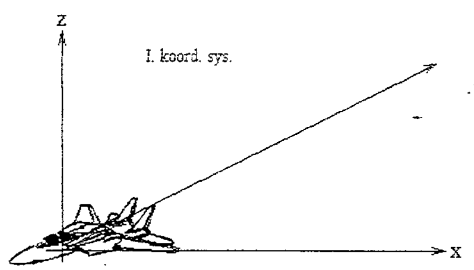

Fig. 2/a.

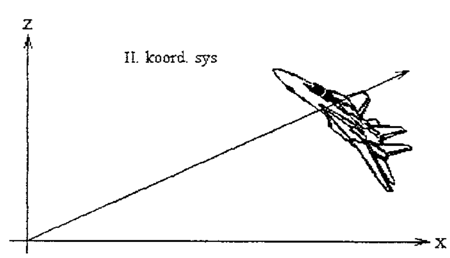

Fig. 2/b.

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In our case, during building up the functional model of the motion analysis grid, the examined object (aeroplane) is replaced by its identifications points. The positions of the identification points of the aeroplane is determined at different instants $t_i \in T$ as starting from the reference points taken outside the aeroplane, and on the basis of those we can deduce the respective motion parameters of the above object with the help of function-approximate method.

2. The motion analysis grid:

The identification point of the aeroplane + reference points outside the aeroplane (camera positions). When building up the functional model of measurements, we should consider two main phases, namely:

1. Replacement of the examined object (aeroplane) by its corresponding identification point.
2. Analysis of the effects caused by the duration of measurements

Points 1 requires further examinations. In this paper it will be not treated. The following considerations are associated with point 2. The number of recordings by camera per seconds and the dynamics of the motion system examined is of such character that the measurements belonging to the individual occasions can be considered as belonging to one instant. Accordingly, the above functional model will be as follows:

A measurement-vector Y_{ji} i=1,2, ...k and a co-ordinate-vector Y_{ji} i=1,2, ...k are assigned to each instant t_{ji} i=1,2,...n $t_{ji} \in T_j$. T_j is the period of the jth experiment.

In the functional model described above, the measurements belonging to the individual occasions can be quantities characterising the motion can be defined on the basis of the corrected quantities after the correcting compensation of the grid has been performed.

The phases of correcting the motion analysis grid are the any correction

1. Preliminary correction
2. Actual correction
3. Interpretation of corrected quantities.

The photogrammetic motion analysis grid involves the discrete sampling of the stochastic motion over the predetermined time-interval $t_\lambda \in [t_0;T]$.

The motion analysis grid shown in Fig.1/a. and 1/b. makes possible the approximate 6D determination of any identified point of the aeroplane by the solution of a conditional extermization problem belonging to each J=1,2, ...M; $t_\lambda \in [t_0;T]$ instant of the examination.

With the knowlidge of the co-ordinate systems represented in Fig.2/a and 2/b. (when the 2/a co-ordinate system is such cordinate system, wich is moving with the aeroplane) vector $\underline{\tilde{Y}}_J(t_\lambda)$ in (1) can be derived for each instant of examination with the help of a mathematical model similar to those used above.

Measuring vector:

$$\underline{\tilde{Y}}_J(t_\lambda) = [x_1(t_\lambda); x_2(t_\lambda); x_3(t_\lambda); \varphi_1(t_\lambda); \varphi_2(t_\lambda); \varphi_3(t_\lambda);] \in R^6 \tag{1}$$

$t_\lambda \in [t_0;T]$ j=1,2, ... M; λ=1,2, ... k index set.

When $x_i(t_\lambda)$: i=1,2,3 the centre of gravity coordinate of airplane for the discreet time.

$\varphi_i(t)$: i=1,2,3 are the realization in the time of the angle swing, what is round the inert wain course of airplane.

M: Number of the experiment.

Note: $\underline{\tilde{Y}}_J(t_\lambda)\,\forall t_\lambda$ λ=1,2, ... k is burdened with error in the λ=1,2, ... k time.

On the basis of these experimental results, let us fire the transitional probability description of the real random vector-variable (1) the following way:

3. The first task is the filteration of the measuring results

At any instant of the motion, there is always a threshold vector ε which can not be exceeded by the system due to its mechanical properties.

Conclusion:

No frequency vectors of greater norm than that of the boundary frequency vector ω can exist among the realizations.

That is all the time of moving is the $\underline{\varepsilon}\left(\dot{\underline{\tilde{Y}}}\right)$ limit vector.

$$\underline{\varepsilon}\left(\dot{\underline{\tilde{Y}}}\right) = f\left(\dot{\underline{\tilde{Y}}} \,\middle|\, i = 1,2,...6\right) \tag{2}$$

The execution of the filteration.

Let's perform the next expansion for the all examination result.

$$\underline{\tilde{Y}}_J(t_\lambda) = \begin{cases} \underline{\tilde{Y}}_J(t_\lambda), & if \quad t \in [t_0;T] \\ 0, & if \quad t \in (-\infty;+\infty)\setminus[t_0;T] \end{cases} \tag{3}$$

It is true for all coordinate function of $\underline{\tilde{Y}}_J(t_\lambda)$ measuring vector:

$$\underline{Y}_J(t_\lambda) \rightarrow Y_{Ji}(t_\lambda) \in C^d[t_0;T] \text{ ;}d_{min}=2;\ i=1,2 \ldots 6 \tag{4}$$

The consequence of (4).

Because: 1. $[t_0;T]$ is compact set.

2. $\tilde{Y}_{Ji}(t)$ are limited and single-valued functions i=1,2, ...6; J=1,2, ...M

Therefor:

For the every $g \in L^1[t_0;T]$ functions are:

(5)

$$\int_{-\infty}^{+\infty} \widetilde{Y}_{Ji}(t) g(\tau - t) dt = \int_{0}^{T} \widetilde{Y}_{Ji}(t) g(\tau - t) dt \qquad \begin{matrix} i = 1,2,...6 \\ J = 1,2,..M \end{matrix}$$

As a consequence of (4), the (5) discrete convolution is in force for any discrete distribution of time-interval $[t_0;T]$.

$$\sum_{K}^{\infty} \widetilde{Y}_{Ji}(t) g(\tau - t) = \sum_{i=1}^{N} C_i \widetilde{Y}_{Ji}(t_{K-1}) \qquad (6)$$

For above task let's apply the binomial filteration (it is filteration on the basis of convolution).

The generation function of binomial filteration:

$$C_{-i} = C_i = \frac{1}{2^{2m}} \binom{2m}{m - |i|} \qquad (7)$$

The next function will be in consequence of the filteration.

(8)

$$\widetilde{Y}_{Ji}(t_\lambda) \rightarrow \text{binomial filteration} \rightarrow Y_{Ji}(t_\lambda) \ \forall J; \forall i; \forall \lambda$$

By applying binomial filteration to (1) and all J, a set of vectors is obtained which continuous $\underline{Y}_J(t_\lambda)$ realizations that will not contain frequencies higher than the boundary frequency (produced by the measurement errors) determined by (2).

4. The interpolation

The next problem to be solved is the proper qualitative interpolation of the set of points 6D determined by, which should be performed separately for each co-ordinate. The quality of interpolation is predetermined by mechanical properties which are shown in (4).

The quality of integration:

$$Y_{Ji}(t_\lambda) \rightarrow Y_{Ji}(t) \in C^d[t_0;T] \qquad \begin{matrix} J = 1,2...M \\ i = 1,2...6 \end{matrix} \qquad (9)$$

Let's apply SPLINE interpolation.

With the use of the above screening and interpolation:

The discrete measurement results (loaded with measurement error) of each experiment can be represented by screened, continuous realizations belonging to functional class $C^d[t_0;T] \ \forall J; \forall i$.

5. The analitical approximation of the transitional probability functional for the above process.

Let's perform the approximation of the transitional probability function on the class the Gauss density functions.

The next is true:

The $\underline{Y}_J(t)$ J=1,2, ...M functions enclose in the $\Omega \in R^6$ bounded and closed set in the every points of $[t_0;T]$, that so: $\underline{Y}_J(t)\big|_{[t_0;T]} \subset \Omega \subset R^6 \ \forall J$.

Since - as our experimental results show - we have no information about the distribution properties of the random vector variate $\underline{Y}_J(t)$ outside the set Ω therefore the behaviour the set Ω of the approximated density function "f" can be accepted for us only as a hypothesis.

Let's examine the quality of the approximation.

Let's define the next function class.

Let be:

G≡{the class of the independent Gauss-vectors dencity function}

That is: (10)

$$g(\underline{x};\underline{\delta}^2;\underline{m}) = \frac{1}{(2\pi)^n \prod_{J=1}^{n} \delta_J^2} \exp\left\{ -\sum_{J=1}^{n} \delta_J^2 (x_J - m_J)^2 \right\}$$

Let's apply extension of the G function class to the other function class.

(11)

$$\widetilde{G} \equiv \widetilde{g} \in \widetilde{G} \ \Big| \ if: \ 1. \ \widetilde{g} \equiv \lim_{n \to m} \sum_{i=1}^{m} C_i g_i(\underline{x};\underline{\delta}_i^2;\underline{m}_i)$$

$$2. K \in R^+ \text{ that } \widetilde{g}\big|_{R^n} \leq K$$

$$C_i \in R^+ \ \forall i \ ; \ m \in N^+ \ or \ m = +\infty$$

Note: The necessary and satisfactory condition of the secund property is:

$\underline{a} \in l_2$ Hilbert space

$$|\underline{a} \in [a_1;a_2...]; \ a_i = \frac{C_i}{(2\pi)^n \prod_{J=1}^{n} \delta_{iJ}^2} \qquad (12)$$

Let's apply the fixing for the every function of the $\widetilde{G}$.

$$\widetilde{G}' \equiv \begin{cases} \widetilde{g}' \equiv \widetilde{g} \ if \ x \in \Omega \\ 0 \qquad if \ x \in R^n \setminus \Omega \end{cases} \qquad (13)$$

After these can understand the following:

Next true are: (14/1)

1. If $\underline{p}_1;\underline{p}_2 \in \overline{\Omega}$ and $\underline{p}_1 \neq \underline{p}_2$ when is true that
   It is $\widetilde{g}' \in \widetilde{G}' \Rightarrow \widetilde{g}'(\underline{p}_1) \in \widetilde{g}'(\underline{p}_2)$
2. If $\underline{p} \in \Omega$ when is true that (14/2)
   It is $\widetilde{g}' \in \widetilde{G}' \Rightarrow \widetilde{g}'(\underline{p}_1) \neq 0$
3. If $\widetilde{g}_i';\widetilde{g}_J' \in \widetilde{G}' \Rightarrow$ when is true that (14/3)

$\forall \tilde{g}_i'; \tilde{g}_J' \in \tilde{G}'$, so

$$\tilde{g}_i'; \tilde{g}_J' = = \left\{\sum_k C_i g_{iK}\left(\underline{x}; \underline{\delta}_{iK}^2; \underline{m}_{iK}\right)\right\}\left\{\sum_k C_J g_{JK}\left(\underline{x}; \underline{\delta}_{JK}^2; \underline{m}_{JK}\right)\right\}$$

is true that:

$$\underline{m}^* \in R^n \text{ and } \operatorname{diag} \underline{\underline{C}}^* \;\Big|\; \underline{\underline{C}}^* \equiv \underline{E}\delta^{*2}$$

$$\tilde{g}_i'\left(\underline{x}; \underline{\delta}_i^2; \underline{m}_i\right)\tilde{g}_J'\left(\underline{x}; \underline{\delta}_J^2; \underline{m}_J\right) = \tilde{g}'\left(\underline{x}; \underline{\delta}_i^{*2}; \underline{m}^*\right)$$

4. If $\tilde{g}_i'; \tilde{g}_J' \in \tilde{G}'$ when is true that (14/4)

$\tilde{g}_i' + \tilde{g}_J' \in \tilde{G}'$

Proof : in the 3

Conclusion:

The consequence of the (14/1), (14/2), (14/3), (14/4) and STONE - WEIERSTARSSE thesis, that every in the Ω continuous real function can approximate, with function of set of $\tilde{G}$ on the Ω set.

That is:

In the every $t_\mu; t_\lambda \in [t_0; T]$ points transitional probability function can approximate with function of $\tilde{G}$ on the Ω set.

Let's solve the previous task with the follow:

Test function

$$\tilde{f}\left(\underline{c}; \underline{m}_i; \underline{\delta}_i^2; \underline{x}\right) \equiv \sum_{i=1}^{m} C_i^2 g_i\left(\underline{m}_i; \underline{\delta}_i^2; \underline{x}\right) \qquad (15/1)$$

Maximum – Likelihood procedure (15/2)

The (15/1) and (15/2) together define the condition – limit value problem.

(17/a)

$$\Phi = \Phi_{aim} + \lambda \Phi_{con} \Rightarrow \text{maximum}\left(\underline{m}_i; \underline{\delta}_i^2; \underline{c} \;\middle|\; i = 1,2,\ldots m\right)$$

$$\Phi_{aim} = \prod_{J=1}^{M}\left(\sum_{i=1}^{m} C_i^2 g\left(\underline{m}_i; \underline{\delta}_i^2; \underline{x}_J\right)\right) \qquad (17/b)$$

$$\Phi_{con} = 1 - \sum_{i=1}^{m} C_i^2 \quad \left\|\underline{\underline{C}}\right\|_{E^n or\, l_2} = 1 \qquad (17/c)$$

The (17/b) assure, that (15/1) test function will be density function in the continuous function class from the solution of the conditional-limit value problem are determined by (17/a), (17/b),(17/c).

Let's apply maximum-Likelihood thesis:

The Likelihood function will be next:

(18/a)

$$L\left(\underline{X} \times \underline{Y}; \underline{c}; \underline{m}_i; \underline{\delta}_i^2 \;\middle|\; i = 1,2,\ldots m\right) \equiv \equiv \prod_{j=1}^{M}\left(\sum_{i=1}^{m} C_i^2 g_i\left[\underline{Y}_J(t_\lambda); \underline{Y}_J(t_\mu); \underline{\delta}_i^2; \underline{m}_i\right]\right)$$

$$t_\lambda; t_\mu \in [t_0; T]$$

$$t_\lambda \geq t_\mu$$

$$J = 1,2,\ldots M$$

where:

G≡{the class of the independent Gauss-vectors dencity function}

$$g\left(\underline{X} \times \underline{Y}; \underline{\delta}^2; \underline{m}\right) = \frac{1}{(2\pi)^{2n} \prod_{J=1}^{2n} \delta_J^2} \exp\left\{-\sum_{J=1}^{2n} \delta_J^2 (r_J - m_J)^2\right\}$$

$$\underline{r} \in \underline{X} \times \underline{Y}$$

The modified Likelihood function will be next:

(18/b)

$$L'\left(\underline{X} \times \underline{Y}; \underline{c}; \underline{m}_i; \underline{\delta}_i^2 \;\middle|\; i = 1,2,\ldots m\right) = = L\left(\underline{X} \times \underline{Y}; \underline{c}; \underline{m}_i; \underline{\delta}_i^2 \;\middle|\; i = 1,2,\ldots m\right) + \lambda\left(1 - \sum_{i=1}^{m} C_i^2\right)$$

The space of the parameters:

$$A \equiv \underbrace{\left[\underline{c} \times \underline{m}_i \times \underline{\delta}_i^2 \;\middle|\; i = 1,2,\ldots m\right]}_{\underline{H}} \times \lambda \qquad (19)$$

The next true are:

If: 1, The required probability function is in the $\tilde{G}$ space that is:

$$\tilde{g} \in \tilde{G} \equiv \lim_{n\to\infty} \sum_{i=1}^{m} C_i \tilde{g}_i\left(\underline{x}; \underline{\delta}_i^2; \underline{m}_i\right) \qquad (20/1)$$

2, The $\underline{\Theta}^* \in A$ is the required parameter vector (20/2)

3, $K \in [L_1; L_2; \ldots; L_{m+2m\,n+1}]$; n=12 (20/3)

where:

$$L_i \equiv \left[\underline{H}; \lambda; \underline{Y}_J(t_\lambda); \underline{Y}_J(t_M)\right] = \frac{\partial \log L}{\partial H_i}$$

4, $$\frac{\partial^2 \log L}{\partial H_i \partial H_J} = L_{iJ} = -M_{iJ} \;\Big|\; i, J = 1,2,\ldots \qquad (20/4)$$

when:

1, If $\underline{K}(\Theta)\big|_{\Theta\in A} = \underline{0}$ when: (21/1)

$$\lim_{m}\lim_{m\to\infty} p\left\{\left\|\underline{\Theta}-\underline{\Theta}^*\right\|\le\varepsilon\right\}=1;\quad \forall\varepsilon\in R^+$$

(21/2)

2, $\lim_{m}\lim_{m\to\infty} p\left\{\left|M_{iJ}-I_{iJ}\left(\underline{\Theta}^*\right)\right|\le\varepsilon\right\}=1;\ \forall\varepsilon\in R^+$

where: $\underline{\underline{I}}\left(\underline{\Theta}^*\right)$, Fischer information matrix

Let's construct $t_\mu;t_\lambda\in[t_0;T]$ have transitional density functions.

Because it is true, that:

$$f\left(\underline{Y}_{t\mu}\mid\underline{Y}_{t\mu-1}\right)=\frac{f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y})}{f\left(\underline{Y}_{t\mu-1}\right)} \qquad (22)$$

$$f(\underline{y})=\int_{R^6} f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y})d\underline{x}$$

The f(y) density function may procedure with integrate of $f(\underline{X};\underline{Y})$ on the R^6.

6. The connection of the constructed transitional density function and diffusion process.

Can verify the following:
If: $Y_{Ji}(t)$ are diffusion process ∀J; i=1,2, …6.

Then: ∀ε∈R+; ∀x∈R6; ∀t<s and t,s∈ $[t_0;T]$

$$\int_{\|Y-X\|>\varepsilon} p(t,\underline{x},s,d\underline{y})=o(s-t) \qquad (23)$$

Let's examine, that in our case wether (23) can realised for the stochastic process is represented by (1) measuring vector.

let be: $t_\lambda\in[t_0;T]\mid\lambda=\{1,2,\ldots n\}$ index set (24)

$$\lim_{m}\max_{\lambda}\left(t_\mu-t_{\mu-1}\right)=0$$

Because (1) $Y_{Ji}(t)\ \forall J;\forall i$ are probability and continuous functions, therefor they have not second-rate rent.
Then is true:

(25)

$$\lim_{n\to\infty}\sum_{\mu=1}^{n} p\left\{d\left[\underline{Y}_J(t_\mu),\underline{Y}_J(t_{\mu-1})\right]>\varepsilon\right\}=0\quad\forall\varepsilon\in R^+$$

Happen the properties of the "p" probability and "d" distance functions, that strongly true:

(26)

$$\lim_{(\mu-1)\to\mu} p\left\{d\left[\underline{Y}_J(t_\mu),\underline{Y}_J(t_{\mu-1})\right]>\varepsilon\right\}=0\quad\forall\mu;\mu-1\in[t_0;T]$$

If: are the follow density functions.

$$t_\mu;t_{\mu-1}\Rightarrow f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y}) \qquad (27/a)$$

then: The (26) is equal to the next equation.

(27/b)

$$\int_{\|\underline{Y}(t_\mu)-\underline{Y}(t_{\mu-1})\|>\varepsilon} f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y})d\underline{x}d\underline{y}=o\left(t_\mu-t_{\mu-1}\right)$$

Because the next equation is true:

(28)

$$\int_{R^n} f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y})dv=f_{\underline{Y}(t_{\mu-1})}(\underline{X})\ge 0\ \mid\ \forall\underline{X}\in R^n$$

Because: $f_{\underline{Y}(t_{\mu-1})}(\underline{X})>0$ and continouos function

when: (29)

$$\int_{\|\underline{Y}(t_\mu)-\underline{Y}(t_{\mu-1})\|>\varepsilon}\frac{f_{\underline{Y}(t_\mu)\underline{Y}(t_{\mu-1})}(\underline{X};\underline{Y})}{f_{\underline{Y}(t_{\mu-1})}(\underline{X})}d\underline{x}d\underline{y}=o\left(t_\mu-t_{\mu-1}\right)\ \forall\varepsilon\in R^+$$

On the basis of above is true, that in the continuous realisation functions case the (23) surely true.
Questions:
Density function, which is approximated by above process wether perform the (23).
The answer is Yes, because are true next:
The $\underline{Y}_J(t)\ t\in[t_0;T]$ is continuous function, that is $t_\mu\to t_\lambda\to\underline{Y}_J(t_\mu)\to\underline{Y}_J(t_\lambda)\ \forall J$
So it is true:

$$\forall t_\mu;t_\lambda\in[t_0;T] \qquad (30)$$

$\underline{Z}_{t_\mu;t_\lambda}\equiv\left\{\underline{Y}_J(t_\mu),\underline{Y}_J(t_\lambda)\right\}\in R^{12}$ hypervector.

$$\lim_{t_\mu\to t_\lambda}\underline{Z}_{t_\mu;t_\lambda}\to\underline{Z}_{t_\mu;t_\lambda} \qquad (31)$$

when:
1, It is $\underline{Z}_{t_\mu;t_\lambda}\ \forall t_\mu;t_\lambda\in[t_0;T]$
2, The $\lim_{t_\mu\to t_\lambda}\underline{Z}_{t_\mu;t_\lambda}$ in the $\underline{Y}_J(t_\mu)$ and $\underline{Y}_J(t_\lambda)$ subspace indicate the same points.

The can sequence of above:

$$\lim_{t_\mu\to t_\lambda}\underline{Z}_{t_\mu;t_\lambda}\to B\subset\beta \qquad (32)$$

where:

β is hyperplane in the R^{12} and true

$$\beta = \{\forall \underline{r} \in \underline{Y}_J(t_\mu) \times \underline{Y}_J(t_\lambda) \mid \underline{nr} = \underline{0} \;\; \underline{n} = \{1,1,\ldots,1\}\}$$

The properties of the statistical functions are known from the statistics theory:

$$\underline{t} = \frac{1}{n}\sum_{i=1}^{n} \underline{Z}^i_{t_\mu;t_\lambda} \qquad (33)$$

$$\lim_{n\to\infty} p\left\{\left\| \frac{1}{n}\sum_{i=1}^{n} \underline{Z}^i_{t_\mu;t_\lambda} - \underline{t}\right\| \le \varepsilon\right\} = 1 \;\; \forall \varepsilon \in R^+$$

Where: 1, the $\underline{t}$ is the consintens and undeformed estimate of the probable value.
2, the δ^2 is the assimptotic undeformed estimate of the square of standard deviation.

So that is shown:

$$\lim_{t_\mu \to t_\lambda} \underline{t} \to B \subset \beta \qquad (34)$$

$$\lim_{t_\mu \to t_\lambda} \underline{\underline{C}} \to \underline{\underline{E}} \qquad \underline{\underline{C}}\text{: covariant matrix}$$

From the (33) and (34) is sown, that in $\lim_{t_\mu \to t_\lambda}$ case the sample concentrate to β hiperplane.
For the vector of the probable value of the conditional density function is true:

$$\lim_{t_\mu \to t_\lambda} \underline{m} \to m^* \in \beta$$

For the vector of the square of standard deviation of the conditional density function is true, that:
$\lim_{t_\mu \to t_\lambda} \underline{\underline{C}} \to \underline{\underline{E}}$ is in the subspace wich is perpendicular to β.

Conclusion: will be realised (23) equation.
Let's examine:
Let be: $t_\lambda \in [t_0;T]$; λ: index set.
We look for its probability, that examined system across $A(t_\lambda) \subset R^6$ sets comes from $A(t_1) \subset R^6$ to $A(t_n) \subset R^6$.
The Kolmogorov – Chapman equation gives the answer for the task.
That is:

$$(35)$$
$$p[t_1;\underline{Y}(t_1),t_n;\underline{Y}(t_n) \in A] =$$
$$= \int_A \int_{R^6} p[t_{n-2};\underline{Y}(t_{n-2}),t_{n-1};\underline{Z}]\, p[t_{n-1};\underline{Y}(t_{n-1}),t_n;\underline{Y}(t_n)]\, d\underline{Z}$$

Wher:
$p[t_{n-2};\underline{Y}(t_{n-2}),t_{n-1};\underline{Z}]$ can produced recursive from the known $p[t_\mu;\underline{Y}(t_\mu),t_\lambda;\underline{Y}(t_\lambda)]$ transitional density function
Arise the next question:
The $p[t_\mu;\underline{Y}(t_\mu),t_\lambda;\underline{Y}(t_\lambda)]$ wether possible continuous extend to $[t_0;T]$.
The $L'(\underline{H};\lambda;\underline{Y}_J(t_\mu) \times \underline{Y}_J(t_\lambda))$ L. function there can picks maximum on the $t_\lambda;t_\mu \in [t_0;T] \Rightarrow \underline{Y}_J(t_\mu) \times \underline{Y}_J(t_\lambda) \in R^{12}$ sample space, where is true for (12).

$$K\left[L_1;L_2;\ldots;L_{m+2mn+1}\right]\Big|_{H'\times\lambda} = \underline{0} \qquad (36)$$

Where: $H' \times \lambda$ is set of points, where the Likelihood function picks maximum.

Transform the (36) in the following way:

$$F(A;B) \equiv \sum_{i=1}^{m+2mn+1} K\left[L_1;L_2;\ldots;L_{m+2mn+1}\right]\Big|_{H'\times\lambda} = 0 \qquad (37)$$

where $A \equiv \left\{ \begin{matrix} \underline{c} \times \underline{m}_i \times \delta_i^2;\lambda \; i = 1,2,\ldots,m \\ J = 1,2,\ldots,M \end{matrix} \right\}$ (38)

$$B \equiv \{\underline{Y}_J(t_\mu) \times \underline{Y}_J(t_\lambda) J = 1,2,\ldots,M\} \qquad (39)$$

where are true:

(40)

1, It is F function wich : $F'_A(A;B)\big|_{\underline{a}\times\underline{b}} = 0$ and continuous

2, It is $\{F'_A(A;B)\big|_{H'\times\lambda}\}^{-1}$ function and continuous

Therefor consequence of the implicit function thesis that:
$f : \varepsilon_{\underline{b}} \to \varepsilon_{\underline{a}}$ is continuous function. (41)
Where are true:

(42/a)
1. $\left\| \underline{Y}_J(t_\mu) \times \underline{Y}_J(t_\lambda)\right\| < K_1 |t_\mu - t_\lambda| \;\; \forall t_\mu;t_\lambda \in [t_0;T]$

(42/b)
2. $\left\| f(\underline{Y}_J) - f(\underline{Y}_J^*)\right\| \le K_2 \left\| \underline{Y}_J - \underline{Y}_J^*\right\| \;\; \forall \underline{Y}_J;\underline{Y}_J^* \in \varepsilon_{\underline{b}}$

The consequence of (41), (42/a), (42/b) it is true that:

$$\left\| f[\underline{Y}_J(t_\mu)] - f[\underline{Y}_J(t_\mu)]\right\| \le K_3 |t_\mu - t_\lambda| \qquad (43)$$

if: $t_\mu \in \varepsilon_{t_\mu}$; $t_\lambda \in \varepsilon_{t_\lambda}$
The consequence of 43 it is true that:

The wanted continuous extension doesn't on the $[t_0;T]$ on the basis of discrete measuring data.

The vertical – probability description of the stochastic process in the $[t_0;T]$ time

Given: $t_\lambda \in [t_0;T]$ λ: index set, discrate division

$\tilde{\underline{Y}}_J(t_\lambda)$ $\forall$J measurement vectors are at $t_\lambda \in [t_0;T]$ disposal. After filteration (3.chapter) and interpolation (4.chapter) $\underline{Y}_J(t)$ will be from $\tilde{\underline{Y}}_J(t_\lambda)$ $\forall$J.

Let's apply result of 4. chapter for vertical probability description of the stochastic process with following:

Let be the test function similar to (15/1).

That is: (44)

$$\tilde{f}(\underline{c};\underline{m}_i;\underline{\delta}_i^2;\underline{x};t_\lambda) \equiv \sum_{i=1}^{m} C_i^2 g_i\left[\underline{m}_i(t_\lambda);\underline{\delta}_i^2(t_\lambda);\underline{x}(t_\lambda)\right]$$

Where:

$$g_i(\underline{m}_i;\underline{\delta}_i^2;\underline{x};t_\lambda) \in G;\ \underline{x} \in R^6$$

The Likelihood function is: (45/a)

$$L_{t\lambda}(\underline{X};\underline{c};\underline{m}_i;\underline{\delta}_i^2 \mid i = 1,2,\dots m) \equiv$$

$$\equiv \prod_{i=1}^{M}\left(\sum_{i=1}^{m} C_i^2 g_i\left[\underline{Y}_J(t_\lambda);\underline{\delta}_i^2;\underline{m}_i\right]\right) \quad t_\lambda \in [t_0;T]$$

The modified Likelihood function will be next: (45/b)

$$L'_{t\lambda}(\underline{X};\underline{c};\underline{m}_i;\underline{\delta}_i^2 \mid i = 1,2,\dots m) =$$

$$= L_{t\lambda}(\underline{X};\underline{c};\underline{m}_i;\underline{\delta}_i^2 \mid i = 1,2,\dots m) + \lambda\left(1 - \sum_{i=1}^{m} C_i^2\right)$$

The space of the parameters:

$$A \equiv \underbrace{\left[\underline{c} \times \underline{m}_i \times \underline{\delta}_i^2 \mid i = 1,2,\dots m\right]}_{\underline{H}} \times \lambda \qquad (46)$$

The (45/b) equation similar to 4.chapter is the limit value problem.

The solution of above problem can be according to description of 4. chapter.

That is:

If: 1. $\tilde{g} \in \tilde{G} \equiv \lim_{m\to\infty} \sum_{i=1}^{m} C_i g_i(\underline{x};\underline{\delta}_i^2;\underline{m}_i)$ (47)

2. (20/2); (20/3); (20/4) are realised (where n=6 in (20/3))

The next is true:

The solution of (46) limit value problem give solution vector to $\forall t_\lambda \in [t_0;T]$.

That is: (48)

$$U \equiv \left\{\left(\mathop{X}_{i=1}^{m} \underline{m}_i(t_a)\right) \times \left(\mathop{X}_{i=1}^{m} \underline{\delta}_i^2(t_a)\right) \times \left(\mathop{X}_{i=1}^{m} c_i(t_a)\right)\right\}$$

$$U \equiv R^b;\ b = 2mn + m;\ \underline{p}(t_\lambda) \in U$$

Because (44) test function is in the class of parametrical density function therefor (48) $\underline{p}(t_\lambda)$ solution vector give density function – which is approximate the task – for $\underline{Y}_J(t_\lambda) \in R^6$; $t_\lambda \in [t_0;T]$; J=1,2, ...m measuring vector set.

Question

Density function wether possible continuous extend to:

The task can solve with taking to 2 subset (α,β) of U set in (48).

That is:

Look for the continuous expansion of the density function. Let's apply two different interpolations.

α, $\mathop{X}_{i=1}^{m} c_i$ subspace (49)

Because (17/c) is true:

$$\underline{c}(t_\lambda) = [c_1(t_\lambda), c_2(t_\lambda), \dots c_m(t_\lambda)] \in R^m;\ \|\underline{c}\| = 1\ \forall t_\lambda \in [t_0;T]$$

Let's approximate $\underline{c}(t_\lambda)$ (49) vectors in the $t_\lambda \in [t_0;T]$ λ: any index set on the spherical shell, continuously.

β, $\left(\mathop{X}_{i=1}^{m} \underline{m}_i\right) \times \left(\mathop{X}_{i=1}^{m} \underline{\delta}_i^2\right)$ subspace (50)

Any the continuous interpolation procedure should apply.

The application of the α, and β, determine the next approximating probability.

For the any $B \subset R^6$ set: (51)

$$p\{\underline{Y}(t_\lambda) \in B \mid t = t_\lambda\} \approx \int_B \sum_{i=1}^{m} c_i(t_\lambda) g_i\left[\underline{m}_i(t_\lambda); \underline{\delta}_i^2(t_\lambda); x\right] dx$$

In that case if use examine of the spherical province B.

Then:

The transformational function of 6D spherical coordinate system.

(52)

$$x_1 = r\cos\varphi_1 \qquad \varphi_J \in [0;\Pi]\ J = 1,2,3,4$$

$$x_2 = r\sin\varphi_1\cos\varphi_2$$

$$x_3 = r\sin\varphi_1\sin\varphi_2\cos\varphi_3 \qquad \varphi_3 \in [0;2\Pi]$$

•

•

$$x_b = r\sin\varphi_1\sin\varphi_2 \dots \sin\varphi_5$$

Then (52) with B set

$$p\{\underline{Y}(t) \in B | t = t_S\} \approx \tag{53}$$

$$\approx \int\limits_{W(r;\varphi_1;i=1,2,\ldots 5)} \sum_{i=1}^{m} c_i(t_S) g_i \{\underline{m}_{ts}(r,\varphi_1 \ldots), \delta^2(r,\varphi_1 \ldots) x_J(r,\varphi_i \ldots)\} *$$

$$\det \left\{ \frac{\sigma(x_1, x_2, \ldots, x_6)}{\sigma(r, \varphi_1, \ldots, \varphi_6)} \right\} dr d\varphi_1 \ldots d\varphi_5$$

7. Summary

The above process perform the next:

If the required density function is on the $\widetilde{G}$ function class, then:

In the $n \to \infty$ and $m \to \infty$ case the processis convergent.

The continuous extension from the discrete $t_\lambda \in [t_0;T]$ (λ is index set) does not on the above process.

The analysed stochastics process and above process perform the (21).

The composition of empiric probability function of process in the examination times don't apply because of little examination number.

Above procedure in the little examination number case too can describe the vertical probability of process (in depend quality of experiment results).

On the basis of the result of 6. chapter (52), (54) probabilities can describe to any $A \subset R^6$ set and $A \subset R^6$ spherical set.

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