PREDICTIVE ADAPTIVE CONTROL OF A NON-LINEAR TIME-VARYING AIRCRAFT SYSTEM

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Abstract

An aircraft that is described by a non-linear, time-varying system is transferred from an initial state to a final state in a certain number of steps which result from the discretization of the entire (time-, fuel- or energy-optimized) trajectory in a sequence of points defining elementary trajectories.

The aircraft is guided from one point to another by a finite-time control vector obtained for linear systems with a quadratic performance criterion. The control time interval is chosen such that the set of accessible states of the aircraft almost always lies in the set that may be obtained by freezing the linearization in the initial point of the elementary trajectory.

At the end of the control interval the state is measured and then allows the determination of the control vector, provided that the difference between the aircraft's state and the predetermined state lies inside a tolerable error window. If this condition is not met a parameter identification is carried out.

An application is made to the in-plane, accelerated climb of a hypothetical supersonic aircraft.

I. Introduction

One of the major problems of the flight of an SST is the accelerated climb through the supersonic Mach number range. For such an aircraft flying at high altitude and therefore exhibiting relatively large time constants it may be interesting to investigate the performance of a predictive control system. This will not be done by reducing repetitively the terminal target set of the time-to-go trajectory (by a human pilot)[1], but rather by guiding the aircraft automatically along some nominal trajectory in a certain number of steps.

In this case optimal control methods with finite-time control duration play an important rôle in anticipating the trajectory for some specified future state of the vehicle: the predicted vehicle control vector is then determined by a fast-time model. For reasons of simplicity and computation time saving optimal control methods for linear systems are used which minimize a quadratic cost functional. The optimal steering function for the model is converted to real time for steering of the actual vehicle. This can be done by using some control law satisfying Erzberg's perfect model following conditions[2] or, for low noise vehicle environment and state measurement situations, by feeding back the vehicle state only at certain time intervals, i.e., by controlling the vehicle by means of the model control vector in the open-loop manner, provided that certain requirements for the cost functional are satisfied in order to reduce the divergence between the actual trajectory and the predicted one to a minimum. This error-tolerant control system also benefits from the trajectory divergence by using it for parameter adjustment with little computation load.

The guidance and stabilization system consists of an off-line determination of a minimal trajectory used as a stored flight plan and of the on-line fast-time control vector computation and parameter identification, which may be performed by an onboard computer (fig. 1).

II. Problem Formulation

Let the dynamics of the vehicle be described by a non-linear, time- and parameter-varying, twice differentiable, first order differential equation system:

$$\dot{x}(t)=f(x,u,a,t),\ x(t_0)=x_0, \qquad (1)$$
$$y(t)=C_1x(t) \quad ,\ u(t_0)=\bar{u}_0$$
$$t\in[t_0,t_t],(t_f>t_0)$$

where x is the n-dimensional state vector, u an r-dimensional control vector, y an m_1-dimensional observation vector ($m_1\leq n$) and a a p-dimensional parameter vector. If the vehicle dynamics is developed into a Taylor's series about the nominal trajectory

$$(x,u,t)_{no} \qquad (2)$$

then the linear part represents the model dynamics of the vehicle

$$\delta\dot{x}(t)=A_k\ \delta x(t)+B_k\ \delta u(t)+D_k\ \Delta a \qquad (3)$$
$$\delta y(t)=C_2\ \delta x(t)$$

frozen at some point $(x,t)_k$, which lies along the nominal trajectory (2) that extends between the two extremal points $(x,t)_0$ and $(x,t)_f$ from and to which the vehicle has to be transferred. δx, δu and δy are the perturbations of the previously defined vectors, Δa accounts for some constant parameter variation and A, B, C_2 and D are matrices with appropriate dimensions. The problem is now to steer the vehicle with the control law

$$u(t)=\bar{u}_k+\delta u(t) \quad (4)$$

from $(x,u,t)_{no_k}$ to a neighborhood close to $(x,u,t)_{no_{k+1}}$ with $l_R/U_o \ll (t_{k+1}-t_k) \ll (t_f-t_o)$ (l_R/U_o being the time needed to cover one reference lenght l_R of the vehicle at the speed U_o) provided $\delta u(t)$ minimizes the quadratic performance criterion (prime denotes transpose)

$$J_k=\min\Big[\delta x'_{k+1}F_k\delta x_{k+1} + \int_{t_k}^{t_{k+1}}(\delta x'(t)Q_k\delta x(t)+\delta u'(t)R\delta u(t))dt\Big] \quad (5)$$

where F, Q and R are weighting matrices so as to maintain satisfactory trajectory and flying qualities.

III. Control Strategy

A. The nominal trajectory

The finite set y_{no} of the connectable points $(y,u,t)_{no_k}$, $y_{no_k}(m_1 x1)$, $m_1<n$, $u_{no_k}(r_1 x1)$, $r_1 \leq r$, $k=1, N_{no}$, resulting from the discretization of the minimal trajectory profile (2) is obtained for a simplified non-linear model of the vehicle (considered, e.g., as a mass-point system) where u_{no} is the steering vector determining the speed at which the entire flightpath has to be covered.
This two-boundary value problem is solved off-line by some method (here Balakrishnan's epsilon technique)[3] such that the nominal trajectory is either time-, fuel- or energy-optimized.

B. The elementary trajectory

The conditions for possible and best open-loop vehicle-model following during the k-th elementary trajectory can be stated as follows:

1) for some useful control domain the future accessible state domain of the model issueing from the measured vehicle state $(\tilde{x},\tilde{t})_k$ must almost always lie in that of the vehicle. This condition implies small initial (vehicle) state measurement and model parameter errors.
From the point $(\tilde{x},\tilde{t})_k$ and for all times in the k-th control period 2a) the autonomous model and 2b) the free vehicle must be stable.
When allowing a certain maximum distortion value between the vehicle and model accessible state domains as defined with condition 1), the time needed to reach this distortion gives the maximum predicted control duration $\hat{t}_{max_{k+1}}-\tilde{t}_k$ for the elementary trajectory initiated at $\tilde{t}_k$. This is a measure for the validity of the model freezing assumption. So far it is evident that control law (4) enables the vehicle to be transferred from $(\tilde{x},\tilde{t})_k$ to the vicinity of the predicted state $(\hat{x},\hat{t})_{k+1}$.
3) the divergence of the vehicle and model trajectories reduces to a minimum during the specified control interval if

$$J_k(\delta u(t))=\min_{(Q,F)_k,R\ \delta u(t)}[\min J]_k \quad (6)$$

The vehicle thus remains in its "linear tube" when the weighting matrices of the cost functional (5) are such that min J_k reaches a minimum value for some fixed or reference coefficients of the matrix R as an example.
The future end point of the nominal trajectory $(\hat{y},\hat{t})_{no_{k+1}}$ ist now taken as the point next to $\hat{t}_{max_{k+1}}$ and the corresponding vehicle state $\hat{x}_{k+1}$ is obtained according to the relation (^{t} denotes pseudo-inverse)

$$\hat{x}_{k+1}=[C_1{}^t\ \hat{y}_{no}+C_2{}^t\ \bar{y}]_{k+1} \quad (7)$$

where $\bar{y}_{k+1}$ is the predicted steady-state vehicle vector corresponding to $\hat{y}_{no_{k+1}}$, so that the k-th initial conditions for the model dynamics (3), with dropped sensitivity matrices, are

$$\delta x(\tilde{t}_k)=\tilde{x}_k-\hat{x}_{k+1} \quad (8)$$

Two types of finite-time control vectors are considered for this regulator problem.

1. a continuous unbounded control vector with free terminal state

$$\delta u_k(t)=-[R^{-1}B'K(t)]_k\ \delta x(t) \quad (9)$$

where $K_k(t)$ is the solution of the Riccati matrix equation

$$\dot{K}_k(t)=[-K(t)A-A'K(t)+K(t)BR^{-1}B'K(t)-Q]_k \quad (10)$$

integrated backward from $K_k(\hat{t}_{k+1})=F_k$ to $t=t_k$.

2. a piece-wise continuous control vector which is obtained by the so-called direct convex feedback optimal control method[4] and is a linear function of the initial conditions when the controls are not constrained,

$$\delta u_k(t)=-[V(t)G]_k\ \delta x(\tilde{t}_k) \quad (11)$$

where the matrix

$$V_k(t)=[\text{diag } v_1(t),\ \text{diag } v_2(t),\ \ldots,\ \text{diag } v_N(t)]_k,\ (r\ x(N\ x\ r))$$

has the property

$$\int_{\tilde{t}_k}^{\hat{t}_{k+1}}[V(t)V'(t)]_k dt=[\tau_k I],\ (r\ x\ r)$$

and $\tau_k=T_k/N$, i.e., the control interval $T_k=\hat{t}_{k+1}-\tilde{t}_k$ is divided (in this case) into

N equal subintervals of lenght τ_k, and G_k is a constant ((N x r)x n) matrix (see Appendix I).

C. The entire trajectory

The terminal point $\hat{x}_{k+1}$ of the k-th elementary trajectory is not an equilibrium state for the vehicle along the nominal trajectory. Thus a fixed end state for the control vector computation is not required. Furthermore, in order to avoid too large control actions and therefore large acceleration peaks at the connection of two trajectories, the finite-time control vector (which ensures attainability of the intermediate final state at the specified control time) is prematurely interrupted at some time $t_{k+1}<\hat{t}_{k+1}$ at which the next control vector $u_{k+1}(t)$ is applied after a new initialization of the control system at $\tilde{t}_{k+1}=t_{k+1}-\Delta t_{k+1}$ (Δt_{k+1} is the computation time of $u_{k+1}(t)$, see figure 2).

The entire control maneuver is then realized by the succession of control vectors with incomplete duration provided the vehicle-model errors remain in a tolerable error window.

IV. Parameter Adjustment Method

It is now assumed that the system definition parameters a (which are intimately related to each other to give the elements of the state and control matrices, the latter being known with some accuracy at t_k) vary slowly during T_k and remain in a bounded parameter set A_p during the entire observation period t_f-t_o. If for $t\in[t_k,t_{k+1}]$ the vehicle model error becomes larger than the tolerable error, then a new control vector has to be computed at $\tilde{t}_{k+1}$ ($<\hat{t}_{k+1}$) with certain corrected parameter values. Obviously, the time required to perform the parameter adjustment must not exceed too much the control vector computation time. Therefore, a fast adaptation method must be found, based on the solution continuity of ordinary differential equations with regard to parameters and utilizing only a small number of parameters in the case of a multivariable system.

As shown in Appendix II, the p definition parameters can be reduced to p_r classes of parameters, the variation of any one of the parameters of the same class leading the system state to the same sector. Stating it another way, the variation gives the same sign combination of the state values defining the kinematical relation for the trajectory divergence. Inversely, a certain trajectory divergence can be corrected by changing only one, i.e., the representative parameter of that class. Further, when several of the parameters belonging to at least two of the p_r classes are varied from their nominal values, then a qualitative identification test yields the dominant error parameter a_d. Next, the value of the dominant error parameter Δa_d responsible for the vehicle-model state error ex at t_{k+1} is given by the solution of the system resulting from the difference between model differential systems with and without sensitivity terms, where both models are driven by the same last control vector $\delta u_k(t)$: the trajectory divergence is thus compensated by a constant parametric deviation Δa_{k+1} by means of the last existing model values.

Since some of the system definition parameters are identified rather than the system matrix elements, it is of course possible to adjust the vehicle to the model parameters and to keep them in a certain domain or to let them obey a specified law (e.g., the static stability margin) as in the CCV-concept.

The error vector being the difference of the vehicle and model state vectors, it is obvious that linearization and freezing operations add to parameter variation effects, so that the true dominant error parameter may not always be identified. But the adjusted one always corrects the trajectory divergence completely, provided conditions III 1) to III 3) hold for the next control period.

V. Application and Results

The theory is applied to the accelerated climb of an SST-type aircraft. The state and control vectors of the in-plane rigid-body aircraft dynamics are respectively:

$$x'=(u,w,q,\theta,Z), \quad n=5 \tag{12}$$

u,w: body-axis speed vector
θ: pitch angle
q: pitch velocity
Z: altitude

$$u'=(\delta_t,\delta_z), \quad r=2$$

δ_t: throttle command position
δ_z: elevator angle

For the supersonic high altitude flight the atmosphere density gradient is taken into account[5] so that the pair (A,B) of the model and control matrices is controllable. Moreover, allowing a certain dynamic pressure error between vehicle and model, the limiting distortion condition becomes (see III B) for a negligeable altitude difference

$$2\Delta\hat{u}_{k+1}/\tilde{u}_k\leq\varepsilon, \quad 0<\varepsilon<1 \tag{13}$$

and the k-th predicted maximum control duration is given by (using a_{11} and b_{12} of the matrices A_k and B_k):

$$T_k=a_{11_k}^{-1}\left(\frac{a_{11}}{b_{12}}\bigg|_k -\frac{\Delta\hat{u}_{k+1}}{\Delta\delta_t}+1\right) \tag{14}$$

where $\Delta\hat{u}_{k+1}=u_{no_{k+1}}-\tilde{u}_k$

$\Delta\delta_t \;=\delta_{t_{no_{k+1}}}-\bar{\delta}_{t_k}$

and $u_{no_{k+1}}$ is that speed value in the nominal trajectory point set which lies closest to $\hat{u}_{k+1}$, as determined by relation (13).

The choice of the weighting matrices of the quadratic functional (5) determines
- the vehicle-model following performances as stated by condition (6),
- the non-violation of the control domain (maximum throttle command) when the control vector is not constrained.

A suitable choice for the guidance task of carrying out the transfer from the initial to the final point of the elementary trajectory is found to be F=diag F=0 with only $f_1, f_5 \neq 0, >0$, which are the co-factors of the speed and altitude increases, respectively. The shape of the trajectory depends on the stabilization, i.e., on the vehicle behavior about its center of mass. Q=0 is chosen with an n-2(=3) significant element positive-definite ($Q_{n-2}>0$) core, the elements being the co-factors of the state values defining the short period motion:

$$Q=\text{diag}[0,q_2,q_3,q_4,0].$$

Although the rôle of the state weighting matrix Q is to minimize the effects of the non-linear and particularly the second order terms of the series development of the vehicle equations of motion (1), Q is not taken to be the Hessian matrix of the system.

The control weighting matrix is set equal to the unit matrix, R=I, and has to prevent excessive amplitudes of the control maneuvers as well as the violation of the throttle command limit $\delta_{t_{max}}=1$.

For any speed and altitude increases, it is found that only Q has to be adapted, F remaining constant over the entire flight path. So Q can be computed off-line together with and according to the nominal trajectory, so that only little changes are needed due to parametric errors between vehicle and model. Furthermore, the weighting matrices are the same for both optimal control methods.

Figure 3a shows three elementary trajectories for the continuous control vector case, starting from the level flight conditions at the altitude of 12 km and at the three Mach numbers M=1.2, 1.6, 2. in the plane (δM, ∂Z) for δM_o = -.12, δZ_o = -50 m. Figure 3b shows the corresponding control vectors (δ_t, δ_z) for the control duration of 17.9 s for which a small trajectory divergence is obtained although about 20% dynamic pressure difference between initial model and final vehicle values are tolerated.

Elementary trajectories initiated at the level flight conditions M_o = 1.2 and Z_o = 12 km obtained for a stepwise-continuous control vector with 5 subintervals for different final altitudes ranging from δZ_o = 0 to -150 m and for the same Mach number increase as in the previous case are shown in figure 4a. With a duration of 17.9 s computed without prescribed final altitude, it appears according to figure 4b that for δZ_o = -150 m the first control subinterval lies on the throttle command limit. Straight lines joining the same control vector subintervals for the different terminal altitudes show the proportionality with the initial conditions when the control vector is free.

Figure 5a shows an entire time-optimized trajectory starting and ending at the level flight conditions for the initial Mach number M_o = 1.2 and altitude Z_o = 12 km and for the final Mach number M_f = 2. and altitude Z_f = 18 km. The only information the control system gets from the nominal trajectory point set, is speed, altitude and flight-path angle values. For an effective total flight duration of 233 seconds (compared to the 224.5 seconds of the pre-determined trajectory duration, the difference is due to the fact that the aircraft lags during the last acceleration phase towards Mach number 2), the vehicle state is measured 14 times only. The control time interval for each elementary trajectory is interrupted 5 seconds before completion and the fast-time system computation is set to 1 second. In this case, for which perfect knowledge of the vehicle parameters is assumed, the speed and altitude vehicle-model errors are kept within ±10 m/s and ±40 m, respectively (figure 5b).

In that particular case of a 5th order characteristic equation, three classes of parameters are found (according to the kinematic condition δZ) to which the qualitative test vector ε_a is related in the following manner:

$\varepsilon_{a_1}=1$: static stability margin variation $\Delta x/x$ dominates

$\varepsilon_{a_2}=1$: mass stability margin variation $\Delta m/m$ dominates

$\varepsilon_{a_3}=1$: lift coefficient margin variation $\Delta c_z/c_z$ dominates

and the logical ε_a yields (see Appendix II)

$$\varepsilon_a = 1/3 \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \text{sgn } ey_1 \qquad (15)$$

where (sgn ey_1)' = (sgn ew, sgn eq, sgn ez) denotes the sign vector of the angle of attack, pitch velocity and altitude errors, respectively. Furthermore, ε_{a_1} accounts for trim value errors, ε_{a_2} for air density changes and dynamic pressure measurement inaccuracies, ε_{a_3} for vertical wind shear as well as for aerodynamic distortion. For the nominal case M_o = 1.2, Z_o = 12 km, the relative sensitivity ratio is

$$-\Delta x/x : -\Delta m/m : \Delta c_z/c_z = 1:.35:4\cdot 10^{-3}.$$

Figure 6 illustrates the dominant error parameter space. The triangle is the visible part of the iso-error plane ($\delta y = \delta z$=6 m), which represents the altitude divergence (after 20 s) for t>0, and which also indicates the zones of the parameter dominances. Finally the identification process is demonstrated for two examples where identification occured one second before the end of the first elementary trajectory due to an initial mass error ($\Delta m/m$ = .008: vehicle heavier than the model, figure 7a) and a static margin error ($\Delta x/x$ = -.001, figure 7b). The corrective identification effect becomes evident on the next elementary trajectory.

VI. Concluding Remarks

Concerning the system performances it can be said that 1) elementary trajectory errors are not additive in the case of the entire trajectory, i.e., that the final state errors do not propagate from one elementary trajectory to the next one, 2) control vector dilation inaccuracies (when playing it back in real time) do not induce trajectory divergence but only a difference between effective and predicted elementary final states.

If strong perturbations or too abrupt parameter changes occur, the actual control task can be accomplished with reduced control intervals in order to smooth the trajectory divergence. If the latter can not be kept in the interior of the error window, the predicted control vector has to be played back in closed-loop form or the control interval has to be shortened again until the actual control system degenerates into a dual one, for which it may not be possible to separate control and estimation.

References

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Appendix I: Convex Feedback Optimal Control Method

The theory is best explained with the aid of the linear time-varying system

$$\dot{x}(t) = A(t)\, x(t) + B(t)\, u(t),$$
$$x(t_o) = x_o,\ t \in [t_o,\ t_o+T]$$
$$x(n \times 1) \quad u(r \times 1)$$

which has to be transferred to

$x(t_o+T) = 0$ free (for the purpose of the present study)

by means of the piece-wise continuous control vector of the form

$$u(t) = -V(t) G\, x_o$$

if u(t) is not bounded, so that the cost functional (5) (with adequate notation) is minimized.

Let all $u_i(t)$, i = 1, r, be decomposed simultaneously into N constant subintervals

$u_{ij}(t)$, j = 1, N of equal lenghts

$\tau_j = \tau$, $(N\tau = T)$.

The unit-pulse functions v_j

$$v_j(t) = \begin{cases} 1 & \text{for } t_{j-1} \leq t < t_j \\ 0 & \text{otherwise} \end{cases}$$

are orthogonal over $t \in [t_o,\ t_o+T]$, i.e.,

$$1)\ v_k(t)\, v_j(t) = \begin{cases} 0 & \text{for } k \neq j \\ v_j(t) & \text{for } k = j \end{cases}$$

$$2)\ \int_{t_o}^{t_o+T} v_k(t)\, v_j(t) dt = \begin{cases} 0 & \text{for } k \neq j \\ \tau & \text{for } k = j \end{cases}$$

so that

$$u(t) = \sum_{j=1}^{N} v_j\, u_j \text{ or } u(t) = V(t)U$$

where

$$V(t) = [v_1(t)I_r,\ v_2(t)I_r,\ \ldots,\ v_N(t)I_r] \qquad ,(r \times rN)$$
$$U' = [U_1,\ U_2,\ \ldots,\ U_N], \qquad ,(1 \times rN)$$

If u(t) = V(t)U is introduced into the solution of the differential system, then

$$X(t) = X_o(t,\ t_o)x_o + X(t,\ t_o)U$$

where

$$X_o(t,t_o) = \Phi(t,t_o) = e^{A(t-t_o)} \qquad ,(n \times n)$$
$$X(t,t_o) = \int_{t_o}^{t_o+T} \Phi(t,t')\ B(t')\ V(t')dt' \qquad ,(n \times rN)$$

these two matrices are continuous over $t \in [t_o,\ t_o+T]$.

The cost functional becomes

$$J(u(t)) = x_o'\ J(0)x_o + 2U'\beta x_o + U'CU.$$

1) $J(u(t))$ is quadratic in U.
2) β and C do not depend on the initial conditions x_o.
3) In the actual case F, Q > 0 and R > 0 (R can be identically zero, provided that one of the matrices F or Q does not vanish entirely), the symmetrical matrix C (rN x rN) is non-singular,

therefore $J_u = 2(CU + \beta x_o) = 0$

and $u(t) = -V(t)\, G\, x_o$

where $G = C^{-1}\beta$, (rN x n).

The expressions for β and C are

$$\beta = X'(t_o+T,t_o)FX_o(t_o+T,t_o)+\int_{t_o}^{t_o+T} X'(t,t_o)\, Q(t)X_o(t,t_o)dt \quad ,(rN \times n)$$

$C=J(F)+J(Q)+J(R)$, with

$$\left.\begin{aligned} J(F)&=X'(t_o+T,t_o)FX(t_o+T,t_o)\\ J(Q)&=\int_{t_o}^{t_o+T} X'(t,t_o)Q(t)X(t,t_o)dt\\ J(R)&=\int_{t_o}^{t_o+T} V'(t)R(t)V(t)dt \end{aligned}\right\} (rN \times rN),\ \text{symmetric}$$

For a bounded control vector of the type

$|u(t)|$, $|U| \in M$

if, after computation of U it is found that

$|U_{i,j}| > M_i$, $i = 1, r$, $j = 1, N$

the computation is performed again with

$U_{i,j} = M_i$

until all

$|U_{i,j}| \leqslant M_i$.

Appendix II: Qualitative Identification Test and Adjusted Dominant Parameter Value

Consider the following linear stationary differential system

$\delta\dot{x}(t)=A\delta x(t)+D\,\Delta a$, $\delta x(t_k)=0$, $t\in[t_k,\hat{t}_{k+1}]$

$\delta y(t)=C\delta x(t)$, $\delta x(n \times 1)$

, $\Delta a(p \times 1)$, constant, not identically zero

, δy scalar

then
1) there exist p_r, $p_r < p$, classes of parameter $\Delta a \in A_p$ leading $\delta x(t)$ to one amongst 2^n-1 of its 2^n distinct sectors when $\delta y(t_1)=\text{const}, > 0$, $t_k < t_1 \leqslant t_{k+1}$

2) $p_r = 2^{n_1}-1$ is given by the sgn combinations of the n_1 states satisfying

$\delta\dot{y}(t_1) = C\, A\, \delta x(t_1) > 0$

i.e., the number of the non-zero elements of the row of A corresponding to $\delta y(t_1)$

3) when simultaneously several of the p parameters Δa, belonging to at least two of the p_r classes are different from zero, then sgn $\delta x(t)$ is given by the dominant parameter a_d. When applied to the actual problem, the dominant error parameter is given by the following qualitative identification test

$\varepsilon_a=1/1\ H\ \text{sgn}\ \{C_1\ ex_k\}$, $\varepsilon_a\ (p_r \times 1)$

, $H\ (p_r \times 1)$

, $C_1\ (1 \times n)$,

$1=n_1+1\leqslant n$,

$1\leqslant p_r$.

The elements of ε_a are logical; H is the transition matrix of the parametric errors.

Then the state error $ex(\tilde{t}_{k+1})$ is compensated with $\Delta a_{d_{k+1}}$=const by means of the differential system formed with the difference of the model dynamics frozen at $\tilde{t}_k$ (3) with and without sensitivity terms and driven by the same last control vector $\delta u_k(t)$, i.e.,

$$\Delta a_{d_{k+1}}=\delta y(\tilde{t}_k)\left[C\int_{\tilde{t}_k}^{\tilde{t}_{k+1}} \Phi(t,t')D_{r_k}\, \varepsilon_a\, dt'\right]^{-1},$$

where

$\Phi(t,\tilde{t}_k)$, (n x n), is the fundamental matrix of the stationary system for $t \in [\tilde{t}_k,\tilde{t}_{k+1}]$.

D_r, (n x p_r), is the sensitivity matrix reduced to the number of the p_r representative parameters,

$\delta y(\tilde{t}_k)=-C\ ex_k$.

$(A,B)_{k+1}$ and $\bar{y}_{k+1}$ (eq. 7) are then computed using the new adjusted value

$a_{d_{k+1}} = a_{d_k} + \Delta a_{d_{k+1}}$.

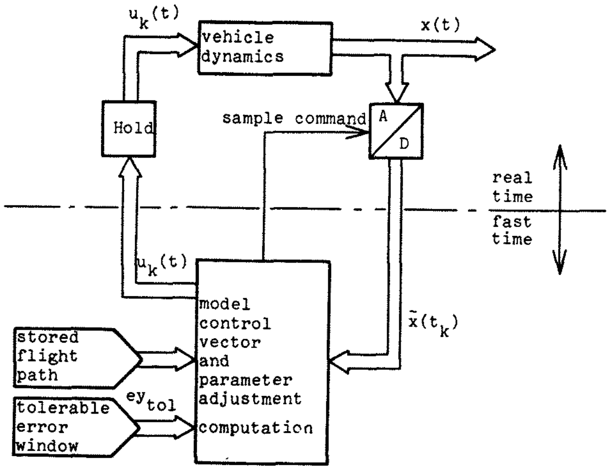

Fig. 1 Block diagram of the predictive control system

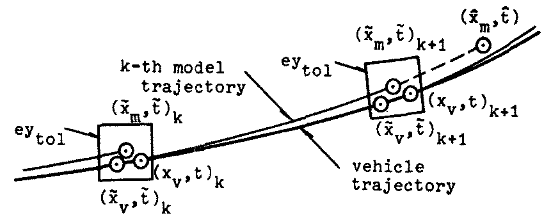

Fig. 2

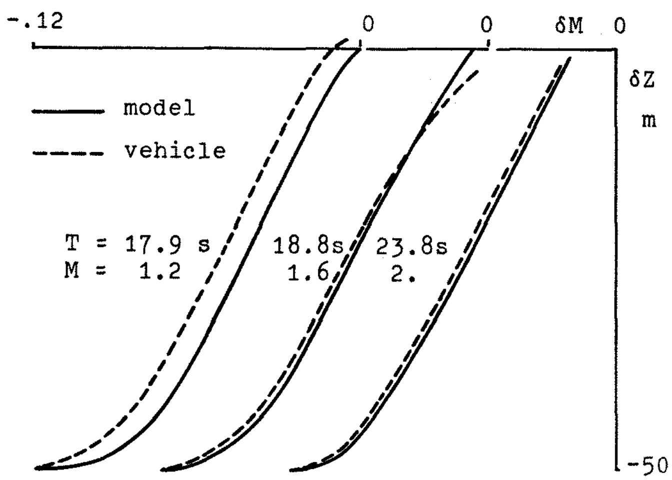

Fig. 3a Elementary trajectories for Z_o = 12 km and $\delta_{t_{no}}$ = .9

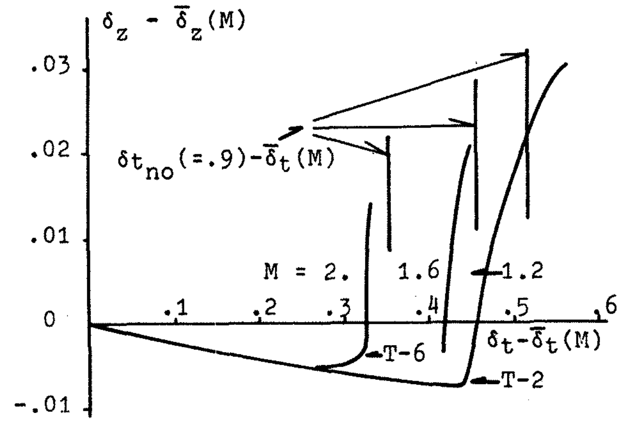

Fig. 3b Continuous control vectors

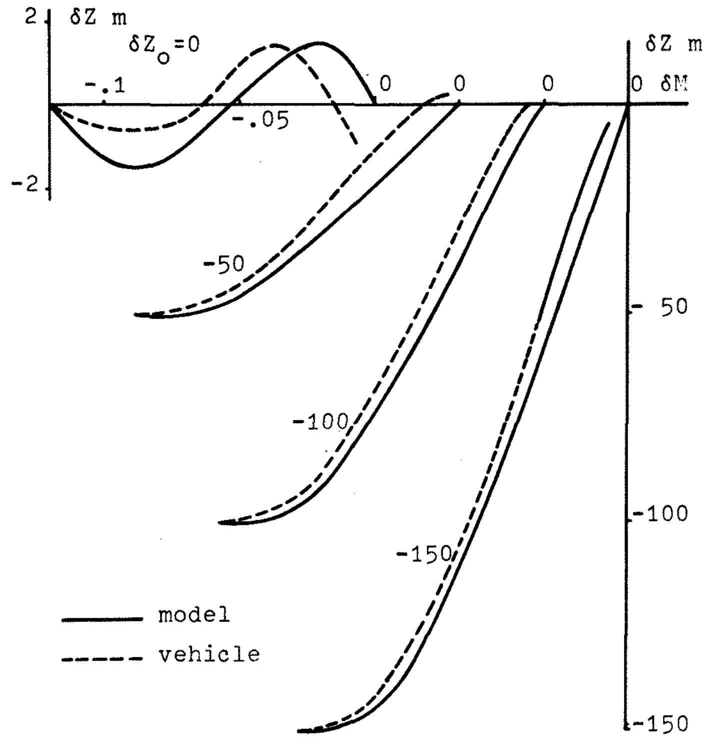

Fig. 4a Elementary trajectories for different δZ_o at Z_o = 12 km, M_o = 1.2 and $\delta_{t_{no}}$ = .9

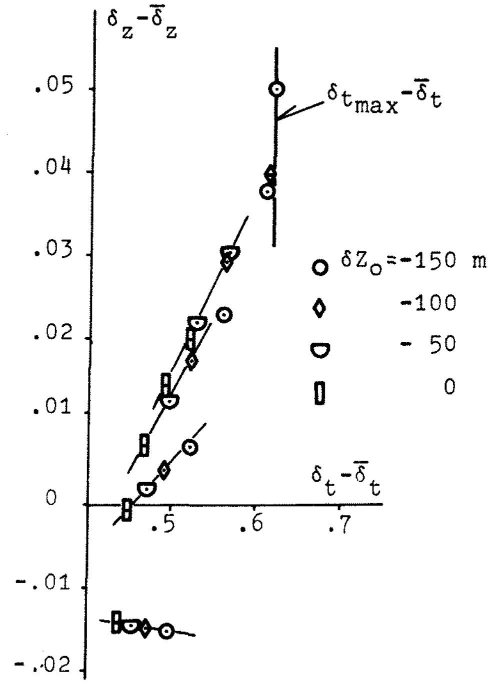

Fig. 4b Step-wise continuous control vectors (N.= 5, T = 17.9 s)

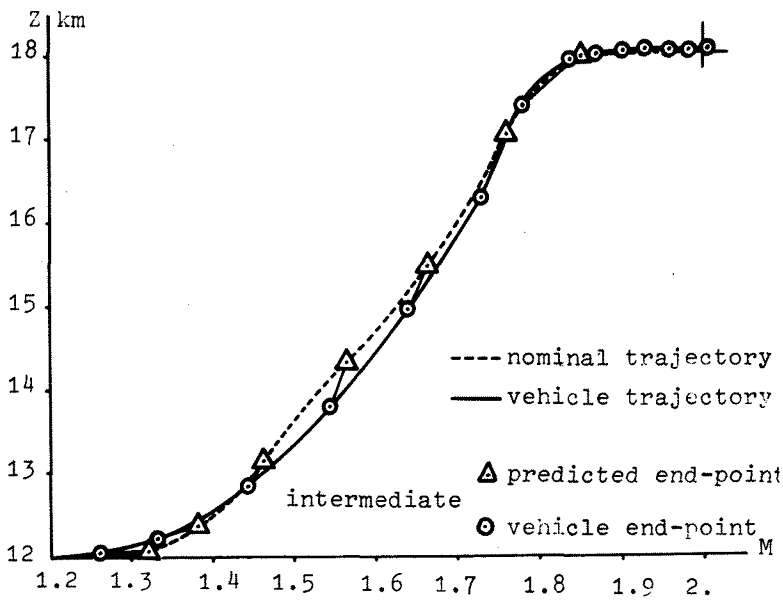

Fig. 5a Entire time-optimized trajectory ($\delta_{t_{no}}$ = .9; M_o = 1.2, Z_o = 12 km; M_f = 2, Z_f = 18 km; final vehicle state after 14 steps: M = 2.011, Z = 18.074 km; continuous control vector)

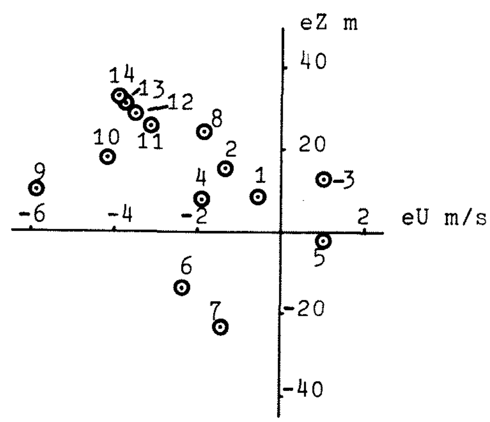

Fig. 5b Speed and altitude vehicle-model errors at the end of each step for the entire time-optimized trajectory

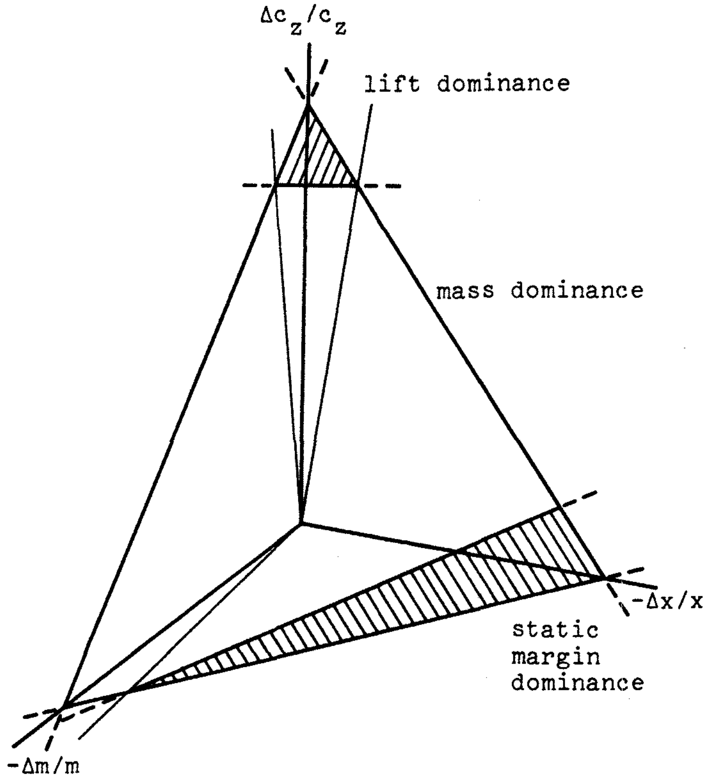

Fig. 6 Visible part of the iso-error plane ($\delta z,t$ = const, > 0) in the p_r-class parameter space

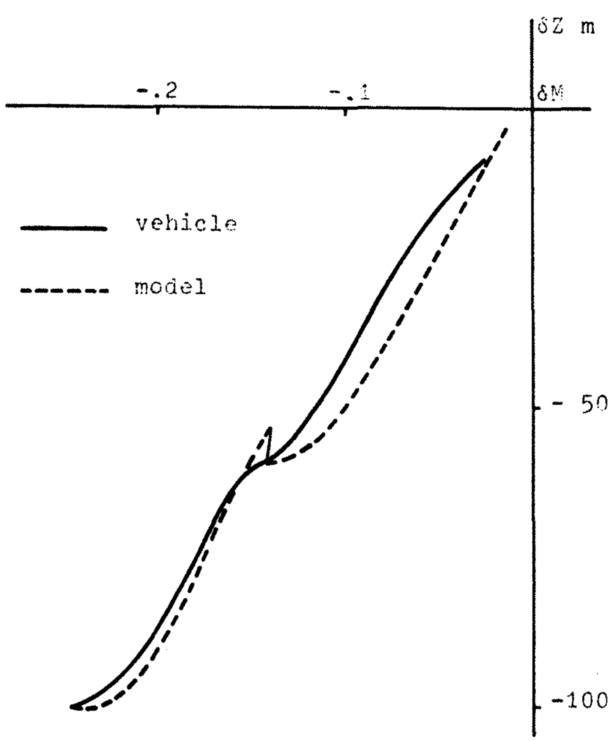

Fig. 7a

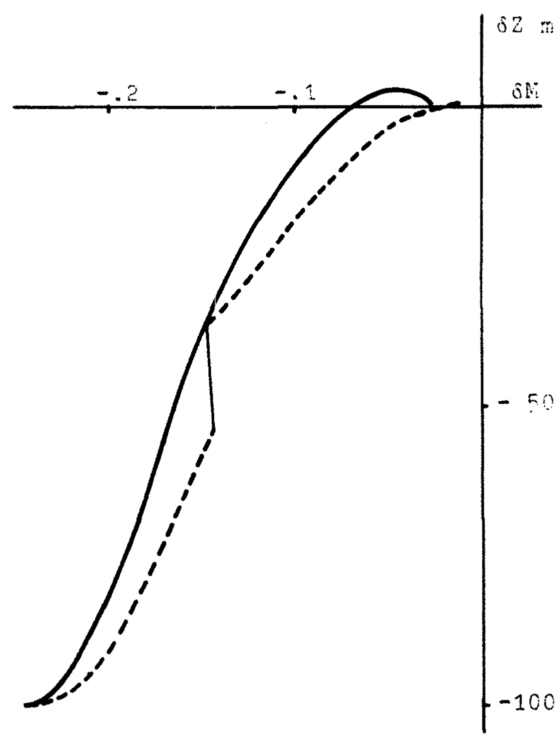

Fig. 7b

Examples of trajectories with parameter adjustment occuring at the end of the first step due to initial mass error (fig. 7a) and static margin error (fig. 7b) for Z_o = 12 km, M_o = 1.2 and $\delta_{t_{no}}$ = .9