33RD CONGRESS
OF THE INTERNATIONAL COUNCIL
OF THE AERONAUTICAL SCIENCES
STOCKHOLM, SWEDEN, 4-9 SEPTEMBER, 2022

IDENTIFICATION OF THE COMPLETE AERODYNAMIC MODEL OF A SUBSCALE FLIGHT TESTING

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Abstract

Subscale aircraft have been used for decades to design new aircraft and evaluate new design techniques. The acquisition of in-flight data from subscale aircraft is already possible today, such as a manned or fullscale aircraft. Thus, more reliable flight simulators are built for flight quality analysis and control design. This work aims to implement a data acquisition and processing system, with the objective of identifying the complete dynamics of a subscale aircraft, model Cessna 182.

Keywords: acquisition and processing system, system identification, subscale, flight testing

1. Introduction

Several works have been developed in recent years with the objective of identifying dynamic models through data obtained in flight tests. In general, the study is divided into two phases, the first consisting of the identification of the longitudinal model, as presented by [1] and [2], and followed by the identification of the laterodirectional model, as presented by [3]. This work presents the identification of the complete model of a subscale aircraft, in which the stability derivatives of the dynamics of the latero-directional movement and the longitudinal movement are estimated at the same time in the same algorithm. The aircraft adopted in this study is a scale model of the Cessna 182, presented in Figure 1. This aircraft is a commercial subscale model, characterized by: a wingspan of 2.05 m, length of 1.62 m, weight of 4 kg, and a combustion engine propulsion system.

Figure 1 – Subscale Cessna 182.

The identification of the complete model provides a greater accuracy between the model and the real dynamics of the aircraft, as considerations of decoupling the longitudinal and latero-directional flight modes are not performed. Thus, this work proposes the identification of the coefficients of the flight dynamics of a complete model, using the data acquired with a Cessna 182 subscale aircraft.

2. Methodology

The identification technique adopted for this work is the M4V (Maneuver, Model, Method, Measures and Validation) [4], and the algorithm procedure is shown in Figure 2.

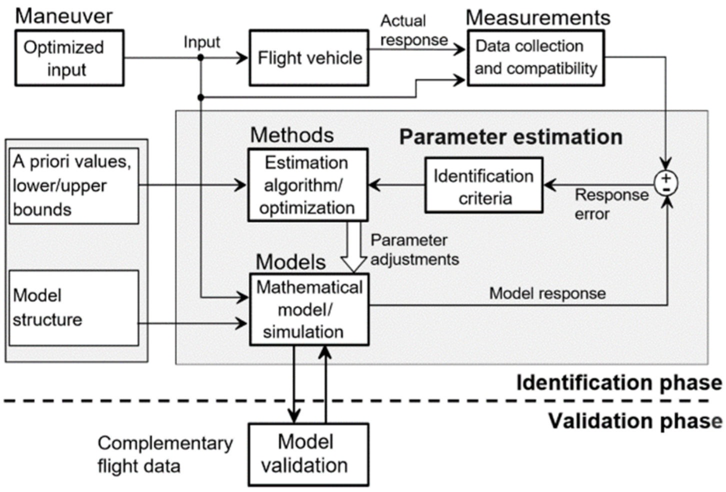

Figure 2 – M4V basic of flight vehicle system identification [4].

2.1 Maneuvers

Initially, the maneuvers that will be used to excite the coupled flight modes of the full dynamics are specified. For each flight mode that needs to be excited, a specific sequence of maneuvers is performed [5], which must result from a study of the interaction between the maneuvers, their operating times and the desired modes. Examples of maneuvers that can be used to excite each flight mode are shown in Figure 3. Maneuvers such as descending and ascending spirals are recommended for the excitation of the coupled modes of the two dynamics. In this work, different maneuvers will be applied to evaluate which one has the best result to be adopted in the identification process.

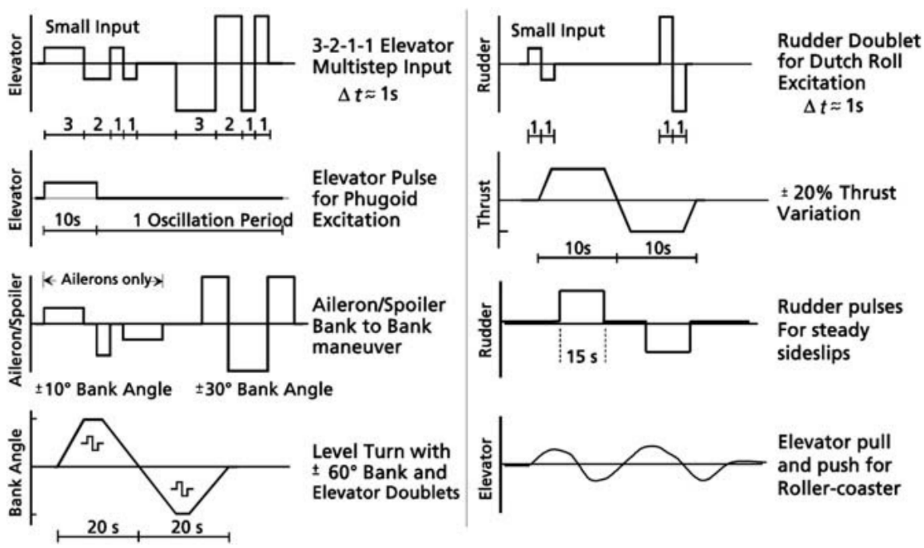

Figure 3 – Maneuvers recomandate to applied [4].

2.2 Aircraft Dynamic Model

The 6D aircraft motion is modeled by the following set of kinematics and dynamics state space equations:

$$\dot{u} = rv - qw - g sin(\theta) + \frac{F_x}{m}, \tag{1}$$

$$\dot{v} = -ru + pw + gsin(\phi)cos(\theta) + \frac{F_y}{m}, \tag{2}$$

$$\dot{w} = qu - pv + gcos(\phi)cos(\theta) + \frac{F_z}{m}, \tag{3}$$

$$\dot{\phi} = p + qsin(\phi)tan(\theta) + rcos(\phi)tan(\theta), \tag{4}$$

$$\dot{\theta} = qcos(\phi) - rsin(\phi), \tag{5}$$

$$\dot{\psi} = qsin(\phi)sec(\theta) + rcos(\phi)sec(\theta), \tag{6}$$

$$\dot{p} = (c_1 r + c_2 p)q + c_3 L + c_4 N, \tag{7}$$

$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7 M, \tag{8}$$

$$\dot{r} = (c_8 p + c_2 r)q + c_4 L + c_9 N, \tag{9}$$

where the variables, u, v and w represent the velocities written in the in the body reference frame (Xb, Yb, Zb), where the Xb axis is aligned with the nose along the body of the aircraft, the Yb axis is directed towards the right wing of the aircraft and the Zb axis so as to form the right-handed system, and the center of the coordinate system is fixed at the aircraft's center of gravity (CG). The mass of the aircraft is represented by m, and the angular velocities around the body axes are given by: p, q and r. The Euler angles ϕ, θ and ψ, are the rolling, pitching and yaw angles respectively.The parameters c_1 to c_9 are associated to the moments of inertia of the aircraft, and the specification of these parameters are described in Fischer [6]. The Fx, Fy and Fz are the forces applied in the directions of the frame of reference, and L, M and N are the moments respectively.
The coefficients equations of forces and moments generated, respectively, in the x, y and z axes are defined by [7], as:

$$C_X = -C_D\,cos(\alpha) + C_L sin(\alpha), \tag{10}$$

$$C_Y = C_{y\beta}\beta + C_{y\delta_a}\delta_a + C_{y\delta_r}\delta_r + (C_{yp}p + C_{yr}r)\frac{b}{V_t}, \tag{11}$$

$$C_Z = -C_{D_0}\,sin(\alpha) + C_L cos(\alpha), \tag{12}$$

$$C_D = C_{D_0} + \frac{1}{\pi e AR}C_L^2, \tag{13}$$

$$C_L = C_{L_0} + C_{L\alpha}\alpha + C_{L\delta_e}\delta_e + C_{Lq}q\frac{\bar{c}}{V_t}, \tag{14}$$

$$C_l = C_{l\beta}\beta + C_{l\delta_a}\delta_a + C_{l\delta_r}\delta_r + (C_{lp}p + C_{lr}r)\frac{b}{V_t}, \tag{15}$$

$$C_m = C_{m_0} + C_{m\alpha}\alpha + C_{m\delta_e}\delta_e + C_{mq}q\frac{\bar{c}}{V_t}, \tag{16}$$

$$C_n = C_{n\beta}\beta + C_{n\delta_a}\delta_a + C_{n\delta_r}\delta_r + (C_{np}p + C_{nr}r)\frac{b}{V_t}, \tag{17}$$

where these moments are used to compute the resultant forces and moments on the body. Where $\bar{c}$ is the mean aerodynamic chord, b is the wingspan δ_a, δ_e and, δ_r, are the respective deflections in the aileron elevator and rudder. And α and β are the angle of attack and the sideslip angle. Therefore, the objective is to estimating the following parameters:

$$\begin{aligned}\Theta_{par} = [&C_{D_0}\ C_{L_0}\ C_{L\alpha}\ C_{L\delta_e}\ C_{L_q}\ C_{y\beta}\ C_{y\delta_a}\ C_{y\delta_r}\ C_{yp}\ C_{yr}\ C_{l\beta}\ C_{l\delta_a}\\ &C_{l\delta_r}\ C_{lp}\ C_{lr}\ C_{m_0}\ C_{m\alpha}\ C_{m\delta_e}\ C_{m_q}\ C_{n\beta}\ C_{n\delta_a}\ C_{n\delta_r}\ C_{np}\ C_{nr}].\end{aligned} \tag{18}$$

In total 25 parameters will be estimated to obtain the complete dynamics of the aircraft.

2.3 Method

The structure of the Output Error Method (OEM) identification is presented in the Figure 4.

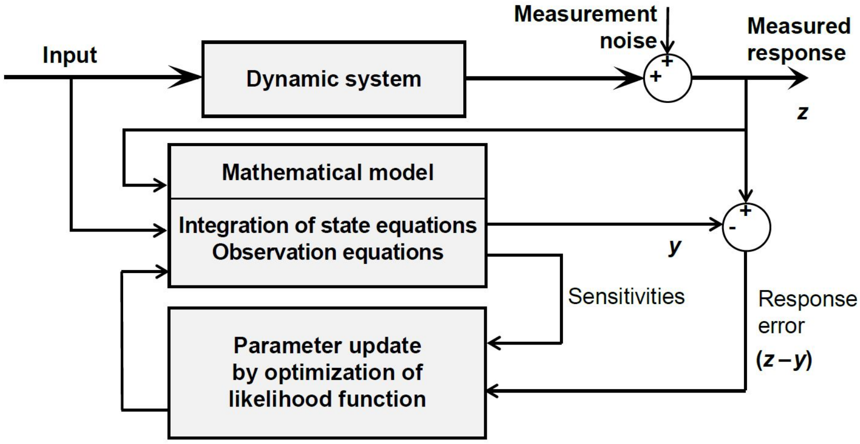

Figure 4 – Schematic block of Output Error Method [4].

The Output Error Method minimizes a cost function that is dependent on the estimated aerodynamic parameters, see Equation 19, and dependent on the covariance of the estimation error. It is adopted the Levengberg-Marquart algorithm as the numerical optimization strategy, which is a variation of the Gauss-Newton method. It consists of minimizing a cost function J, in order to have the maximum likelihood between the data collected and those obtained by the simulation of the model. Where θ is the vector of parameters to be identified and R is the measurement error covariance. The cost function J is given by:

$$J(\Theta) = \frac{1}{2} n_y N + \frac{N}{2} ln(det(\mathbf{R})) + \frac{N n_y}{2} ln(2\pi). \tag{19}$$

2.4 Measures

The data system acquisition used is a Pixhawk 2.4.8 flight controller hardware, see Figure 5, and the software installed is the open-source ArduPilot, working with the Plane model. The data acquisition system records the required parameters that will be used in the identification: accelerations, attitude angles, true air speed, inertial speed, servo command signals, angle of attack and sideslip angle. This system is low cost and has a good quality for data acquisition [8].

2.5 Validation

In the process of validation, it was verified the plausibility of the identified parameters for the proposed model. Another method used to validation is the analyses of parameter error covariance matrix given by P, expressed by:

$$P \approx \left\{ \sum_{k=1}^{N} \left[\frac{\partial_y(t_k)}{\partial \Theta} \right]^T R^{-1} \left[\frac{\partial_y(t_k)}{\partial \Theta} \right] \right\}^{-1}. \tag{20}$$

The elements of main diagonal of P represents the standard deviation of the estimation of the parameters and is bounded by the limits of Cramer-Rao (CR) [9]. Where t_k is the time points at each sampled measurement.

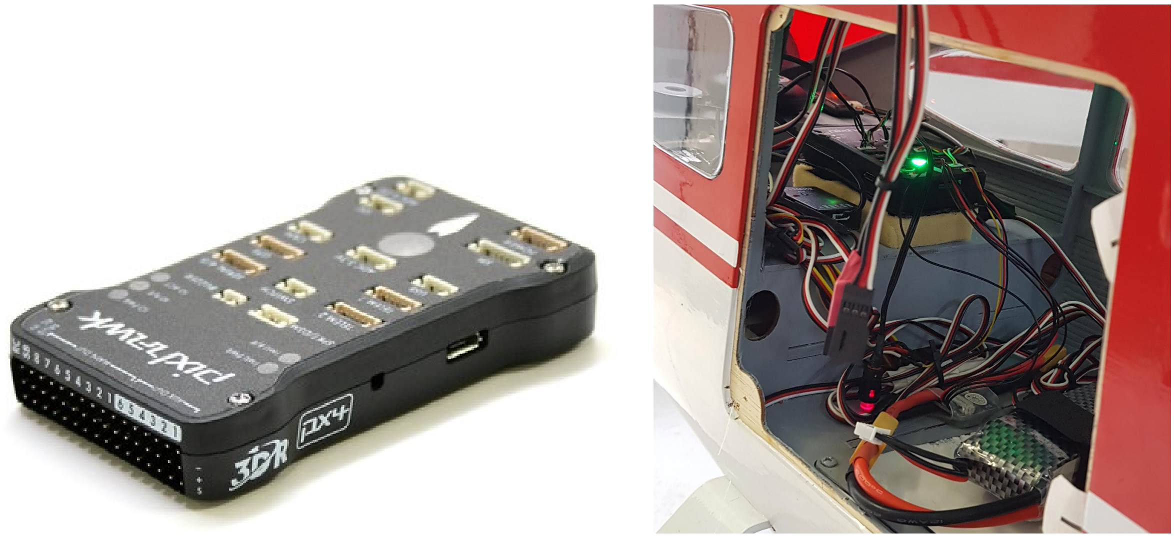

Figure 5 – Pixhawk flight controller [10] and the Pixhawk intaled in the Cessna

In:

$$CR = \sigma_{\theta_i} = \sqrt{p_{ii}}, \tag{21}$$

the p_{ii} are the elements of main diagonal of P and the CR values should be lower then 20%, to reach an acceptable accuracy [11]. In another analysis, the Goodness of Fit method was applied, which is used to analyze the residual error of the measurements, and is calculated by the following equation [4]:

$$\sigma_i = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[z_i(t_k) - y_i(t_k)]^2}, \quad i = 1, 2, ..., n_y. \tag{22}$$

An additional validation of the results is the evaluation of the correlation of the measured output with the estimated one, which is calculated with a normalized cross-covariance function [12] given by:

$$\rho_{zy} = \frac{\sum_{i=1}^{N}[(z_i(t_k) - \frac{1}{N}\sum_{i=1}^{N} z_i(t_k))(y_i(t_k) - \frac{1}{N}\sum_{i=1}^{N} y_i(t_k))]}{\sqrt{\sum_{i=1}^{N}[(y_i(t_k) - \frac{1}{N}\sum_{i=1}^{N} y_i(t_k))^2]}\sqrt{\sum_{i=1}^{N}[(y_i(t_k) - \frac{1}{N}\sum_{i=1}^{N} y_i(t_k))^2]}}, \tag{23}$$

and the desired value for this parameter should be greater than 70%.

3. Results

This Section presents and discusses the results of the data acquisition, the identification process, and the validation of the identified model. After the installation of the electronic system for the flight control and the data acquisition, the flight campaign was carried out. We emphasize that the maneuvers were performed by a pilot, which may imply an inaccuracy in the maintenance of the times between the commands of the excitation maneuvers. These differences can influence the identification process.

3.1 Manuevers

To select the maneuvers that were used in the identification algorithm, they are taken in maneuvers that excite all flight modes, that is, doublet maneuvers applied in the Elevator, for the short period excitation, doublet maneuvers applied in the Rudder for Dutch roll excitation, inputs applied to the aileron for short-period excitation, in addition to maneuvers applied to different surfaces in order for the model aircraft to perform a downward expiratory and thereby activate the concomitant lateral and longitudinal dynamics. The Figure 6 shows the maneuvers that were used in the identification.

It is observed that the maneuvers are concatenated in sequence in order to carry out in a single execution of the algorithm the identification of all the parameters of interest. In addition, different starting points were used for each maneuver, so the algorithm is not penalized by sudden variations in the junction points of the maneuvers.

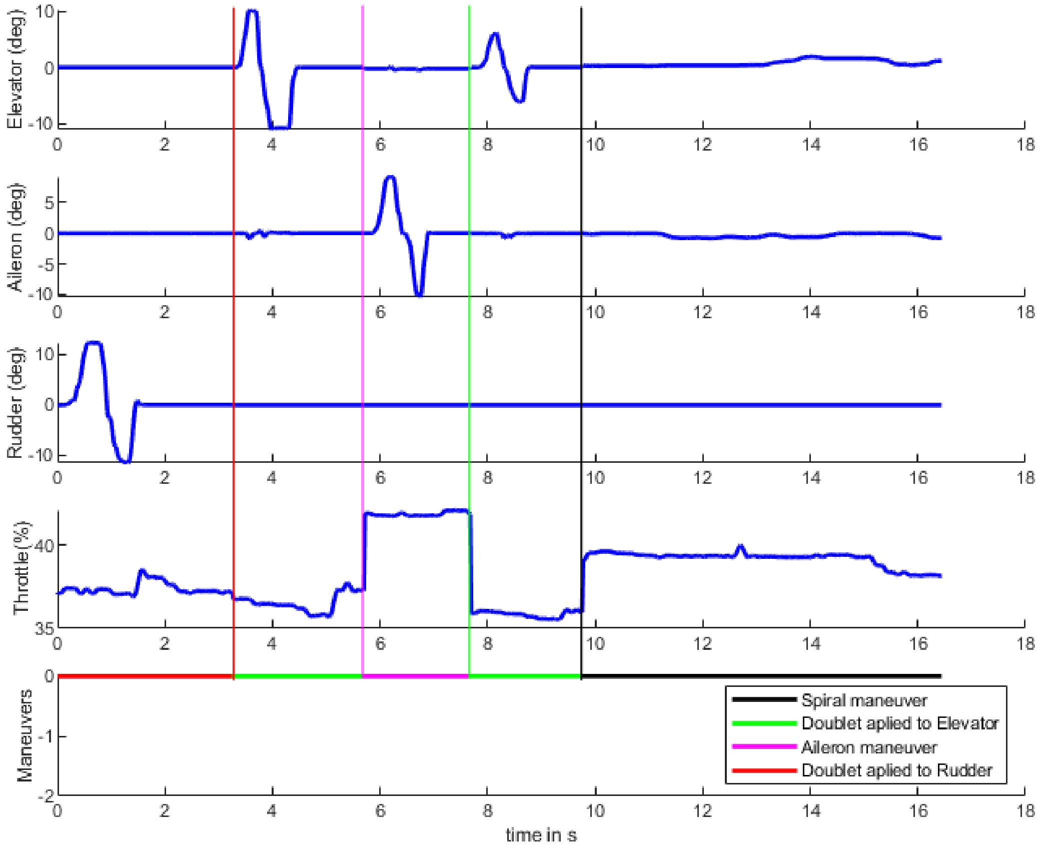

Figure 6 – Maneuvers used to identified.

3.2 Identification Results

The comparison of the results obtained by the identification algorithm with the measured data of the observed variables are presented in Figure [7]. The values obtained for each of the stability derivatives identified are presented in Table [1]. It is observed that the OEM method was able to execute and converge to a result that satisfies the convergence criteria, which culminated in a cost J=-8740 and R=5.97e-14.

The identification algorithm was implemented in MatLab®, and before starting the identification a random search method was applied in order to guarantee that the identification is at a global minimum, this random search method is described in [13].

3.3 Validation

For validation, a sequence of three different maneuvers were applied, following the following order, a doublet applied to the rudder, a doublet applied to the elevator and a rolling sequence excited by the input applied to the aileron. These maneuvers used in validation are different maneuvers from those used in identification, thus increasing the reliability of the data obtained.

Figure [8] shows the graphs of the observed variables comparing the measured and estimated data. In Table [1] are presented the results obtained in the validation and the results of the CR criterion. It is observed that the derivatives related to the longitudinal movement had excellent results, being below 12%, most of the derivatives of the latero-directional dynamics were within the expected values, which is below 20%, but the derivatives C_{lr}, $C_{n_{\delta}a}$, C_{np} and C_{n_r} had a bad performance, leaving with values above 200% in the worst case.

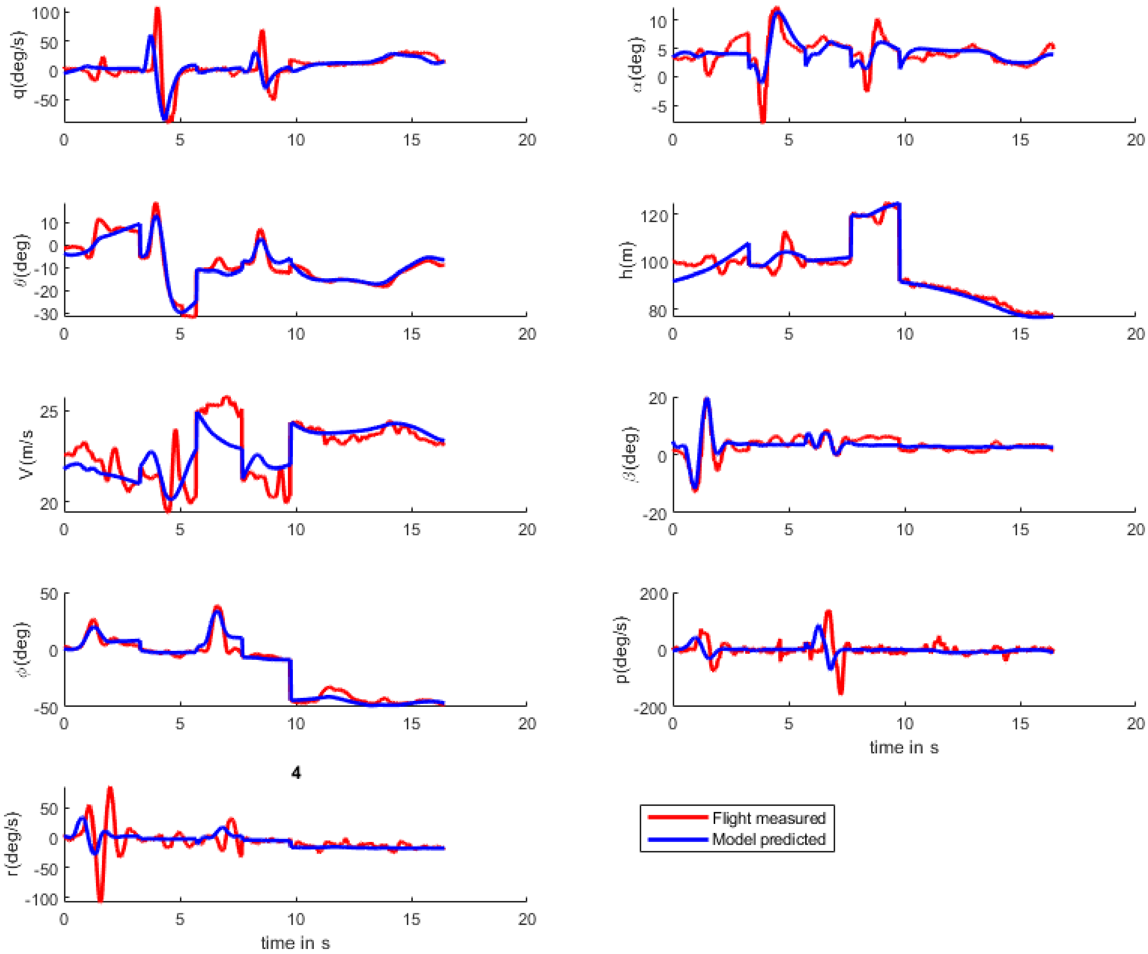

Figure 7 – Results of estimation by output-error method.

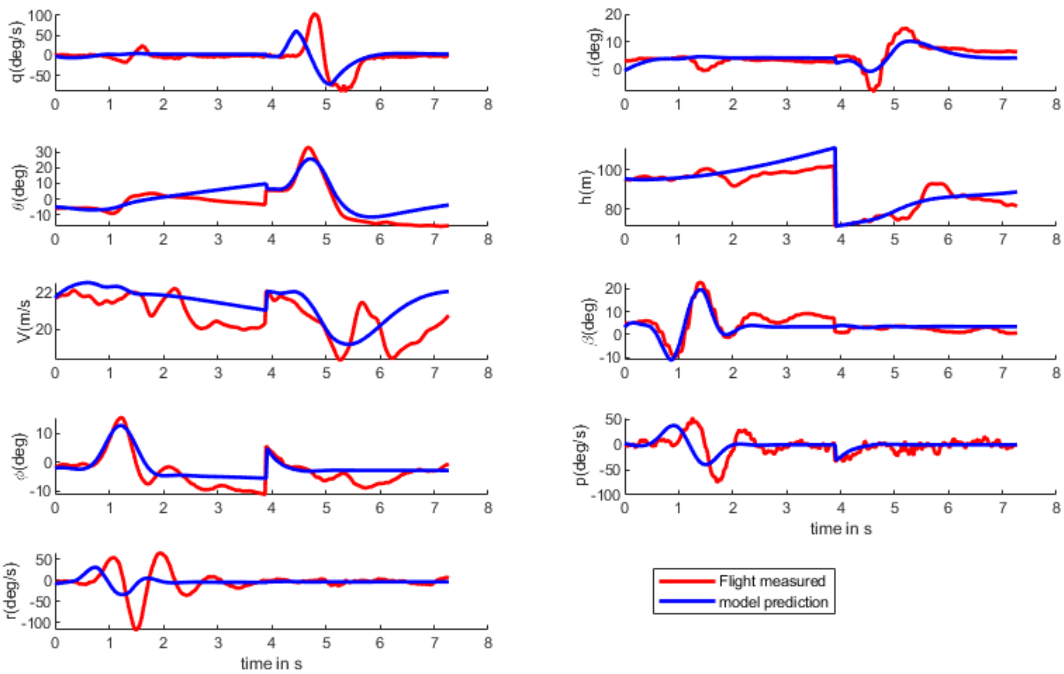

Figure 8 – Results of the validation.

Table 1 – Identified parameters and CR

Latero-diretcional Parameter	Value	CR(%)	Longitudinal Parameter	Value	CR (%)
C_{y_β}	-2.9715	3.06	C_{L_0}	0.3631	3.21
$C_{y\delta_a}$	-0.3030	20.25	C_{L_α}	0.4800	9.16
$C_{y\delta_r}$	0.5244	11.10	$C_{L_d e}$	-1.4925	5.86
C_{y_p}	0.9748	9.06	C_{L_q}	2.1878	4.56
C_{y_r}	-4.2938	2.29	C_{D_0}	0.2520	0.89
C_{l_β}	-0.0498	11.02	C_{m_0}	-0.0144	12.82
$C_{l\delta_a}$	0.0785	6.17	C_{m_α}	0.0940	6.99
$C_{l\delta_r}$	0.0073	34.79	$C_{m_d e}$	1.0698	2.24
C_{l_p}	-0.0683	5.72	C_{m_q}	-5.4094	1.80
C_{l_r}	0.0100	91.22			
C_{n_β}	0.1606	4.60			
$C_{n\delta_a}$	-0.0061	81.15			
$C_{n\delta_r}$	0.0667	5.89			
C_{n_p}	0.0038	144.20			
C_{n_r}	0.0071	213.53			

In the Goodness of Fit presented in Table[2], validation criteria that evaluates the residue, the results obtained are consistent and with a good representation for all the observed variables.
For the cross-covariance criterion presented in Table [3], the results are acceptable above 70%, which is desired for most variables except for p, q and r. It was shown that the estimation of the variables of the coplet model using the OEM was not effective in estimating the derivatives realigned with these variables.

Table 2 – Goodness of Fit results

Parameter	Value	unit
q	0.041	degrees/s
α	0.0067	degrees
θ	0.0410	degrees
h	1.8614	m
V	0.2329	m/s
β	0.0084	degrees
ϕ	0.0441	degrees
p	0.0318	degrees/s
r	0.0215	degrees/s

Table 3 – Crossed covariance

Parameter	Value
q	33%
α	78 %
θ	87%
h	92%
V	82 %
β	83 %
ϕ	90%
p	9 %
r	5 %

4. Conclusion

This work is a first step towards the application of systems identification to estimate the derivatives of the complete nonlinear model of a subscale aircraft. The results showed that it is possible to obtain the derivatives of the complete model at once, even with coupled motions. However, an in deep investigation is required to improve the estimation of some parameters, specially for the latero-direction motion.

We highlight the need to improve the quality of the signals related to the estimation of p, q and r states. In future works it is important to assess how the OEM can obtain better results for the estimation of parameters related to these states. The use of direct measures of alpha and beta can also contribute to these estimates, adding an air-data-boom to obtain these variables can contribute to the improvement of the results. In this experiment, they are being obtained through estimation.

An additional implementation that is planned is activation of the maneuvers through programmed inputs in the controller, thus removing the uncertainty of input delay generated by the pilot when generating the maneuver.

Another possibility is the change in the identification algorithm, such as the Filter Error Method, which is an option when process noises and disturbances are considered.

An important consideration that was observed is that the greater the number of variables involved in the identification process, the more difficult the OEM has to converge and obtain good results, so it would be a good strategy to estimate some derivatives by different methods and keep them fixed during the process achieving better results.

Despite these results, which were somewhat out of the ordinary, in general the M4V methodology proved to be an excellent solution for applications in identification processes.

5. Acknowledgment

Acknowledgment to grant 2019/08468-8, São Paulo Research Foundation (FAPESP), which provides the resources for acquiring the model aircraft as well as the electronic system.

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7. Copyright Statement

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