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LPV/LFR CONTROL FOR ACTIVE FLUTTER SUPPRESSION OF A SMART AIRFOIL MODEL

Renan L. Pereira[1], Pedro J. González[2] & Flávio J. Silvestre[3]

[1]Instituto Tecnológico de Aeronáutica
[2,3]Technische Universität Berlin

Abstract

This paper focuses on a novel gain-scheduling state-feedback control strategy to cope with the active flutter suppression of a smart airfoil model. Unlike other gain-scheduled control approaches in the literature, this one allows combining the polytopic and linear fractional representations into a particular framework to increase the degrees of freedom of a control design. The synthesis conditions have been derived using parameter-dependent Lyapunov functions associated with a static full-block multipliers concept to obtain a less conservative condition such that all time-varying parameters of the linear system may be incorporated. The distinctive gain-scheduled controller is established from existence linear matrix inequalities (LMIs) conditions, where the main steps to obtain such parameter-dependent controller are given. Finally, a smart airfoil model is used to show the effectiveness of the proposed control method in terms of variation of the airspeed.

Keywords: Gain-scheduled controller, Active flutter suppression, Smart airfoil model, LMIs.

1. Introduction

Currently, modern aircraft designs have been focused on the reduction of structural weight to improve fuel efficiency and reduce enviromental impact [1, 2, 3]. However, the reduction of the structural weight may lead to reduced stiffness that in turn to critical issues. For instance, the wings become flexible and can cause undesired coupling of rigid body dynamics and elastic deformation through aerodynamic forces and aircraft control surface resulting in a phenomenon called flutter [4]. Flutter in aircraft is an oscillation caused by the interaction of aerodynamic forces, structural elasticity, and inertial effects that due to increasing airflow speed, the structural damping becomes insufficient to tolerate the high vibration caused by the aerodynamic forces [5, 6]. As a result, the flutter suppression problem in aircraft has been a relevant research topic in the aeronautical application for decades and many methods in control literature concerning the suppression of flutter phenomenon have been proposed.

The active flutter suppression received great attention in the past through the Active Flexible Wing (AFW) program [7] and Benchmark Active Controls Technology (BACT) wing, in which several control laws as optimal control, multi-rate control, nonlinear control and robust control using performances criteria were employed. In recent years, the literature shows that the synthesis of multivariable controllers that rely on the use of linear parameter-varying (LPV) strategies have been very attractive to solve this problem. This statement is due to the ability to adequately represent some classes of nonlinear systems using a finite set of linear models on a convex hull. To the best of our knowledge, the first contribution using LPV controllers for active flutter suppression problem was published by [8], where a gain-scheduled controller design for active flutter suppression problem using linear fractional representation (LFR) was proposed. It is worth mentioning here that LFR is a powerful description to represent plants subject to nonlinearities and time-varying parameters [9, 10]. Unlike other representations, as gridding modeling [11], the stability is guaranteed, as well as present more general parameter dependencies and uncertainties than the polytopic approach [12, 13]. However,

many of the proposed LPV/LFR approaches employ the concept of quadratic stability. In these approaches, a single parameter-independent Lyapunov function is used, an assumption that generally leads to conservative results [14, 15]. Such conservative results occur because of the absence of restrictions on how fast the parameters may vary [16, 17].Therefore, the search for methods that allow dealing with bounded rates of parameter variation in terms of parameter-dependent Lyapunov (PDL) functions for LFR becomes crucial. In this case, as far as the authors know, there are a few works regarding gain-scheduled controllers design conditions using PDL functions for LFR systems. Most of these works use sophisticated tools such as integral quadratic constraint (IQC) [18] to obtain gain-scheduling synthesis conditions, an approach that can lead to a higher computational effort.
In this context, the paper consists in providing novel gain-scheduled state-feedback controllers to cope with the active flutter suppression of a smart airfoil model. The synthesis conditions are formulated using a generalization of the discrete-time conditions derived in [19, 20] for application in the continuous-time domain. The main difference among the strategies is due to the difficulty in dealing with the time-derivative problem of the PDL functions. Herein, the time-derivative Lyapunov matrix is denoted as belonging to polytopic convex sets, in which the rates of the time-varying parameters vector are chosen to be bounded to allow the existence of a finite LMI condition. In order to evaluate the effectiveness of the proposed condition, the smart airfoil model developed by [21] will be used. This model also was employed in other applications, for instance [22] in which polytopic LPV models were developed. Taking into account the detailed mathematical modeling presented in that work, we generalize one of these polytopic LPV models for LFR.
The remaining of this paper is given as follows. Section 2 presents the smart airfoil model in details. Section 3 addresses the main contribution of this paper, where novel gain-scheduling state-feedback control strategies for LPV/LFR systems are derived. Section 4 illustrates the LFR model of the smart airfoil model, as well as the results of the proposed methods. Section 5 presents our conclusions.

1.1 Notation

The paper notation is mostly standard. $\mathbb{R}^{n\times n}$ is the $n\times n$ dimensional Euclidean space. A^T and A^{-1} denotes, respectively, the transpose and the inverse of matrix A. $A\succ 0$ means that A is a positive-definite matrix; similarly, $A\prec 0$ means that A is a negative-definite matrix. diag$\{.\}$ represents a diagonal matrix with the specified elements and the symbol $\bullet$ represents the transpose elements in the respective symmetric positions.

2. Smart airfoil model

This section closely follows the LPV modeling of a smart airfoil presented in [22]. There the linearized airfoil model is expressed by two differential equations given by

$$\left\{ \begin{array}{c} \begin{bmatrix} m+M & Mx_\alpha \\ Mx_\alpha & I_\alpha \end{bmatrix} \begin{bmatrix} \ddot{h}(t) \\ \ddot{\alpha}(t) \end{bmatrix} + \begin{bmatrix} K_h & 0 \\ 0 & K_\alpha \end{bmatrix} \begin{bmatrix} h(t) \\ \alpha(t) \end{bmatrix} = \begin{bmatrix} 0 \\ mg \end{bmatrix} y(t) + F(t) \\ m\ddot{y}(t) = mg\alpha(t) + u(t) \end{array} \right. \tag{1}$$

where the aerodynamic loading is $F(t)$, m is the moving mass and M the airfoil mass. In this model, we can note that the position of the mass $y(t)$ is assumed to be a control input to airfoil mass, as well as the variable $u(t)$ is given as the control input to the moving mass m. These airfoil movements are illustrated in Figure 1 . Taking into account that the aerodynamic loading $F(t)$ is

$$F(t) = q_a c C_{L_\alpha} \left(\begin{bmatrix} \frac{-1}{V} & 0 \\ \frac{e}{V} & 0 \end{bmatrix} \begin{bmatrix} \dot{h}(t) \\ \dot{\alpha}(t) \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 0 & e \end{bmatrix} \begin{bmatrix} h(t) \\ \alpha(t) \end{bmatrix} \right) \tag{2}$$

and replacing the expression (2) in (1), we obtain a full smart airfoil model. However, it is important to keep in mind that in this study a nondimensionalized model is desired. In this case, using a simple change of variable approach such that $\tau = \omega_\alpha t$, $\bar{e} = e/b$, $\bar{g} = g/\omega_\alpha^2 b$, $\bar{h} = h/b$, $\bar{V} = V/\omega_\alpha b$, which b is a typical section semi-chord, we get the nondimensionalized equations of motion for the smart airfoil

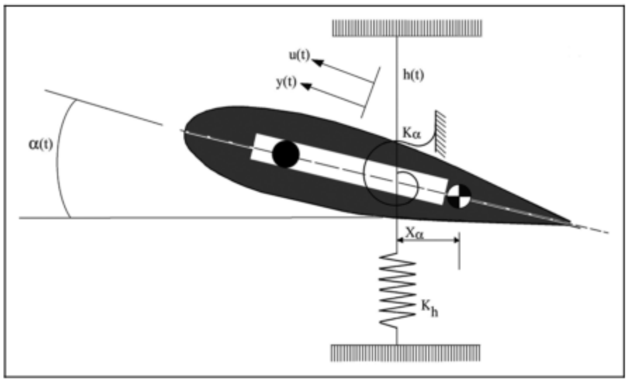

Figure 1 – Configuration of the smart airfoil system [22].

model. Therefore, the smart airfoil model can be rewritten as

$$\begin{cases} \begin{bmatrix} 1+\beta & \bar{x}_\alpha \\ \bar{x}_\alpha & \bar{r}_\alpha^2 \end{bmatrix}\begin{bmatrix}\ddot{\bar{h}}(\tau)\\ \ddot{\alpha}(\tau)\end{bmatrix} + \begin{bmatrix} \frac{2\bar V}{\mu} & 0 \\ \frac{-2\bar V\bar e}{\mu} & 0\end{bmatrix}\begin{bmatrix}\dot{\bar h}(\tau)\\ \dot\alpha(\tau)\end{bmatrix} + \begin{bmatrix} \frac{\omega_h^2}{\omega_\alpha^2} & \frac{2\bar V^2}{\mu} \\ 0 & \frac{-2\bar V^2 \bar e}{\mu}+\bar r_\alpha^2\end{bmatrix}\begin{bmatrix}\bar h(\tau)\\ \alpha(\tau)\end{bmatrix} = \begin{bmatrix}0\\ \beta\bar g\end{bmatrix}\bar y(\tau) \\ \ddot{\bar y}(\tau) = \bar g\alpha(\tau)+\bar u(\tau)\end{cases} \tag{3}$$

By using some mathematical manipulations, we can transform the descriptor equations denoted in (3) into a standard differential equation given by

$$\begin{cases} \begin{bmatrix}\ddot{\bar{h}}(\tau)\\ \ddot{\alpha}(\tau)\end{bmatrix} + \begin{bmatrix} \frac{-2\bar r_\alpha^2\bar V}{q_a\mu} - \frac{-2\bar V\bar e\bar x_\alpha}{q_a\mu} & 0 \\ \frac{2\bar x_\alpha\bar V}{q_a\mu} + \frac{2\bar V\bar e(1+\beta)}{q_a\mu} & 0\end{bmatrix}\begin{bmatrix}\dot{\bar h}(\tau)\\ \dot\alpha(\tau)\end{bmatrix} + \begin{bmatrix} \frac{-\bar r_\alpha^2\omega_h^2}{q_a\omega_\alpha^2} & \Upsilon \\ \frac{\bar x_\alpha\omega_h^2}{q_a\omega_\alpha^2} & \Gamma\end{bmatrix}\begin{bmatrix}\bar h(\tau)\\ \alpha(\tau)\end{bmatrix} = \begin{bmatrix}\frac{-\bar x_\alpha\beta\bar g}{q_a}\\ \frac{(1+\beta)\beta\bar g}{q_a}\end{bmatrix}\bar y(\tau) \\ \ddot{\bar y}(\tau) = \bar g\alpha(\tau)+\bar u(\tau)\end{cases} \tag{4}$$

where the variables Υ, Γ and q are

$$\Upsilon = \frac{-2\bar r_\alpha^2\bar V^2}{q_a\mu} - \frac{2\bar V^2\bar e\bar x_\alpha}{q_a\mu} + \frac{\bar r_\alpha^2\bar x_\alpha}{q_a}, \Gamma = \frac{2\bar x_\alpha\bar V^2}{q_a\mu} + \frac{2\bar V^2\bar e(1+\beta)}{q_a\mu} + \frac{\bar r_\alpha^2(1+\beta)}{q_a}, q_a = -(\bar r_\alpha^2(1+\beta) - \bar x_\alpha^2) \tag{5}$$

and the values of its parameters are defined as in Table 1. In this study case, it can be seen that when the flutter phenomenon occurs, both pitching angle α and the plunging displacement h are feedback such that the position of the mass m is properly adjusted to achieve a reduced flutter. The

Parameter	Value	Parameter	Value
μ	152	$b\ [m]$	0.127
$\bar e$	0.35	$\omega_\alpha\ [rad.s]$	64.1
$\bar x_\alpha$	0.25	$\omega_h\ [rad.s]$	55.9
$\bar r_\alpha^2$	0.388	β	0.01

Table 1 – Parameter and values of the smart airfoil model.

crucial variable to understand the flutter phenomenon is the airspeed $\bar V$ of the smart airfoil. In this case, as the velocity/airspeed is available in real-time, the variable under variation can be adopted as a time-varying parameter. From [22], two time-varying parameters were considered in terms of the airspeed given by the normalized variables $\bar V$ and $\bar V^2$ such that an operation region may be built to cope with the flutter problem. However, it is important to highlight that both time-varying parameters are defined taking into account the same value of airspeed $\bar V$, in which is measured or available in real-time. This assumption can lead to a bad use of the gain-scheduled controllers designed because we will have two controllers operating in two vertices that never will occur in the real world since the linear combination of the airspeed relies on just two extremum points. In this case, an approximation method to deal with the variable $\bar{V}^2$ should be adopted. Although it is clear that an LPV model can be obtained using such approximation in terms of $\bar{V}$, we currently have some frameworks to deal with this issue, for instance, the gridding, polytopic, polynomial, and LFR structures. Differently from [22], we will use the LFR structure to encompass the control theory applied in this paper, as well as a larger range of airspeed lying in the region $\bar{V} \in [4 \;\; 8]$ is adopted. This assumption can be seen in Figure 3, where the eigenvalues evolution with increasing flow velocity is characterized.

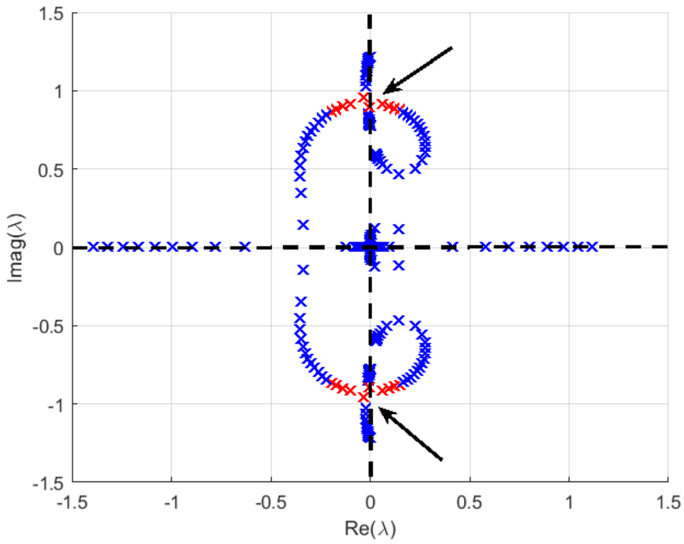

Figure 2 – Eigenvalues evolution with increasing flow velocity for $\bar{V} \in [4 \;\; 8]$.

3. Results

The main concern of this section consists in presenting with relative simplicity the problem statement and the LMI-based conditions for the synthesis of gain-scheduling state-feedback controllers using a general class of LPV systems given by LFR. The proposed conditions rely on the use of scaling matrices that can be reinterpreted as a full-block multipliers description to obtain stabilizing and $\mathscr{L}_2$ controllers.

3.1 Stabilizing problem

Consider the continuous-time LPV system given by

$$\dot{x}(t) = \mathscr{A}(\theta(t))x(t) \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is the states vector and the $\mathscr{A}(\cdot) \in \mathbb{R}^{n\times n}$ is a fixed function of a time-varying parameters vector $\theta(t) = [\theta_1(t), \ldots, \theta_m(t)]^T$ in which is available in real-time and takes values in the polytope $\Omega_N \in \mathbb{R}^m$. Among all the possible mathematical representations of the system (6), we will deal with the LFR [23]:

$$G_{yu} : \begin{cases} \dot{x}(t) = Ax(t) + B_q q(t) \\ p(t) = C_p x(t) + D_{pq} q(t) \end{cases} \tag{7}$$

$$q(t) = \Delta(\theta(t))p(t) \tag{8}$$

where the system G_{yu} in (7) define the linear time-invariant system and $\Delta(\theta(t)) \in \mathbf{\Delta}$ is the time-varying uncertainty structure given by $\mathbf{\Delta} = diag(\theta_1 I_{s_1}, \ldots, \theta_1 I_{s_m})$ that combined satisfy an upper linear fractional transformation $G_{yu}(\Delta(\theta)) = \mathscr{F}_u(G_{yu}, \mathbf{\Delta})$. Herein, the parameter-dependent system $G_{yu}(\Delta(\theta))$ is denoted as an LTI system with the time-varying parameters present in a feedback loop as a diagonal block, as can be seen in Figure 3. Taking into account that $\mathbf{\Delta} = \Delta(\theta)$ for all $\theta \in \Omega_N$ containing the

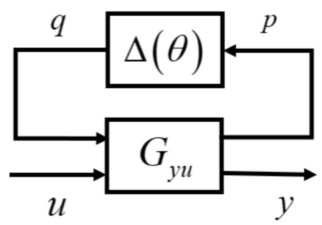

Figure 3 – LFR block diagram of a time-varying parameter $G_{yu}(\Delta(\theta))$ system.

origin, then $\Delta(\theta)$ may be defined by convex combination of Δ_i, $i = 1,\dots,N$, such that

$$\Delta(\theta) = \sum_{i=1}^{N} \xi_i(t)\Delta_i : \xi_i \geq 0, \sum_{i=1}^{N} \xi_i(t) = 1 \tag{9}$$

be satisfied and the variable $\max(s_1,\dots,s_m)$ corresponds to the LFR degree of system for $p(t)$ and $q(t)$ with same dimensions ($n_p = n_q$). Moreover, the representation (7)$-$(8) is described by appropriate matrices $A \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times n_q}$, $C_p \in \mathbb{R}^{n_p\times n}$ and $D_{pq} \in \mathbb{R}^{n_p\times n_q}$, where the well-posed condition $\det(I - D_{pq}\Delta(\theta)) \neq 0$ for all $\theta \in \Omega_N$ is ensured. In this case, the system $G_{yu}(\Delta(\theta))$ can be rewritten as

$$G_{yu}(\mathbf{\Delta}) : \begin{cases} \dot{x}(t) = Ax(t) + B_q\mathbf{\Delta}p(t) \\ p(t) = C_p x(t) + D_{pq}\mathbf{\Delta}p(t) \end{cases} \tag{10}$$

In order to properly address the state-feedback synthesis condition for this class of system, let us first introduce a stability condition based on parameter-dependent Lyapunov function. A novel stability condition for (10) may be given as follows.

Lemma 1 (Stability condition) *Consider the LFR system in* (10)*. If there exist symmetric positive definite matrices $P(\mathbf{\Delta}) \in \mathbb{R}^{n\times n}$ and matrix $\Pi \in \mathbb{R}^{n_p\times(n+n_p)}$ such that the following LMI hold:*

$$\Psi_P + \Phi_\Delta^T \Pi_P + \Pi_P^T \Phi_\Delta \prec 0 \tag{11}$$

where the matrices Ψ_P, Φ_Δ with appropriate dimensions are given by

$$\Psi_P = \begin{bmatrix} A^T P(\mathbf{\Delta}) + P(\mathbf{\Delta})A + P(\dot{\mathbf{\Delta}}) & P(\mathbf{\Delta})B_q\mathbf{\Delta} \\ \bullet & 0 \end{bmatrix}, \; \Phi_\Delta = \begin{bmatrix} C_p & -I + (D_{pq}\mathbf{\Delta}) \end{bmatrix} \tag{12}$$

then system (10) *is asymptotically stable for all $\theta \in \Omega_N$.*

Proof 1 *Consider the following parameter-dependent Lyapunov function*

$$V(x,\mathbf{\Delta}) = x(t)^T P(\mathbf{\Delta})x(t) \tag{13}$$

then the system (10) *is asymptotically stable if there exist symmetric matrices $P(\mathbf{\Delta}) \succ 0$ such that the time-derivative of the Lyapunov function $\dot{V}(x,\mathbf{\Delta}) \prec 0$ along the state trajectory of the system is given by*

$$[Ax(t) + B_q\mathbf{\Delta}p(t)]^T P(\mathbf{\Delta})x(t) + x^T(t)P(\mathbf{\Delta})[Ax(t) + B_q\mathbf{\Delta}p(t)] + x^T(t)P(\dot{\mathbf{\Delta}})x(t) \prec 0 \tag{14}$$

which is equivalent to

$$\begin{bmatrix} x(t) \\ p(t) \end{bmatrix}^T \begin{bmatrix} A^T P(\mathbf{\Delta}) + P(\mathbf{\Delta})A + P(\dot{\mathbf{\Delta}}) & P(\mathbf{\Delta})B_q\mathbf{\Delta} \\ \mathbf{\Delta}^T B_q^T P(\mathbf{\Delta}) & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} \prec 0. \tag{15}$$

Now adopting the conventional scaling technique in terms of full-block multipliers introduced by [13, 12], for any real matrices $\mathbb{G}_\Delta$ and $\mathbb{H}_\Delta$ of compatible dimensions, we get

$$\begin{aligned} x^T(t)\mathbb{G}_\Delta p(t) &= x^T(t)\mathbb{G}_\Delta C_p x(t) + x^T(t)\mathbb{G}_\Delta(D_{pq}\mathbf{\Delta})p(t) \\ p^T(t)\mathbb{H}_\Delta p(t) &= p^T(t)\mathbb{H}_\Delta C_p x(t) + p^T(t)\mathbb{H}_\Delta(D_{pq}\mathbf{\Delta})p(t) \end{aligned} \tag{16}$$

whereby using symmetric matrices properties, the expression (16) *can be recast as*

$$\begin{bmatrix} x(t) \\ p(t) \end{bmatrix}^T \begin{bmatrix} \mathbb{G}_\Delta C_p + C_p^T \mathbb{G}_\Delta^T & \mathbb{G}_\Delta (D_{pq}\mathbf{\Delta}) - \mathbb{G}_\Delta + C_p^T \mathbb{H}_\Delta^T \\ (D_{pq}\mathbf{\Delta})^T \mathbb{G}_\Delta^T - \mathbb{G}_\Delta^T + \mathbb{H}_\Delta C_p & \mathbb{H}_\Delta (D_{pq}\mathbf{\Delta}) + (D_{pq}\mathbf{\Delta})^T \mathbb{H}_\Delta^T - \mathbb{H}_\Delta - \mathbb{H}_\Delta^T \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \end{bmatrix} = 0 \tag{17}$$

Combining the constraints in (15) *and* (17) *into a single stability condition, such that one can obtain the inequation* (11) *for* $\Pi_P = \begin{bmatrix} \mathbb{G}_\Delta^T & \mathbb{H}_\Delta^T \end{bmatrix}$ *as a static multiplier description. Hence, the proof is concluded.*

It can be seen that this characterization for the stability condition of continuous-time LPV/LFR systems has some particularities in terms of computational complexity to deal with the time-derivative of the Lyapunov function. To solve such a problem, we will address later some concepts about the variation rates of the parameters $\theta(t)$ to achieve the stabilizing control conditions. By making use of these approaches, novel existence conditions to obtain gain-scheduled controllers for LPV/LFR systems may be established.

3.1.1 LMI-based conditions for state-feedback controllers

Given Lemma 1, it is possible to obtain an equivalent dual condition. It is well-known that LFR systems can be transformed using the duality property, such that certain issues can be solved using more suitable formulations. The dual stability condition of Lemma 1 is denoted as follows.

Corollary 1 (Dual Stability condition) *Consider the LFR system in* (10)*. If there exist symmetric positive definite matrices* $W(\mathbf{\Delta}) \in \mathbb{R}^{n\times n}$ *and matrix* $\Pi \in \mathbb{R}^{n_q \times (n+n_q)}$ *such that the following LMI hold:*

$$\Psi_W + \Phi_\Delta^T \Pi_W + \Pi_W^T \Phi_\Delta \prec 0 \tag{18}$$

where the matrices Ψ_W*,* Φ_Δ *with appropriate dimensions are given by*

$$\Psi_W = \begin{bmatrix} AW(\mathbf{\Delta}) + W(\mathbf{\Delta})A^T - W(\dot{\mathbf{\Delta}}) & W(\mathbf{\Delta})C_p^T \\ \bullet & 0 \end{bmatrix}, \; \Phi_\Delta = \begin{bmatrix} (B_q\mathbf{\Delta})^T & -I + (D_{pq}\mathbf{\Delta})^T \end{bmatrix} \tag{19}$$

then system (10) *is asymptotically stable for all* $\theta \in \Omega_N$*.*

As this paper deals with the active flutter suppression of a smart airfoil, a stabilizing condition in terms of a state-feedback strategy can be posed. Therefore, consider the following LFR system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_q\mathbf{\Delta}p(t) + B_u u(t) \\ p(t) &= C_p x(t) + D_{pq}\mathbf{\Delta}p(t) + D_{pu}u(t) \end{aligned} \tag{20}$$

where $B_u \in \mathbb{R}^{n\times n_u}$ and $D_{pu} \in \mathbb{R}^{n_p \times n_u}$ allow describing the parameter-dependent input matrix $B(\mathbf{\Delta})$ in terms of fixed matrices. Now adopting the state-feedback control law $u(t) = K_x(\mathbf{\Delta})x(t)$ for the system (20), the closed-loop system may be described by

$$G_x(\mathbf{\Delta}) : \begin{cases} \dot{x}(t) = (A + B_u K_x(\mathbf{\Delta}))x(t) + B_q\mathbf{\Delta}p(t) \\ p(t) = (C_p + D_{pu}K_x(\mathbf{\Delta}))x(t) + D_{pq}\mathbf{\Delta}p(t) \end{cases} \tag{21}$$

We can notice that to obtain a stabilizing state-feedback condition for the system (20), a simple change of variable can be used. However, even doing such mathematical manipulation, the time-derivative problem of the parameter-dependent matrix W is still an unsolved problem. In this case, we resort to variation rates of the parameter $\theta(t)$ to solve this issue. Therefore, taking into account that

$$\dot{\mathbf{\Delta}} = \sum_{j=1}^{M} \lambda_j(t) h_j : \lambda_j \geq 0, \sum_{j=1}^{M} \lambda_j(t) = 1 \tag{22}$$

as a result, we get

$$W(\dot{\mathbf{\Delta}}) = \sum_{i=1}^{N} \dot{\xi}_i(t) W_i = \sum_{i=1}^{N}\sum_{j=1}^{M} \mu_j(t) h_i^j W_i = W(h_j) \tag{23}$$

where more details on this representation can be found in [24]. We can see that $\dot{\mathbf{\Delta}} = \dot{\Delta}(\theta(t))$ exists and satisfies the differential inclusion $\dot{\theta}(t) \in \Omega_M$, such that Ω_M is also a convex polytope set $co\{h_1, \ldots, h_M\}$. Moreover, it is important to point out that the vectors used to define Ω_M can not be chosen arbitrarily. In this sense, such vectors can be built from the bounded rate of parameter variation, i.e. the magnitude of the time-derivative of $\dot{\theta}(t)$.
As a result, a finite LMI-based condition for state-feedback control of continuous-time LPV/LFR systems can be derived. It can also be seen that combining both LFR and polytopic representations may lead to powerful and straightforward synthesis conditions. The following theorem addresses the resulting combination between the LFR and polytopic frameworks.

Theorem 1 (Gain-scheduling control condition) *Consider the LFR system in* (20)*. If there exist symmetric positive definite matrices* $W_i \in \mathbb{R}^{n\times n}$ *and matrices* $Z_i \in \mathbb{R}^{n_u \times n}$*,* $\Pi \in \mathbb{R}^{n_q \times (n+n_q)}$ *such that the following LMI hold:*

$$\Psi_{W_{i,j}} + \Phi_{\Delta_i}^{\;T} \Pi_W + \Pi_W^T \Phi_{\Delta_i} \prec 0 \tag{24}$$

where the matrices $\Psi_{W_{i,j}}$*,* Φ_{Δ_i} *with appropriate dimensions given by*

$$\Psi_{W_{i,j}} = \begin{bmatrix} AW_i + W_i A^T + B_u Z_i + Z_i^T B_u^T - W(h_j) & W_i C_p^T + Z_i^T D_{pu}^T \\ \bullet & 0 \end{bmatrix}, \; \Phi_\Delta = \begin{bmatrix} (B_q \Delta_i)^T & -I + (D_{pq}\Delta_i)^T \end{bmatrix} \tag{25}$$

for all $i = 1, \ldots, N$ *and* $j = 1, \ldots, M$ *then system* (20) *is asymptotically stabilized by a gain-scheduled controller* $K_{xi} = Z_i W_i^{-1}$ *for all* $\theta \in \Omega_N$ *and* $\dot{\theta} \in \Omega_M$*.*

Proof 2 *Using the dual form for stability condition given in Corollary 1 and applying a change of variables* $Z(\mathbf{\Delta}) = K_x(\mathbf{\Delta}) W(\mathbf{\Delta})$ *in order to avoid the coupling among the variables, we prove the sufficient of the condition. Hence, the proof is complete.*

Since a stabilizing condition was provided, a straightforward contribution consists of deriving LMI-based conditions subject to a given performance index. In this case, an appropriate criterion is the induced $\mathscr{L}_2$ performance, once it is desired to tolerate disturbances and noises in the active flutter suppression design.

3.2 Induced $\mathscr{L}_2$ performance

Consider the closed-loop LPV system given by

$$T_{z\omega} : \begin{cases} \dot{x}(t) = Ax(t) + B_q q(t) + B_\omega \omega(t) \\ p(t) = C_p x(t) + D_{pq} q(t) + D_{p\omega}\omega(t) \\ z(t) = C_z x(t) + D_{zq} q(t) + D_{z\omega}\omega(t) \end{cases} \tag{26}$$

$$q(t) = \Delta(\theta(t)) p(t), \tag{27}$$

where $A \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times n_q}$, $B_\omega \in \mathbb{R}^{n \times n_\omega}$, $C_p \in \mathbb{R}^{n_p \times n}$, $D_{pq} \in \mathbb{R}^{n_p \times n_q}$, $D_{p\omega} \in \mathbb{R}^{n_p \times n_\omega}$,$C_z \in \mathbb{R}^{n_z \times n}$, $D_{zq} \in \mathbb{R}^{n_z \times n_q}$ and $D_{zu} \in \mathbb{R}^{n_z \times n_\omega}$. Herein the signals $z(t)$ and $\omega(t)$ correspond to exogenous outputs and exogenous inputs, such that the integers may be defined as $m_p = n + n_p + n_\omega$ and $m_q = n + n_q + n_z$. In this case, analogously to stabilizing problem section, we can represent the closed-loop LFR system in terms of a block diagram, as can be seen in Figure 4.
Therefore, using the well-known Bounded Real Lemma (BRL) for LPV systems, we can ensure that the closed-loop system (26) and (27) is asymptotically stable and satisfy the induced $\mathscr{L}_2$ norm for all $\theta \in \Omega_N$ and $\dot{\theta} \in \Omega_M$. In this sense, an appropriate description for closed-loop system should be posed to facility the comprehension. Following the early section, the $T_{z\omega}(\Delta(\theta))$ system can be rewritten as

$$T_{z\omega}(\mathbf{\Delta}) : \begin{cases} \dot{x}(t) = Ax(t) + B_q \mathbf{\Delta} p(t) + B_\omega \omega(t) \\ p(t) = C_p x(t) + D_{pq} \mathbf{\Delta} p(t) + D_{p\omega}\omega(t) \\ z(t) = C_z x(t) + D_{zq} \mathbf{\Delta} p(t) + D_{z\omega}\omega(t) \end{cases} \tag{28}$$

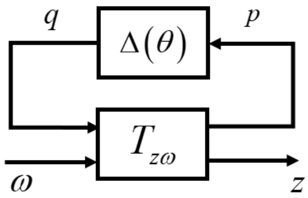

Figure 4 – LFR block diagram of a closed-loop $T_{z\omega}(\Delta(\theta))$ system.

and a novel induced $\mathscr{L}_2$ performance condition for LPV/LFR systems may be established. Differently from the other conditions, such a condition allows coping with the time-derivative Lyapunov function from the bounded rate of parameter variation knowledge in polytopic representation. Thus the novel induced $\mathscr{L}_2$ norm condition for (28) may be given as follows.

Lemma 2 (Induced $\mathscr{L}_2$ norm condition) *Consider the closed-loop LFR system in* (28)*. If there exist symmetric positive definite matrices $P(\mathbf{\Delta}) \in \mathbb{R}^{n\times n}$ and matrix $\Pi \in \mathbb{R}^{n_p \times m_p}$ such that the following LMI hold:*

$$\begin{cases} \min\limits_{P} \gamma, \text{ subject to} \\ \Psi_P + \Phi_\Delta^T \Pi \Xi + \Xi^T \Pi^T \Phi_\Delta \prec 0 \end{cases} \tag{29}$$

where the matrices Ψ_P, Φ_Δ and Ξ with appropriate dimensions are given by

$$\begin{aligned} \Psi_P &= \begin{bmatrix} A^T P(\mathbf{\Delta}) + P(\mathbf{\Delta})A + P(\dot{\mathbf{\Delta}}) & P(\Delta)B_q\mathbf{\Delta} & P(\mathbf{\Delta})B_\omega & C_z^T \\ \bullet & 0 & 0 & D_{zq}^T\mathbf{\Delta} \\ \bullet & \bullet & -\gamma I & D_{z\omega}^T \\ \bullet & \bullet & \bullet & -\gamma I \end{bmatrix}, \Xi = \begin{bmatrix} I & 0 \end{bmatrix} \\ \Phi_\Delta &= \begin{bmatrix} C_p & -I + D_{pq}\mathbf{\Delta} & D_{p\omega} & 0 \end{bmatrix}, \end{aligned} \tag{30}$$

then system (28) *is asymptotically stable and $\mathscr{L}_2$ upper bound can be found from*

$$\|T_{z\omega}(\mathbf{\Delta})\|_{i,2} \leq \inf \gamma \tag{31}$$

Proof 3 *The induced $\mathscr{L}_2$ performance for LPV systems may be found from the bounded real lemma,*

$$\dot{V}(x, \mathbf{\Delta}) + \gamma^{-1} z^T(t) z(t) - \gamma \omega^T(t) \omega(t) \prec 0 \tag{32}$$

where the time-derivative of the Lyapunov function is given by (13) *and γ the upper bound for the $\mathscr{L}_2$ norm. Developing the expression in* (32)*, we obtain*

$$\dot{x}^T(t) P(\mathbf{\Delta}) x(t) + x^T(t) P(\dot{\mathbf{\Delta}}) x(t) + x^T(t) P(\mathbf{\Delta}) \dot{x}(t) + \gamma^{-1} z^T(t) z(t) - \gamma \omega^T(t) \omega(t) \prec 0 \tag{33}$$

which it can be recast in a matricial form, such that

$$\begin{bmatrix} x(t) \\ p(t) \\ \omega(t) \end{bmatrix}^T \begin{bmatrix} A^T P(\mathbf{\Delta}) + P(\mathbf{\Delta})A + P(\dot{\mathbf{\Delta}}) & P(\mathbf{\Delta})B_q\mathbf{\Delta} & P(\mathbf{\Delta})B_\omega & C_z^T \\ (B_q\mathbf{\Delta})^T P(\mathbf{\Delta}) & 0 & 0 & D_{zq}^T\mathbf{\Delta} \\ B_\omega^T P(\mathbf{\Delta}) & 0 & -\gamma I & D_{z\omega}^T \\ C_z & D_{zq}\mathbf{\Delta} & D_{z\omega} & -\gamma I \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \\ \omega(t) \end{bmatrix} \prec 0. \tag{34}$$

By using the same description presented in the previous section for the time-varying parameters, in which for any real matrices $\mathbb{G}_\Delta \in \mathbb{R}^{n\times n_p}$, $\mathbb{H}_\Delta \in \mathbb{R}^{n_p\times n_p}$ and $\mathbb{J}_\Delta \in \mathbb{R}^{n_p \times n_\omega}$, one can deduce from [12] the following conditions:

$$\begin{aligned} &x^T(t)\mathbb{G}_\Delta p(t) - x^T(t)\mathbb{G}_\Delta (C_p x(t) + (D_{pq}\mathbf{\Delta})p(t) + D_{p\omega}\omega(t)) \equiv 0 \\ &p^T(t)\mathbb{H}_\Delta p(t) - \left(x^T(t)C_p^T + p^T(t)(D_{pq}\mathbf{\Delta})^T + \omega^T(t)D_{p\omega}^T\right)\mathbb{H}_\Delta p(t) \equiv 0 \\ &p^T(t)\mathbb{J}_\Delta \omega(t) - \left(x^T(t)C_p^T + p^T(t)(D_{pq}\mathbf{\Delta})^T + \omega^T(t)D_{p\omega}^T\right)\mathbb{J}_\Delta \omega(t) \equiv 0 \end{aligned} \tag{35}$$

Now stacking the vectors x, p and ω in (35) *in order to combine with* (34)*, it is possible to compose* (29) *into a single condition taking into account that $\Pi = \begin{bmatrix} \mathbb{G}_\Delta^T & \mathbb{H}_\Delta & \mathbb{J}_\Delta \end{bmatrix}$ is a general full-block multiplier. Hence, the bounded real lemma for continuous-time LPV/LFR systems is achieved and the proof is complete.*

As aforementioned in the early section, the goal here consists in providing existence LMI conditions for the synthesis of gain-scheduled state-feedback controllers. In this case, an augmented plant with the exogenous signals ω and z with an appropriate control law should be addressed. As a result, it follows the LPV system

$$G_{z\omega}: \begin{cases} \dot{x}(t) = Ax(t) + B_q q(t) + B_u u(t) + B_\omega \omega(t) \\ p(t) = C_p x(t) + D_{pq} q(t) + D_{pu} u(t) + D_{p\omega}\omega(k) \\ z(t) = C_z x(t) + D_{zq} q(t) + D_{zu} u(t) + D_{z\omega}\omega(t) \end{cases} \tag{36}$$

$$q(t) = \Delta(\theta(t))p(t) \tag{37}$$

under a determined state-feedback control law $u(t) = K_x(\mathbf{\Delta})x(t)$ with $K_x(\mathbf{\Delta}) \in \mathbb{R}^{n_u \times n}$ being a parameter-dependent gain. As a consequence, the closed-loop system $T_{z\omega}(\mathbf{\Delta})$ may be recast by

$$\begin{aligned} x(t) =& (A + B_u K_x(\mathbf{\Delta}))x(t) + B_q \mathbf{\Delta}(t)p(t) + B_\omega \omega(t) \\ p(t) =& (C_p + D_{pu} K_x(\mathbf{\Delta}))x(t) + D_{pq}\mathbf{\Delta}(t)p(t) + D_{p\omega}\omega(t) \\ z(t) =& (C_z + D_{zu} K_x(\mathbf{\Delta}))x(t) + D_{zq}\mathbf{\Delta}(t)p(t) + D_{z\omega}\omega(t) \end{aligned} \tag{38}$$

Similarly to stabilizing problem, a dual condition for the induced $\mathscr{L}_2$ norm condition provided in Lemma 2 can be derived. Concerning the brevity of the contributions of this work, we choose to omit this extended condition to present directly the application of the dual condition to obtain the $\mathscr{L}_2$ gain-scheduled state-feedback controller synthesis. In this condition, the same assumptions regarding the time-derivative problem of the parameter-dependent matrix W will be employed. Therefore, the bounded rate of the parameter variation knowledge is crucial to reach feasible solutions. Then the formulation given in (23) to deal with the time-derivative Lyapunov function is addressed. In this sense, it follows the $\mathscr{L}_2$ gain-scheduling control condition in Theorem 2.

Theorem 2 ($\mathscr{L}_2$ gain-scheduling control condition) *Consider the LFR system in* (38). *If there exist symmetric positive definite matrices $W_i \in \mathbb{R}^{n\times n}$ and matrices $Z_i \in \mathbb{R}^{n_u\times n}$, $\Pi \in \mathbb{R}^{n_q\times m_q}$ such that the following LMI hold:*

$$\begin{cases} \min\limits_{W_{i,j}, Z_i} \gamma, \text{ subject to} \\ \Psi_{W_{i,j}} + \Phi_{\Delta_i}^T \Pi \Xi + \Xi^T \Pi^T \Phi_{\Delta_i} \prec 0 \end{cases} \tag{39}$$

where the matrices $\Psi_{W_{i,j}}$, Φ_{Δ_i} and Ξ with appropriate dimensions are given by

$$\Psi_{W_{i,j}} = \begin{bmatrix} W_i A^T + AW_i + B_u Z_i + Z_i^T B_u^T - W(h_j) & W_i C_p^T + Z_i^T D_{pu}^T & W_i C_z^T + Z_i^T D_{zu}^T & B_\omega \\ \bullet & 0 & 0 & D_{p\omega} \\ \bullet & \bullet & -\gamma I & D_{z\omega} \\ \bullet & \bullet & \bullet & -\gamma I \end{bmatrix}, \Xi = \begin{bmatrix} I & 0 \end{bmatrix}$$
$$\Phi_{\Delta_i} = \begin{bmatrix} (B_q\Delta_i)^T & -I + (D_{pq}\Delta_i)^T & (D_{zq}\Delta_i)^T & 0 \end{bmatrix} \tag{40}$$

for all $i = 1, \ldots, N$ and $j = 1, \ldots, M$ then system (38) *is asymptotically stabilized with guaranteed $\mathscr{L}_2$ norm by a gain-scheduled controller $K_{xi} = Z_i W_i^{-1}$ for all $\theta \in \Omega_N$ and $\dot{\theta} \in \Omega_M$. The $\mathscr{L}_2$ upper bound can be found from*

$$\|T_{z\omega}(\mathbf{\Delta})\|_{i,2} \leq \inf \gamma \tag{41}$$

Proof 4 *The proof follows the application of the change of variables $Z(\mathbf{\Delta}) = K_x(\mathbf{\Delta})W(\mathbf{\Delta})$ into the dual $\mathscr{L}_2$ condition that can be easily obtained from the condition* (39) *for the system* (28).

It is important to keep in mind that the proposed method makes use of parameter-dependent Lyapunov functions in combination with scaling techniques to obtain less conservative results, as well as incorporates a time-varying input matrix in its formulation without additional pre-compensators for the gain-scheduling control strategy. This strategy will be addressed in the next section for the active flutter suppression problem of a smart airfoil model.

4. Design and Results

In this section, two active flutter suppression designs addressed by gain-scheduled controller syntheses are provided to illustrate the effectiveness of the proposed method. The first one deals with the stabilization problem by Theorem 1, while the second design handles induced $\mathscr{L}_2$ performance denoted by Theorem 2. Both syntheses were implemented using the following software suites: MATLAB$^{®}$ software, version 9.9.0 R2022b, Yalmip [25] and SeDuMi [26]. As a consequence, an appropriate formulation of the smart airfoil in the LFR should be built. Therefore, the smart airfoil model can be described as

$$G(\mathbf{\Delta}):\begin{cases} \dot{x}(t)=Ax(t)+B_q q(t)+B_u u(t) \\ p(t)=C_p x(t)+D_{pq}q(t)+D_{pu}u(t) \text{ with } x\in\begin{bmatrix} \bar{h} & \alpha & \dot{\bar{h}} & \dot{\alpha} & \bar{y} & \dot{\bar{y}} \end{bmatrix}^T \\ y(t)=C_y x(t)+D_{yq}q(t)+D_{yu}u(t) \text{ and } x_0=\begin{bmatrix} 4\times 10^{-3} & 0 & 0 & 0 & 0.2 & 0 \end{bmatrix}^T \end{cases} \tag{42}$$

$$q(t)=\Delta(\theta(t))p(t) \tag{43}$$

where the state-space representation of this system class is given by

$$\begin{bmatrix} \dot{x}(t) \\ p(t) \\ y(t) \end{bmatrix} = \left[\begin{array}{cccccc|c|c} 0 & -0.0531 & 0 & 0 & 0 & -0.9986 & 0 & 0 \\ 0.0552 & 0 & -0.8942 & 0 & 0.8281 & 0 & -0.1484 & 0 \\ 0 & 0.9986 & 0 & 0 & 0 & -0.0531 & 0 & 0 \\ 0 & 0 & 0.5780 & 0 & -1.8670 & 0 & 0.1882 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0188 & 0 & 0 & 1 \\ \hline 0 & 0.1280 & 0 & 0 & 1.4084 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} x(t) \\ q(t) \\ u(t) \end{bmatrix} \tag{44}$$

in which the time-varying uncertainty structure is $\mathbf{\Delta}=\theta(t)$ with $4\leq\theta(t)\leq 8$ and the bounded rate of variation as $|\dot{\theta}(t)|\leq 4$. Moreover, for this numerical example, the variation of the airspeed $\bar{V}$ was choose to be $\bar{V}=2\sin(2\tau)+6$ in order to evaluate the effectiveness of the proposed conditions, even knowing that in practical experiments a smooth variation of the airspeed is recommended. In Figure 5, the behavior of the airspeed variation and its respective bounded rate of variation is presented.

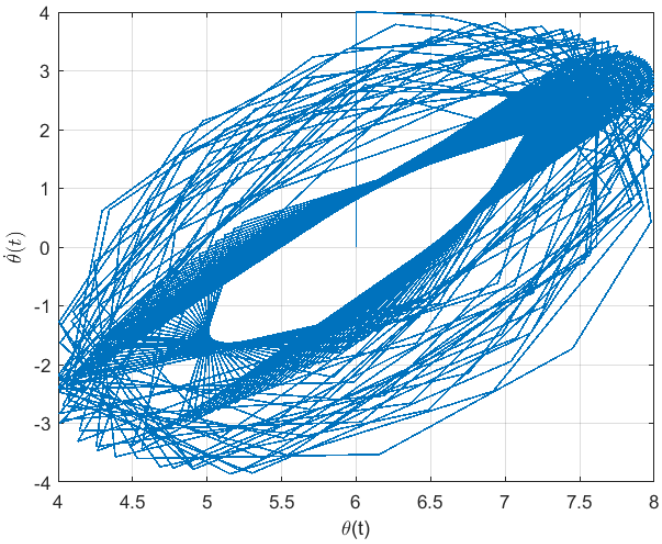

Figure 5 – Time-varying parameters results.

Considering the information about the time-varying parameter θ and its bounded rate variation, the convex sets Ω_N and Ω_M can be built, as well as the convex combinations of the vectors h^j for $j=1,\ldots,M$. Following [24] the derivative of $\theta(t)$ with respect to time, denoted by $\dot{\theta}(t)\in\mathbb{R}^N$ exists and belongs to the convex set such that

$$\Omega_M = \left\{ \delta \in \mathbb{R}^N : \ \delta = co\{h^1, \ldots, h^M\}, \sum_{i=1}^{N} h_i^j = 0, j = 1, \ldots, M \right\}. \tag{45}$$

Therefore, a feasible vector representation of the bounded rate variation $\dot{\theta}(t) \leq 4$ may be given by

$$H = [h^1 \mid h^2\ ] = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \in \Omega_M, \tag{46}$$

in which the sum of the lines of h^j is null. As a result, both LMI-based conditions derived in Theorems 1 and 2 can be solved, yielding two gain-scheduled state-feedback controllers to tolerate the flutter phenomenon in a smart airfoil system. Each one of the controllers designed has its objective. The first parameter-dependent controller tackles the stabilization of the airfoil system while the second ensures the induced $\mathscr{L}_2$ performance. Nevertheless, both controller's structures are the same and can be described as

$$K_x(\Delta(\theta)) = \sum_{i=1}^{2} \xi_i(t) K_{xi} \tag{47}$$

for all $\theta \in \Omega_N$ and $\dot{\theta} \in \Omega_M$. In this sense, by using the Theorem 1, the vertices K_{xi} of the gain-scheduled controller used to deal with the stabilization problem are given by

$$K_{x1} = \begin{bmatrix} 2.6902 & 63.9108 & 22.1527 & 36.4104 & -16.5064 & -2.1995 \end{bmatrix} \tag{48}$$

$$K_{x2} = \begin{bmatrix} 2.8750 & 91.5221 & 49.4587 & 104.9901 & 7.7203 & -2.5142 \end{bmatrix} \tag{49}$$

It can be seen that both plunging displacement and pitching angle were stabilized, as well as the moving mass (Figure 6). However, the settling time for the variables $\bar{h}$ and α has presented a little long (around 50s). Unfortunately, this control design does not allow accomplishing the improvement of the results in terms of performance, only a stabilizing solution. In this case, resorting to Theorem 2, a gain-scheduled controller subject to $\mathscr{L}_2$ performance can be found. For this design, we use the matrices $B_w = \begin{bmatrix} 0 & 0 & 100 & 100 & 0 & 100 \end{bmatrix}^T$, $C_z = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2\times 10^{-5} \end{bmatrix}$ and $D_{pw} = D_{zw} = D_{zq} = 0$. Therefore, following the previous approach, the vertices K_{xi} of the $\mathscr{L}_2$ controller obtained were

$$K_{x1} = \begin{bmatrix} 4.9358 & 474.3635 & 235.9008 & 320.0055 & -69.6902 & -8.1378 \end{bmatrix} \tag{50}$$

$$K_{x2} = \begin{bmatrix} 4.4965 & 530.3966 & 425.5310 & 783.4239 & 202.8148 & -10.4644 \end{bmatrix} \tag{51}$$

Using this theorem, we obtain an upper bound for closed-loop system $\|T_{z\omega}(\boldsymbol{\Delta})\|_{i,2} \leq 0.3689$ and an improvement regarding the settling time for the plunging displacement and pitching angle, keeping almost the same control effort as used for the stabilization design. These comparison results may be verified from Figure 7. Finally, to evaluate the conservatism of the proposed conditions, we compare the result obtained for $\mathscr{L}_2$ performance with the conditions proposed by [12], in which quadratic methods are derived. Using the quadratic approach, i.e. adopting the matrix W in (39) as constant, we get an upper bound for the closed-loop system $\|T_{z\omega}(\boldsymbol{\Delta})\|_{i,2} \leq 0.5914$. Hence, the proposed conditions provide better and less conservative results than existing conditions in the control literature.

5. Conclusion

This paper presented new gain-scheduling control strategies for continuous-time LPV/LFR systems. The importance of these strategies arises as alternative methods in terms of a lack of works on this topic. The distinctive formulation of the proposed controller designs is given by using the combination of the polytopic and LFR descriptions, which allowed encompassing the parameter-dependent Lyapunov functions and static full-block multipliers structures in its characterization. As the LFR framework denotes the LPV system through fixed matrices interconnected to a time-varying diagonal matrix, the association between the polytopic Lyapunov matrices and the state-space representation allowed to become the problem feasible. This fact can be useful in the control literature

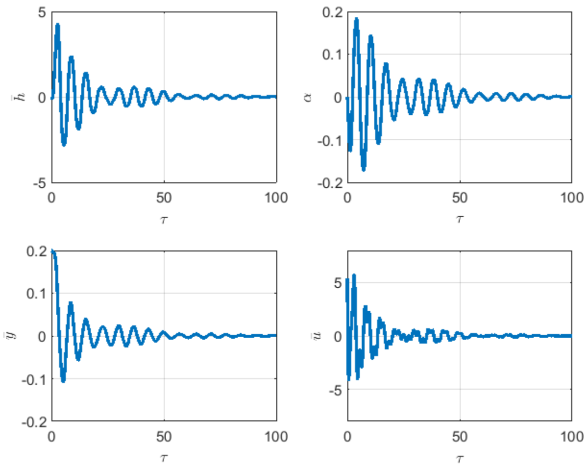

Figure 6 – Time response for the stabilization control design.

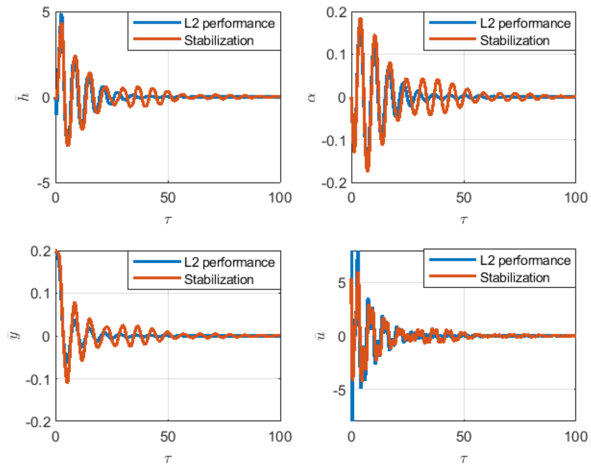

Figure 7 – Time response for the $\mathscr{L}_2$ control design.

since time-varying parameters in all linear system can be addressed without workarounds like pre-compensators. These properties are specifically significant due to real-world applications and the dependency of the time-varying parameters on dynamic system matrices that lead to nonconvex problems. Moreover, it can be seen that the simulation results showed the advantages and effectiveness of the proposed LPV control technique for flutter suppression in a smart airfoil model.

6. Contact Author Email Address

The authors' contact e-mails are: renanrlp@ita.br, pedro.gonzalez@dlr.de and flavio.silvestre@tu-berlin.de

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8. Copyright Statement

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