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A METHOD FOR EVALUATING AIRCRAFT DEPENDENCIES USING THE CLOSEST POINT OF APPROACH METHODOLOGY TO SUPPORT ATM OPERATIONS

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Abstract

The technologies such as enhanced arrival systems support increased traffic density. In the ATM, whilst this innovation promotes the growth of aviation demand, it can lead up to an increasing of ATC workload, due to the definition of separation minima still based on the current surveillance technology and the limitations in traffic operations. The Closest Point of Approach (CPA) is an essential factor that characterizes a potential conflict in a three-dimensional airspace, especially when the plane is preventing the loss of separation or is avoiding occurring collision risks.

This paper aims describing a methodology for the CPA estimation, while calculating appropriate approach geometry parameters, such as the Time to Closest Point of Approach (TCPA), and the Distance at CPA (DCPA). Additionally, it analyses the sensitivity of CPA with respect to the different approach geometries. TCPA and DCPA parameters describe the approach condition with respect to the highest priority vehicle and/or with respect to all the interacting vehicles, and determine the dependency between the probed aircraft and the traffic vehicles detected as potentially interacting with it, resulting useful to support the problem resolution proposal, and to provide, on demand with reference to the probed vehicle, the Controllers (Planning and Executive) with enhanced situational awareness.

Keywords: Closest Point of Approach, Conflict Detection, Air Traffic Management

1. Introduction

The introduction of innovative technologies in ATM contributes to increase the Aviation demand by supporting a higher traffic density with new different typologies of airspace users and aircraft operations. NextGen [1] in the US and European SESAR [2] Programmes aim at achieving innovative systems to ensure aircraft remain functional and collision-free. Free flight concept is a crucial topic in this new perspective of future ATM. Even if free flight will eliminate the needs for ATC by providing pilot with the freedom to select their path and speed in real time, the definition of separation minima between aircraft is still based on the current surveillance performances. By definition, conflict occurs when the distance between two aircraft violates the minimum allowed separation. Particularly, the aircraft separation requirement is defined by a minimum horizontal distance and a minimum vertical distance that the aircraft have to maintain. A safely separation between the pair of aircraft is ensured by a suitable vertical design on entry levels. In addition, according to ICAO regulation [3], two aircraft are considered to be in a conflict if their horizontal separation is less than the minimum aircraft separation and if their vertical separation is less than 1000 ft. The minimum separation standards are given in Table 1.

Table 1 Separation Minima in MN. [3]

Trailing Aircraft	Category	Leading Aircraft		
		Heaving	Medium	Light
	Heaving	4	3	3
	Medium	5	3	3
	Light	6	5	3

These requirements define a protected zone surrounding each aircraft. A loss of separation between aircraft is an overlapping of the aircraft protected zones. A conflict is a predicted loss of separation within a lookahead time interval. The time interval is considered to envisage the path of a probed aircraft and the potential dependencies inside the defined conflict area within the considered time window (ΔT), namely the time horizon, between the aircraft probed and the surrounding controlled traffic sector. Actually, the time interval, within which the prediction vectors are usually visualized on ATC system, can be up to 12 minutes. During the normal planning assurance operations, if a potential problem is detected (e.g. conflict), the current procedures are expected to provide an assessment of entry and exit conditions, and identify the traffic related problems to be reported to tactical controller if need. In a new vision of planning operating mode, that triggering event, if occurs, is identified by automated detection systems, i.e. Conflict Detection (CD) that can contribute to reduce the number of fault short-mid term conflict alert for ATCOs and support the implementation of future free flight concept, so as to reduce the fuel consumption and CO_2 emission. CD is generally analyzed on three different levels: long-term, mid-term, short-term. In long-term, CD involves trajectory planning and airline scheduling. Mid-term conflict detection (MTCD) is generally carry out by ATCO by using semi-automatic tools over a time horizon of 10 minutes. The short-term CD focuses on a time scale of seconds on minutes, within which the detection must be dealt with immediately. ATCO and pilots are assisted by Short Term Conflict Alert (STCA) and Traffic Collision Avoidance System (TCAS) systems. This paper will not focus on the description of above mentioned CD systems, but the MTCD will be considered with the lookahead time of 5-20 minutes. In this case a detection of dependencies among all aircraft in conflict area within the time horizon can be useful to support the problem resolution proposal, maintaining the same current procedures. Such a dependency can be evaluated based on the calculation of the time remaining to reach the closest approach configuration between the probed aircraft and the traffic vehicles at the same altitude, detected as potentially interacting with it in the protected volume around aircraft. Thus, an important key for the flight operations is to evaluate the geometric parameters of a configuration within which a conflict can occur. Closest Point of Approach (CPA) is a key concept in algorithmic aspects for MTCD.

CPA was firstly studied in maritime domain for vessel CD [4,5], and then also for aircraft [6-9]. Furthermore, as a crucial factor for separation assurance and collision avoidance, CPA is widely used in the aeronautical domain to obtain an improved use of the ATM systems and assess the navigational risk [10-15].

This paper aims to propose a data processing methodology for CPA estimation, while calculating appropriate approach geometry parameters, such as the Time to CPA (TCPA), and the Distance at CPA (DCPA). The intent is here to describe the CPA from a mathematical point of view, starting from the trajectory propagation of moving aircraft assumed here as a mass point along a straight trajectory with a constant velocity vector at the same altitude. Additionally, it analyses the sensitivity of CPA with respect to the highest priority vehicles detected as potentially interacting with it, resulting useful to support the problem resolution proposal, and to provide, on demand with reference to the probed vehicle, the planning and executive ATCOs with enhanced situational awareness. A small-scale experiment is carried out through generation of trajectories propagation for many different scenarios within the Italian airspace. The dataset used for the scenarios is based on real trajectories data by observing a specific traffic in a predefined different time interval. This allows to extract a more real aircraft path, and specify the airspace, aircraft type, flight phase, and so on. The scenarios include both extended controlled area and small entry/exit area, in order to test the system performance. The validation of the methodology is focused on the CPA sensitivity analysis and evaluation of the behavior of CPA with respect to different approach geometries, quantifying the DCPA and TCPA variation as a consequence of variation of the surrounding aircraft speeds, and, finally, delivering the early dependencies information provision to the air traffic controllers.

2. Closest Point of Approach (CPA)

In this Section, trajectories propagation is described with reference to their approximation. Then, a description of CPA problem is reported.

2.1 Aircraft trajectories and polyline approximation

In ATM, a trajectory describes the motion of aircraft in a 2 or 3-dimensional airspace. Although in real word aircraft have smooth trajectories, their paths can be analytically built through the linear interpolation of the given waypoints in suitable coordinates system. Thus, the trajectory is discretized by points in order to approximate it through a set of straight-lined segments. A polyline in xy plane approximation of a trajectory connects aircraft positions, sampled at discrete time instances, by line segments. Such a trajectory can be also presented as a sequence $\mathrm{p}\langle(t_1, \vec{v}_1), (t_2, \vec{v}_2), \ldots (t_\mathrm{n}, \vec{v}_\mathrm{n})\rangle$ where $\vec{v}_\mathrm{i}$ represents the position vector of the aircraft at time instance t_i .

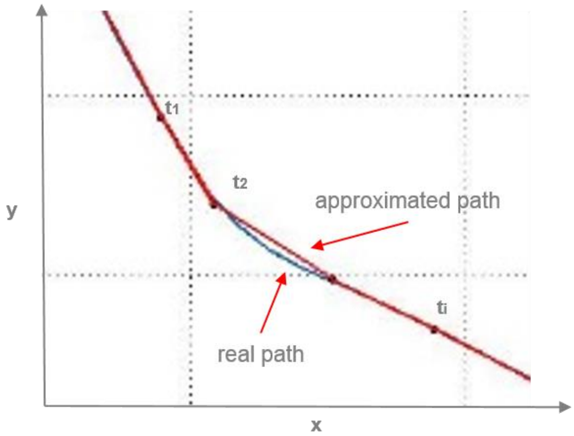

Figure 1 – Trajectory of an aircraft (blue line) vs its polyline approximation (red line).

Many consolidated study and research activities investigated on interpolated methods and related issues [16]. From a mathematical point of view, cubic interpolation technics are usually adopted for aircraft trajectory generation in TMA in order to insure a continuous trajectory curvature [17]. Given a dataset of (n+1) points, it is possible to interpolate them by a polynomial of degree n, defined as p_n that represents the curve of a spline constrained to interpolate those given points. For the trajectory interpolations, a generic polynomial p_n of order n can be written in the following canonic formula:

$$p_n = f(x) = \sum_{i=0}^{n} a_i x^i \qquad (1)$$

with coefficients a_i. The conditions for a polynomial interpolation expressed in the canonical basis are:

$$\sum_{i=0}^{n} a_i {x_j}^i = y_{i,} \qquad {}_{j=0,\ldots,n} \qquad (2)$$

The implementations of interpolation algorithms, such as the piecewise cubic Hermite interpolating polynomial provided many benefits particularly for aircraft arrival phase and operations [18-20].

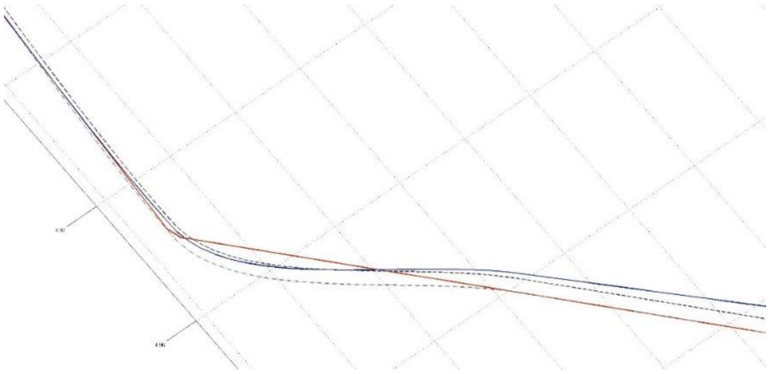

Figure 2 – Trajectory of an aircraft (blue line) vs its polyline approximation (red line) and curved interpolating polynomial techniques (dotted black lines).

The demonstrated procedures allow a curved path, resulting in a predictable and repeatable ground track during a turning. Its implementation preserves the dispersion of tracks.

The analysis of the benefits and issues for different interpolation technics is out-of-scope of the paper. Only trajectories with linear approximations will be considered in this paper. Furthermore, this study is based on flight cruise phase when aircraft speed, heading, and altitude do not change and the path angle is zero along the flight segments. The choice has been done in order to simplify the computational operations, and does not limit the performance of the proposed algorithm if it would be based on a more complicated approximation.

Particularly, in three-dimensional geographic coordinates system, let A = [A_1, A_2… A_N] be the set of all aircraft in a defined time horizon. Initially, flights independently plan their trajectories according to their flight plans without considering mutual interferences. For aircraft A_i and A_j, their 4D trajectories within the sector of interest, can be denoted as lists of waypoints:

$$\begin{aligned} p_i(t_0^i, t_n^i) &= [p_i(t_0^i, \varphi_0^i, \lambda_0^i, h_0^i), p_i(t_1^i, \varphi_1^i, \lambda_1^i, h_1^i), \dots, p_i(t_n^i, \varphi_n^i, \lambda_n^i, h_n^i)] \\ p_j(t_0^j, t_n^j) &= [p_j(t_0^j, \varphi_0^j, \lambda_0^j, h_0^j), p_j(t_1^j, \varphi_1^j, \lambda_1^j, h_1^j), \dots, p_j(t_n^j, \varphi_n^j, \lambda_n^j, h_n^j)] \end{aligned} \quad (3)$$

where $p_i(t_0^i, \varphi_0^i, \lambda_0^i, h_0^i)$ is the first waypoint of A_i at the time t_0^i with coordinates φ^i , λ^i , h^i that are latitude, longitude and altitude respectively, and $p_i(t_n^i, \varphi_n^i, \lambda_n^i, h_n^i)$ is the last waypoint where A_i exits from the boundary of the interested airspace. The waypoint list is a temporal and spatial discretization of the trajectory with an interval ΔT. The inputs necessary for the trajectory prediction based on the flight plan is the sequence of the waypoints and related updates for each aircraft. Therefore, the profiles are generated through the linear interpolation of the pre-defined waypoints on assigned flight levels.

Once 4D trajectories have been generated within the considered time horizon, these are projected in bi-dimensional and vertical plan. Then they are discretized in infinitesimal segments trough a suitable sample depending on the chosen time range length, in order to calculate the distance, denoted as d, between each couple of aircraft at time instant t, progressively along the considered entire time window.

Suppose that aircraft A_i and A_j are flying in the airspace within time windows $[t_0^i, t_n^i]$ and $[t_0^j, t_m^j]$ respectively. If the time windows intersect in a certain time window $[t_p , t_q] = [t_0^i, t_n^i] \cap [t_0^j, t_m^j]$, the flight trajectories could be subjected to potential conflicts. Such a conflict is function of time and coordinates of aircraft as denoted below:

$$C\left(p_i(\mathrm{t}, \varphi_t^i, \lambda_t^i, h_t^i), p_j(\mathrm{t}, \varphi_t^j, \lambda_t^j, h_t^j)\right) = \begin{cases} 0 \; if \; |h_t^i - h_t^j| \geq H \\ 1 \; if \; (|h_t^i - h_t^j| < H) \&\& (d < D) \end{cases} \quad (4)$$

This indicates that if two aircraft lose separation on their waypoints, C is nonzero. H and D are the minimum vertical and horizontal separations respectively. Generally, D=5NM, and H=2000ft if flights are above FL290 and H=1000ft if flights are under FL290.

2.2 Closest Point of Approach (CPA) problem

Potential conflicts within the defined time window $[t_p, t_q]$ occur when the closest possible distance of aircraft, denoted with d_{CPA}, is less than the above predefined distance threshold D. It is therefore necessary to investigate Closest Point of Approach (CPA) to determine the closest distance under the considered approach scenarios in the airspace. In this sub-section the CPA problem is described.

The Figure 3 shows an example of CPA with two moving aircraft Ai and Aj.

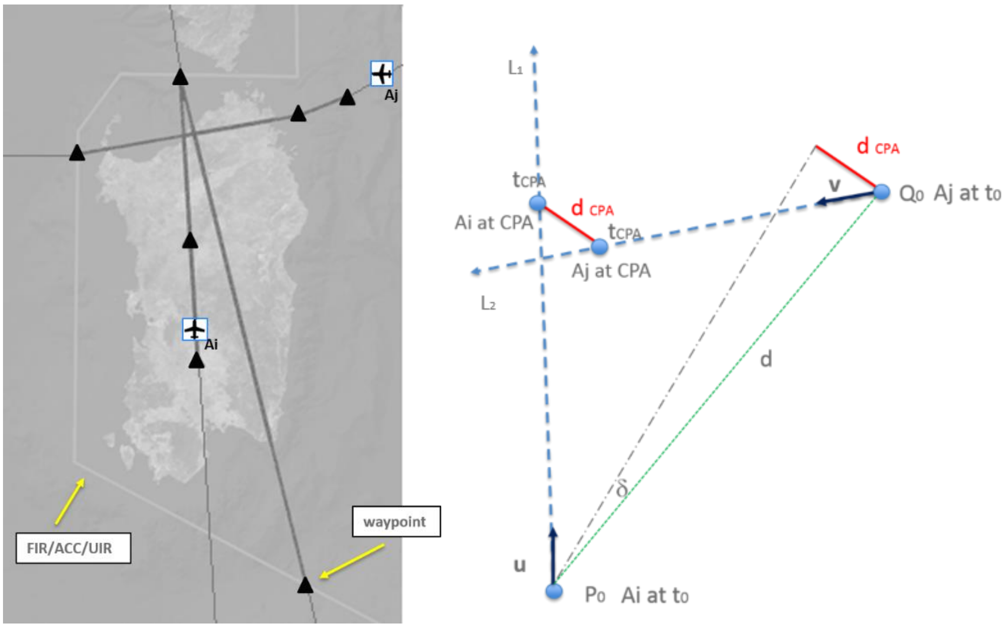

Figure 3 – Example of routes based on pre-defined waypoints (left); illustration of distance at the Closest Point of Approach (right) between 2 aircraft in 2D airspace

As shown in the Figure 3, A_i and A_j are considered as two mass points along the L_1 and L_2 be the trajectory propagations respectively. In other terms they are dynamically changing points whose positions **P**(t) and **Q**(t) are associated to the two aircraft at time t. In the three-dimensional airspace, we refer the CPA to the positions at which two dynamically moving points with metric coordinates x, y and z reach their closest possible distance. These points can be intended as tracks moving in two fixed directions at fixed speeds. Consequently, the two points are moving along two lines in space. Particularly, we only consider the progression starting from the initial position; thus, $\mathbf{P}_0$ and $\mathbf{Q}_0$ are their positions at time $t=t_0=0$, and their velocity vectors per unit of time are **u** and **v** respectively. We can indicate t_{CPA} the time in which the distance between them is minimum. Since they are flying with different speeds and directions, the distance between these two aircraft first decreases and then increases. The equations of motion for these two points are:

$$\mathbf{P}(t) = \mathbf{P}_0 + \mathrm{t} \cdot \mathbf{u} \qquad (5)$$
$$\mathbf{Q}(t) = \mathbf{Q}_0 + \mathrm{t} \cdot \mathbf{v} \qquad (6)$$

which are the familiar parametric equations for the lines.

The two equations are coupled by having a common parameter t. So, at time t, the distance between them is given by:

$$\begin{aligned} \mathrm{d}(t) &= |\mathbf{P}(t) - \mathbf{Q}(t)| \\ &= |\mathbf{P_0} - \mathbf{Q_0} + \mathbf{u} \cdot \mathrm{t} - \mathbf{v} \cdot \mathrm{t}| \\ &= |\mathbf{P_0} - \mathbf{Q_0} + (\mathbf{u} - \mathbf{v}) \cdot \mathrm{t}| = |\mathbf{w}(t)| \end{aligned} \qquad (7)$$

where

$$\mathbf{w}(t) = \mathrm{w}_0 + \mathrm{t} \cdot (\mathbf{u} - \mathbf{v}) \qquad (8)$$

with

$$\mathrm{w}_0 = \mathrm{P}_0 - \mathrm{Q}_0 \quad (9)$$

When the derivative of d(t) with respect to t is equal to 0, the minimum value of d (t) is reached. To eliminate the negative values, D(t) = $d(t)^2$ in place of d(t). It states that d(t) is a minimum when D(t) = $d(t)^2$ is a minimum. Thus, D(t) is calculated as follows:

$$\mathrm{D}(t) = \mathbf{w}(t) \cdot \mathbf{w}(t) = (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) t^2 + 2\mathbf{w}_0 \cdot (\mathbf{u} - \mathbf{v}) t + \mathbf{w}_0 \cdot \mathbf{w}_0 \quad (10)$$

The CPA is achieved when the partial derivative of D(t) with respect to t. is 0. That is, D(t) has a minimum when:

$$0 = \frac{d}{dt}\mathrm{D}(t) = 2t[(\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v})] + 2\mathbf{w}_0 \cdot (\mathbf{u} - \mathbf{v}) \quad (11)$$

Equation (11) can be solved to get the time of CPA (TCPA):

$$T_{CPA} = \frac{-w_0 \cdot (\mathbf{u} - \mathbf{v})}{|\mathbf{u} - \mathbf{v}|^2} \qquad (12)$$

whenever |**u – v**| is nonzero. If |**u – v**| = 0, then the two point tracks are traveling in the same direction at the same speed, and will always remain the same distance apart, so one can use $t_{CPA} = 0$. We can determine D_{CPA} as follows

$$D_{CPA}(P(t), Q(t)) \begin{array}{c} = d(T_{CPA}) \\ = |T_{CPA} * (\mathbf{u} - \mathbf{v}) + \mathrm{w}_0| \\ = \frac{-w_0 \cdot (\mathbf{u} - \mathbf{v})}{|\mathbf{u} - \mathbf{v}|^2} * (\mathbf{u} - \mathbf{v}) + \mathrm{w}_0 \end{array} = |P(T_{CPA}) - Q(T_{CPA})| \quad (13)$$

When $t_{CPA} < 0$, then the CPA has already occurred in the past, and the two tracks are getting further apart as they move on in time.

Once the time remaining to reach the CPA between the probed aircraft and the surrounding traffic vehicles has been calculated, the system finally identifies the aircraft with lower estimated TCPA with respect to the probed aircraft as the highest priority dependency. The output is a sorted list of potentially interacting vehicles. This list of dependencies is set up in such a way that the priority order is inverse to the estimated time to reach the CPA.

The pseudo-code for computing CPA is below:

Table 2 Process for computing CPA

CPA determination		a/c tracks defined by [a/c1(x_{start}, y_{start}, z_{start}), (x_{end}, y_{end}, z_{end}), (vx_{ac1}, vy_{ac1}, vz_{ac1})], [a/c2(x_{start}, y_{start}, z_{start}), (x_{end}, y_{end}, z_{end}), (vx_{ac2}, vy_{ac2}, vz_{ac2})],	
Step	Title	Process	Output
1.	Trajectories generation	Propagate automatically trajectories	Set of aircraft tajectories
2.	CPA_time():	Compute the time of CPA for two tracks	The time at which the two tracks are closest
3.	CPA_distance():	Compute the distance at CPA for two tracks	The distance for which the two tracks are closest
4.	Dependencies functionality	Return the aircraft with lower estimated TCPA/DCPA with respect to the probed aircraft	The aircraft with lower estimated TCPA/DCPA

2.3 Data processing

This section shows how to compute the CPA among aircraft that are dynamically moving. The main intent of the simulations is to evaluate the CPA with respect to the different approach geometries and dependences of interacting vehicles [21-22]. The proposed high-level algorithm concept is shown in Figure 4.

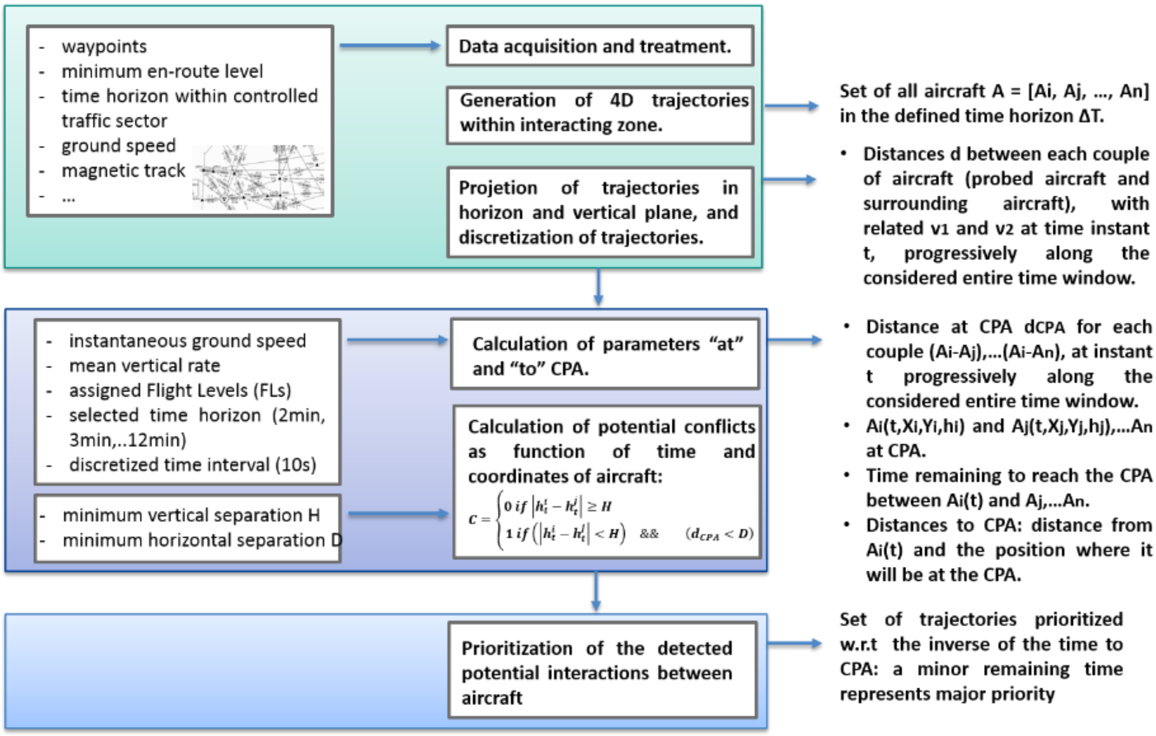

Figure 4 – High-level Process Sequence.

As shown in Figure 4, the system first receives and elaborates the external data. Particularly, three-dimensional coordinates of probed aircraft and interacting aircraft, as well as speed and heading are need inputs for the system. The whole model consists of three main blocks: the first module is dedicated to the trajectory propagation; the second one is focused on the CPA determination; and the third block provides a sorted list of potentially interacting vehicles. It is developed in Matlab/Simulink® [23]. The outputs are the prioritization of dependencies and the information about the specified dependencies.

2.3.1 Trajectories propagation

Since scenarios are a core part of simulation and evaluation of enhanced concepts in ATM, it is important to choose a set of situations that allow a robust feedback on whether an ATM concept can be implemented in the operational environment. The approach here adopted uses a first set of simulation scenarios that are built on the base of a real Italian airspace adopting the waypoints and the flight procedures provided by ATS documentation. The inputs necessary to simulate the planned trajectories consist of the en-route chart standard ATS route network, including significant points and navaids, as well as restrictions on speed and from ICAO minimum wake turbulence separation. The first scenario focuses on a zone of en-route chart standard ATS route network referred to an Italian air space. This chart defines the airways and shows the entry waypoints from which the STARs (Standard Arrival Routes) can be flown. Here it is possible to distinguish the routes and waypoints that are designed with black lines and symbols respectively. Relevant data along each route are:

- the distance, in nautical miles, between two consecutive waypoints,
- the name of airway,
- the minimum en-route level, and
- the magnetic track.

Once these data are been acquired, the treatment of them consist of converting all the coordinates in the desiderated reference system.

All the aircraft trajectories are modelled in three spatial dimensions x, y and z, and it is assumed that the aircraft fly linear trajectories at constant velocities, and no wind conditions. The Figure 5 shows the examples of trajectories propagation based on pre-defined waypoints.

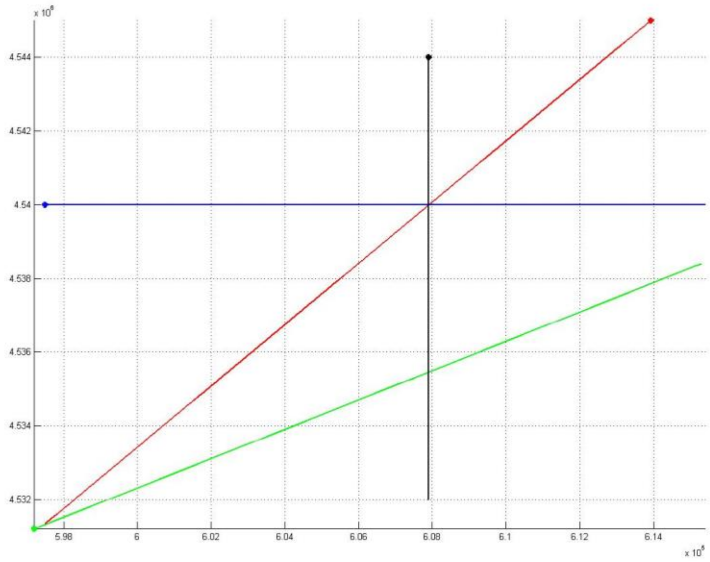

Figure 5 – Simulated propagated trajectories with approach geometries.

2.3.2 CPA computation

Once the first set of scenarios is built, a second set of scenarios is delivered by modifying the waypoints to evaluate the characteristics of the CPA between aircrafts for different approach geometries. In order to test methods and algorithms, the set of simulations are achieved in Matlab ® environment [23].

The analysis is focused on the sensitivity of the TCPA, DCPA varying the approach geometry as well as the velocity parameter. The aim is here to provide an analysis mean on by CPA to support the ATC decisions in the vectoring choices. The CPA parameters are evaluated considering the approach geometry with three different set of crossing angles: right angle (crossing tracks at 90 degrees), acute angles (same tracks) and obtuse angles (reciprocal tracks).

The probed aircraft ac1 is flying on heading 230° (red line in the Figure 5). The following aircraft are analysed:

- ac2 is flying on heading 90°;
- ac3 is flying on heading 68°;
- ac4 is flying on heading 180°.

The first simulation is set at the same velocity of the first aircraft v_{ac1} = v_{acn}= 200 kts. Then, we have supposed that the first aircraft has always speed of v_{ac1} = 200 kts and the other aircraft have different speeds. The following Table 3 reports the results of the distance at closest point of approach and the time at closest point of approach, expressed in meters and seconds respectively, obtained for all aircraft tracks within different sectors in Italian airspace, considering the following velocities: v_{acn} =100-120-200-300-400-500 kts. For this study, different values of cruise altitude and different values of speed have been considered for testing system performance and computation feasibility. Particularly, for low speed we have considered a light traffic. Commercial traffic has been taken into account for high speed.

Table 3 CPA computing with different approach geometries

v_{ac1}=200 kts direction for ac1: 230°				
Heading [°]		**Traffic speed [kts]**	**Distance at closest point of approach [m]**	**Time at closest point of approach [s]**
90		200	13717.6314	27.4606
		100	12374.6159	41.9009
		120	12725.8511	38.1607
		300	14430.8127	19.7386
		400	14860.9342	15.1325
		500	15144.5578	12.1433
68		200	7222.504	51.7576
		100	6205.8670	60.5901
		120	6407.8172	65.4991
		300	7866.2908	40.9017
		400	8290.5127	33.7675
		500	8590.6027	28.7347
180		200	5884.0358	-9.6541
		100	5735.8553	13.4369
		120	5956.895	8.745
		300	4764.3422	-16.6107
		400	3837.7283	-15.2374
		500	3227.8892	-12.9001

The variation of the distance at closest point of approach and the time at closest point of approach with respect to the first case for $v_{ac2}=v_{ac1}$ demonstrates that aircraft speeds have a relatively small effect on CPA. The approach geometry represents the key parameter. Therefore, an additional set of simulations with various traffic scenarios have been performed to support the sensitivity analysis of CPA. For these simulations, we have assumed that both the aircraft are flying at the same altitude and in no wind conditions. For simplifying computations, seconds and meters units are used for time and distance respectively, and speed is in m/s. The analysis is focused on the sensitivity of the TCPA and DCPA varying the approach geometry as well as the velocity parameter. The aim is here to provide an analysis mean on by CPA to support the ATC decisions in the vectoring choices. The CPA parameters are evaluated considering the approach geometry with three different set of crossing angles: right angle (crossing tracks at 90 degrees), acute angles (same tracks) and obtuse angles (reciprocal tracks). The figure 6 below shows an example of crossing, same and reciprocal tracks respectively.

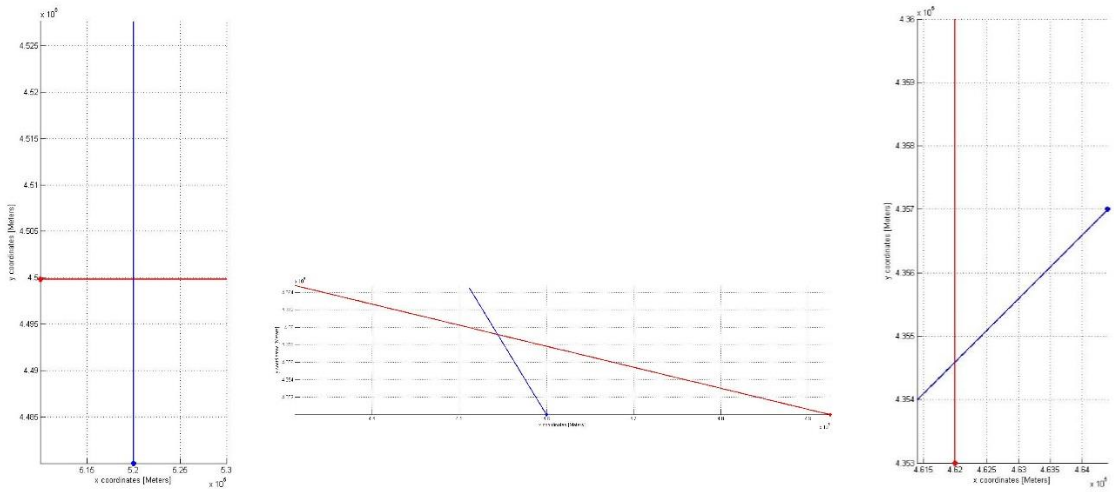

Figure 6 – Simulated trajectories with crossing right angle (left), acute angle (center), obtuse angle (right)

In the first configuration, the first aircraft ac1 is flying on heading 90° and the second aircraft ac2 on heading 180°. The first simulation is set at the same velocity of the first aircraft v_{ac2} = 269 m/s. Then, we have supposed that the first aircraft has always speed of v_{ac1} = 269 m/s and the second aircraft has different speeds: v_{ac2} =249 m/s, 200 m/s, 300 m/s, 400 m/s, 500 m/s. The following Table 4 reports the results of the distance at closest point of approach and the time at closest point of approach, expressed in meters and seconds respectively, obtained for two aircraft tracks at 90°.

Table 4 CPA computing with crossing track at 90°

v_{ac1}=269 m/s	$v_{ac2}<v_{ac1}$ v_{ac2}=200,249 m/s	$v_{ac2}=v_{ac1}$ v_{ac2}=269 m/s	$v_{ac2}>v_{ac1}$ v_{ac2}=300,400, 500 m/s
Distance at closest point of approach [m]	858.2-384.8	212.1	30.5-649.8-1078.4
Time at closest point of approach [s]	13.0-12.1	11.7	11.0-9.1-7.6

For this approach geometry with crossing track at 90 degrees, a marginal change of velocity led up to sensitive variation of distance DCPA. From an ATC prospective, this configuration allows the "1 in 60 rule" to be used. This can assist significantly in controller and pilot decision making and understanding. The "1 in 60 rule" supports the controller which needs to determine the extent of the heading change, according the following rule: "1 Degree offset angle equates to 1 nm displacement at 60 nm from a origin, so the distance off track is equal to the number of degrees off course per distance to station divided by 60".

However, in order to test the feasibility of the tool and evaluate the CPA parameters after the crossing point, we have considered included in the calculations also the configurations, in which the CPA has already occurred in the past, and the two tracks are getting further apart as they move on in time. For this reason, the time is negative. The table below shows just an example of CPA computing. This is important to demonstrate that CPA continues to exist also after the crossing point.

Table 5 CPA computing - CPA has already occurred

v_{ac1}=269 m/s v_{ac2}=200,269,300 m/s	$v_{ac2}<v_{ac1}$	$v_{ac2}=v_{ac1}$	$v_{ac2}>v_{ac1}$
Distance at closest point of approach [m]	261.6	494.9	775.1
Time at closest point of approach [s]	-15.5	-13.6	-12.7

For the second approach geometry, the first aircraft ac1 is flying on heading 280° and the second aircraft ac2 on heading 325°. The following Table 6 reports the results of the distance at closest point of approach and the time at closest point of approach, expressed in meters and seconds respectively, obtained for two aircraft tracks at 45°.

Table 6 CPA computing with same tracks (acute angles)

v_{ac1}=269 m/s	$v_{ac2}<v_{ac1}$ v_{ac2}=200,249 m/s	$v_{ac2}=v_{ac1}$ v_{ac2}=269 m/s	$v_{ac2}>v_{ac1}$ v_{ac2}=300,400, 500 m/s
Distance at closest point of approach [m]	30679.8-29824.0	29486.8	28985.0-27564.4-26431.0
Time at closest point of approach [s]	30.2-32.6	33.1	33.4-32.4-30.3

Here, a marginal increasing of velocity does not led up to sensitive variation of distance DCPA. In order to test the feasibility of the tool and evaluate the CPA parameters after the crossing point, we have considered also a configuration, in which the first aircraft ac1 is flying on heading 132° and the second aircraft ac2 on heading 50°. For this approach geometry, the implementation of CPA formula shows that the CPA has already occurred in the past, and the two tracks are getting further apart as they move on in time. For this reason, the time is negative as for the example in the table below.

Table 7 CPA computing with same tracks (acute angles) - CPA has already occurred

v_{ac1}=269 m/s v_{ac2}=200,269,300 m/s	$v_{ac2}<v_{ac1}$	$v_{ac2}=v_{ac1}$	$v_{ac2}>v_{ac1}$
Distance at closest point of approach [m]	27091.5	27901.9	28004.5
Time at closest point of approach [s]	-22.8	-7.1	-2.0

For the acute angles, the controller can vector the aircraft that is behind. This is usually used when the two aircraft are maintaining altitude and one is considered to be overtaking the other as specified in ICAO Annex 2 [24], which states that "An overtaking aircraft is an aircraft that approaches another from the rear on a line forming an angle of less than 70 degrees with the plane of symmetry of the latter". This is the more convenient choice from ATC perspective as well, since it requires less intervention (there is already some separation).

When aircraft are flying with reciprocal tracks, the crossing angle is an obtuse angle. In this configuration we have supposed that the first aircraft ac1 is flying on heading 360° and the second aircraft ac2 on heading 210°. The following Table 8 reports the results of the distance at closest point of approach and the time at closest point of approach, expressed in meters and seconds respectively, obtained for two aircraft tracks at 150°.

Table 8 CPA computing with reciprocal tracks (obtuse angles)

v_{ac1}=269 m/s	$v_{ac2}<v_{ac1}$ v_{ac2}=200,249 m/s	$v_{ac2}=v_{ac1}$ v_{ac2}=269 m/s	$v_{ac2}>v_{ac1}$ v_{ac2}=300,400, 500 m/s
Distance at closest point of approach [m]	3372.1-3277.0	3242.5	3193.2-3062.3-2963.0
Time at closest point of approach [s]	7.1-6.6	6.4	6.1-5.4-4.8

Aircraft on reciprocal (opposite) tracks are generally manage by vectoring both of them to solve potential conflicts. This method increases the controller workload (due to having more communication exchanges on the frequency) but offers the benefit of less impact on each aircraft trajectory.

Consequently, the increase of the distance flown is usually negligible. In a way, this technique is derived from ICAO Annex 2 [24], which states that "*when two aircraft are approaching head-on or approximately so and there is danger of collision, each shall alter its heading to the right*".

The above descripted simulations demonstrate that the approach geometry has a strong impact on CPA parameters and ATC decision making, while a small increasing of velocity has a small relative impact on the DCPA. Starting from above defined set of scenarios, we have modified the waypoints of the trajectories, in order to generate new tracks remaining in the same allowed airspace and according to ATC rules and constraints, but with different approach geometries and at different speeds. Particularly, we have calculated CPA parameters for 18 different scenarios, and we have evaluated of what is variated the DCPA and TCPA. The following Tables 9-10 show the variation in percentage of the distance at closest point of approach and the time at closest point of approach with respect to the first case for $v_{ac2}=v_{ac1}$ by simulating different set of approach geometries with different crossing angles.

Table 9 Variation in percentage of DCPA and TCPA (acute angles)

Variation [%] of:	$v_{ac2}<v_{ac1}$	$v_{ac2}=v_{ac1}$	$v_{ac2}>v_{ac1}$
Distance at closest point of approach	2.55	-	6.16
Time at closest point of approach	5.13	-	3.80

Table 10 Variation in percentage of DCPA and TCPA (obtuse angles)

Variation [%] of:	$v_{ac2}<v_{ac1}$	$v_{ac2}=v_{ac1}$	$v_{ac2}>v_{ac1}$
Distance at closest point of approach	2.47	-	5.23
Time at closest point of approach	7.01	-	15.09

2.3.3 Dependences and Prioritization

The system performs the prioritization of the detected potential interactions between the probed aircraft and the surrounding traffic under the control of the sector or planned for the sector. This information is elaborated by the tool based on the calculation of the time remaining to reach the Closest Point of Approach (CPA) between the probed aircraft and the traffic vehicles detected as potentially interacting with it: the aircraft with lower estimated time to reach the CPA with respect to the probed aircraft is considered as the highest priority dependency.

The tool performs the calculation and provision of relevant parameters describing the approach condition with respect to the highest priority vehicle and/or with respect to all the interacting vehicles (dependencies), in order to provide the Controllers with better understanding of the approach condition between the probed vehicle and the interacting vehicles, in particular the one with highest priority. The set of calculated parameters of CPA includes the Time to Closest Point of Approach (TCPA), and the Distance at CPA (DCPA).

The dependency assessment with respect to the surrounding traffic is performed by implementing straight-line trajectory propagation, based on the propagation of the aircraft motion according to the vehicle current inertial velocity vector. The Figure 6 shows the proposed separation assurance operating method for low traffic sample.

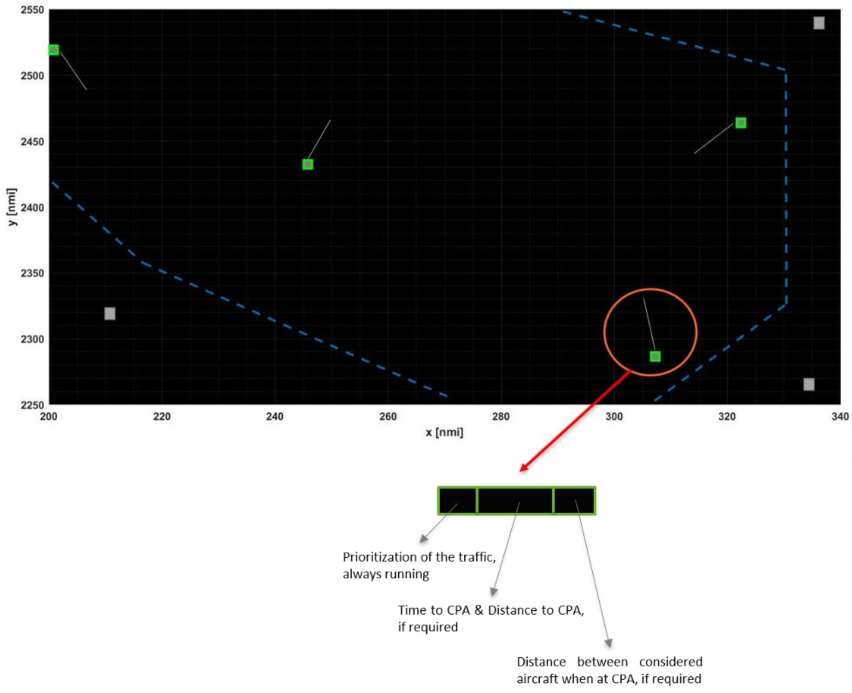

Figure 6 – Proposed separation assurance operating method

Particularly, the Figure 6 shows a small-scale experiment through a simulation of trajectories propagation for commercial aircraft within Italian airspace. To be noted that only 4 aircraft are in a certain sector and they will be considerate in the simulation with respect to the probed aircraft. All aircraft are flying at FL330 with different speed and on different heading. The Tables 11-12 report the heading and speed of probed aircraft and surrounding traffic in the selected sector for two scenarios.

Table 11 Scenario1

Aircraft	Heading [degrees]	Speed [kts]
probed	330	430
1	260	395
2	30	420
3	150	500

Table 12 Scenario2

Aircraft	Heading [degrees]	Speed [kts]
probed	230	300
1	90	300
2	68	400
3	180	400

For these scenarios we have assumed that aircraft information is available as horizontal and vertical components in a three-dimensional airspace. In ATC, speed and distance are typically reported in knots [kts] and nautical miles [nmi]. Fast-time aircraft positions are here used for computing trajectories propagations in the en-route phase. The trajectories of the considered aircraft pair are discretized in segments over the considered time horizon ΔT, so the distance between aircraft is calculated for each discretization point and the discretized d_{CPA} is evaluated as the minimum value of the calculated distances at the considered discretization points. For this simulation we have assumed that the horizontal distance thresholds = 5 nmi; the time to CPA threshold = 600 sec; and the altitude threshold = 1000 ft. In addition to the DCPA and TCPA the tool delivers the early dependencies information provision to the air traffic controllers. This information is elaborated by the system based on the calculation of the time remaining to reach the Closest Point of Approach (CPA) between the probed aircraft and the traffic vehicles detected as potentially interacting with it: the aircraft with lower estimated time to reach the CPA with respect to the probed aircraft is the highest priority dependency. The Tables 13-14 report the results of the simulations as a sorted list of potentially interacting vehicles, intended as a sorted list of dependencies, where the priority order is inverse to the estimated time to reach the CPA. As specified in the previous section, when the CPA has already occurred in the past, the tool reports a negative value of the time, so that the two tracks are getting further apart as they move on in time, and the corresponding potential conflict has not occurred. Then, the prioritization is based on the CPA that will take place.

Table 14 Dependences and prioritization for Scenario1

Aircraft	Heading [°]	Speed [kts]	Prioritization	TCPA [sec]	DCPA [nmi]
probed	330	430	-	-	-
1	260	395	-	-602.1987	159.5136
2	30	420	1	255.5352	77.1240
3	150	500	-	-440.6886	100.9770

Table 15 Dependences and prioritization for Scenario2

Aircraft	Heading [°]	Speed [kts]	Prioritization	TCPA [sec]	DCPA [nmi]
probed	230	300	-	-	-
1	90	300	1	19.7386	7.7920
2	68	400	2	33.7675	4.4765
3	180	400	-	-15.2374	1.7429

3. Conclusion

This paper aims at presenting the mathematical models and algorithm tools for evaluating the Closest Point of Approach (CPA) among aircraft in a three-dimensional airspace, and its related parameters. Additionally, it analyses the sensitivity of CPA with respect to the different approach geometries. For verifying the performance of the algorithm, simulations with various traffic scenarios have been performed. The results demonstrate that, during the convergence of aircraft, the approach geometry has a strong impact on the CPA, especially in flight directions with acute crossing angle, while the increasing of velocity has a small relative impact on the CPA, resulting in variations of DCPA and TCPA lower than 3 %. The consideration of this result can improve the ATC decision-making, and facilitate the air traffic operations by reducing ATC workload. The tool can be used in the route design process, ensuring the safety for air traffic management. Furthermore, the proposed tool is able to deliver the early dependencies information that is useful for investigation of aircraft movement and encounter status, improving the definition and design of new ATC decision making support tools, and increasing the enhanced situational awareness for both pilot and executive and planning ATCs.

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5. Copyright Statement

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