33RD CONGRESS
OF THE INTERNATIONAL COUNCIL
OF THE AERONAUTICAL SCIENCES
STOCKHOLM, SWEDEN, 4-9 SEPTEMBER, 2022

ICAS 2022 SWEDEN

STABILITY OF A LAMINAR BOUNDARY LAYER OVER LOCALISED SURFACE INDENTATION

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Abstract

The stability of a Tollmien-Schlichting (TS) wave over surfaces undergoing localised three dimensional curvature changes such as indentations was studied. All the depression heights, h, and wavelengths, λ, considered here appeared to modulate an artificially excited TS wave, where the perturbation introduced by the moderate cases grew linearly within the depression and further downstream. Severe indentations, where a recirculation bubble was formed appeared to amplify the growth of the convective instability wave significantly. The frequency spectra showed that the non-linear regime was dominated by the harmonic modes. Even if the frequency spectra showed some activities related to the oscillation of the separation bubble in the artificially excited case the instability process was found to be more dominated by the convective instability process. In the absence of artificial forcing for the severest indentation, $h = 2.17$ and $\lambda = 81mm$ a naturally excited mode took over the instability process and dominated the non-linear instability process.

Keywords: Indentation, boundary layer, transition

1. Introduction

In simplified 2D wall bounded flows such as Blasius, Falkner-Skan and Poiseille the Tollmien-Schlichting (TS) wave, initially proposed by Tollmien and later confirmed during the experiment of [1], is found to be a dominant instability mode. Significant progress has been made in understanding the physical process through which this instability wave grows up to a certain extent where it rapidly breakdowns to turbulence. Once excited through the process of receptivity TS waves amplify exponentially during the primary growth stage in a linear process. Followed by a secondary growth stage where the non-linear effects are dominant and present serious challenges both for experiments and modelling. Amongst many others, in the UK early studies by [2] using linear stability theory demonstrated that disturbance introduced at the wall in the form of wave packets can interact with the boundary layer instability waves and grow at a rate which can be predicted by linear stability theory. This type of forcing is normally achieved by surface vibrating ribbon or periodic suction and blowing through the wall as demonstrated by [1]. However, natural forcing of TS waves can occur in experimental facilities with high turbulence intensity and early conundrum linked with the disparity in transition location observed in different experimental facility rapidly led to the belief of the influence of the freestream disturbances on the excitation and growth of instability waves. Freestream disturbances exist either in acoustic or in vortical form, [3] and [4] independently demonstrated that theoretically the acoustic disturbance can couple with the TS waves generated by a receptivity site produced by surface protuberances and in a more recent study [5] showed that vortical disturbances can have similar influences on TS waves generated by two dimensional roughness.

The excitation and growth of instability waves in wall bounded flows is also heavily influenced by wall curvatures and surface protuberances of two and/or three dimensional nature, such as roughness or waviness. Surface curvature causes the boundary layer to accelerate or decelerate while imposing a favourable pressure gradient which damps the growth of the TS wave and an adverse pressure

gradient produces a destabilising effect. On the other hand sufficiently small surface roughness does not affect the baseflow to the same extent as wall curvature changes, but it acts as a receptivity site. Efforts in trying to understand the effect of roughness on transition can be dated back to 1940's during the study of [6] and [7], [8] and [9]. Most of the effort was placed on establishing a criterion for transition based on the Reynolds number based on the roughness height, R_k. This criterion is very useful for engineering and design purposes but is not a model with sufficient representation of the physical mechanism. Further studies by [10] shed some more light in the process of excitation and growth of a TS wave by 2D roughnesses. [11] and later [12] studied 3D roughnesses and suggested that the localised pocket of the disturbances within the roughness region is excited by the freestream turbulence which is further amplified while moving downstream. The demarcation between roughness and waviness lies between the scales of the protuberance where it is viewed as a roughness when it's height and width are of the $O(10^{-1} - 10^{0})$ the thickness of the boundary layer where as a waviness problem is usually when the width or wavelength is of the $O(\geq 10^{0} - 10^{1})$ with generally a sinusoidal or Gaussian waveform. The effect of surface waviness, both convex and concave was initially studied by [13] who derived an empirical critical-criterion for the departure from laminar to turbulent. [14] and co-workers revisited similar problems and proposed a more robust criterion which included the effect of compressibility, pressure gradient and wing sweep. Once again these studies were from a purely engineering approach without great detail of the instability mechanism leading to transition.
The current investigation is aimed at understanding the destabilising effect on a TS-wave developing over a three-dimensional surface deformation, which will be referred as surface indentation from here onwards. In practice this could potentially occur due to unwanted surface protuberance owing to limitation of manufacturing processes or damages to the skin or outer shell of an aerodynamic surface. Indentations were of a cosine bell shape with the maxima recessed to the surface of the flat plate and of a single period. The wavelengths of the indentaions were of the order of 50 to 150 times the displacement thickness of a reference Blasius baseflow and the amplitudes were of the order of 1 to 2 times. These indentations were expected to modify the base flow locally due to the presence of a pressure gradient and perturb the incoming, artificially excited instabilty wave. An account of the stability of TS waves over this particular configuration is lacking in the literature where the focus has been mainly on two dimensional low amplitudes surface waviness, approximately 1 order of magnitude less than the displacement thickness where the local viscous flow is still attached to the surface. The work of [15] could be relevant to the current study especially in the context of the low amplitude (shallow) depressions however the mean flow distortion experienced in the latter's case was considerably benign in comparison to the deep cases presented here where the flow separates locally. The 2D numerical study of [16] was extended to waviness which generated a slightly higher pressure gradient but still limited to the condition of zero skin friction which marks the onset of separation but once again in this study the deepest depressions generated a non-negligible amount of separated flow, as reported by [17] and therefore the interaction between the TS wave and the localised surface curvature induced recirculating bubble had to be considered.

2. The Experiment

The experiment was conducted at a freestream Reynolds number, $Re = 1.2 \times 10^{6}/m$ in the low turbulence wind tunnel at Gaster Laboratory. The tunnel has a test section of $0.90m \times 0.90m \times 4m$ and a turbulence intensity, $Tu \approx 0.006\%$ for a velocity range of $2m/s$ to $20m/s$ at the centre of the empty test section. Hot wire measurements were made along a $0.90 \times 2.0m$ aluminium flat plate mounted between the ceiling and the floor of the tunnel. The plate had a rectangular cut through, starting at $0.5m$ from the leading edge and extending to $0.8m$ to house an aluminium insert panels comprising the 3D surface indentations of varying amplitude, h and aspect ratio, λ. The curvature of the indentation was defined by a 3D cosine bell-shape, similarly to those studied by [17] where the minimum point in the depression is located at $x = 650mm$ from the leading edge. Indentation amplitudes of $h = 0.81mm$, $h = 1.62mm$ and $2.17mm$ and wavelengths of $\lambda = 81mm$ and $\lambda = 162mm$ were studied, but here only the case which generated a self-sustained oscillation has been presented. Changes in surface curvature in the indentation presented some challenges in ensuring accurate velocity profile measurement during hot wire traverse. This was resolved by using a Micro-epsilon optoNCDT 1710-50 laser displacement sensor which was fixed to the x-axis of the traverse and able to track

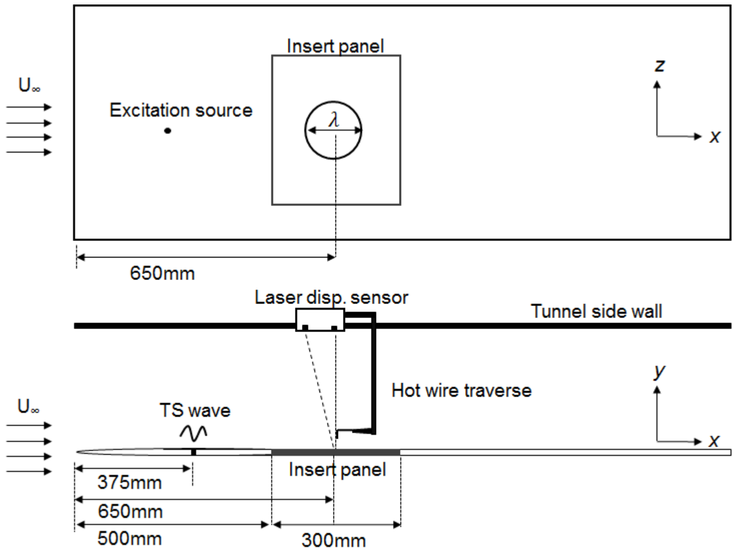

Figure 1 – Schematic illustration of the side view (top) and top view (bottom) of the experimental model and set-up. The side view also illustrates the hot wire traverse mechanism and the laser displacement sensor.

any change in surface curvature. The displacement sensor with an accuracy of $7\ \mu m$ and frequency response of $1kHz$, operates on the principle of laser triangulation. From figure 1, at every measurement station the laser beam was shone through the 5μm diameter hot wire sensor and due to the microscopic nature of the sensor, negligible error of the beam reflecting from the surface arises. This ensured that the laser was always coincident to the position of each wall normal traverse and allowed the hot wire to be positioned at a prescribed reference wall normal location while traversing over the indentation.

Constant temperature anemometry (CTA) technique was employed to capture the mean, U and the fluctuating streamwise velocity component, u', using right angle 55P14 Dantec Dynamics hot wire probe since it allowed for near wall measurements, especially in the minima of the indentation while ensuring minimum intrusive effects to the bubble. The hot wire probe was connected to a DISA (Dantec) M-unit module and the signal was acquired through a PXI 6143 A/D card. The anemometer was set at an overheat ratio of 1.75 and King's law was used for hot wire calibration over a velocity range of 1 to 25m/s. The raw hot wire signal was first passed through an anti-aliasing and secondly through Krohnhite filters at a bandpass frequency of 2Hz to 2kHz. The DC signal was used to resolve the mean velocity component, while the AC signal for the fluctuating component was acquired at a rate of 5kHz for 10 seconds durations. In the presence of an accurate near wall alignment system [19] demonstrated that the total expanded relative uncertainty in hot-wire measurements was approximately 3.9% and in this case since the total drift in temperature was less than $1.5°C$ the uncertainty expected to be even lower.

[20] technique was employed to excite the T-S waves through a point source located at 375 mm from the leading edge of the flat plate, shown in figure 1. The point source consisted of a 0.5 mm diameter hole on the measurement side and a cavity to house a miniature 8Ω acoustic driver. The waveform and frequency of the driver was controlled via LabVIEW. The amplitude of the forcing, was adjusted such that the signal-to-noise ratio was large enough to resolve the T-S wave as accurately as possible but also ensuring the forcing was reasonably small so as to avoid non-linear behaviour occurring prior to the region of the indentation. For the artificially forced case a forcing frequency of 172Hz was selected following [17] and two amplitudes amplitudes were chosen 'low' and 'high' where the later was approximately 5 times larger. In the unexcited case the excitation source was turned-off and the hole was covered to prevent the excitation of any cavity modes that can amplify while propagating downstream.

3. Results and Discussion

3.1 Artificially Excited

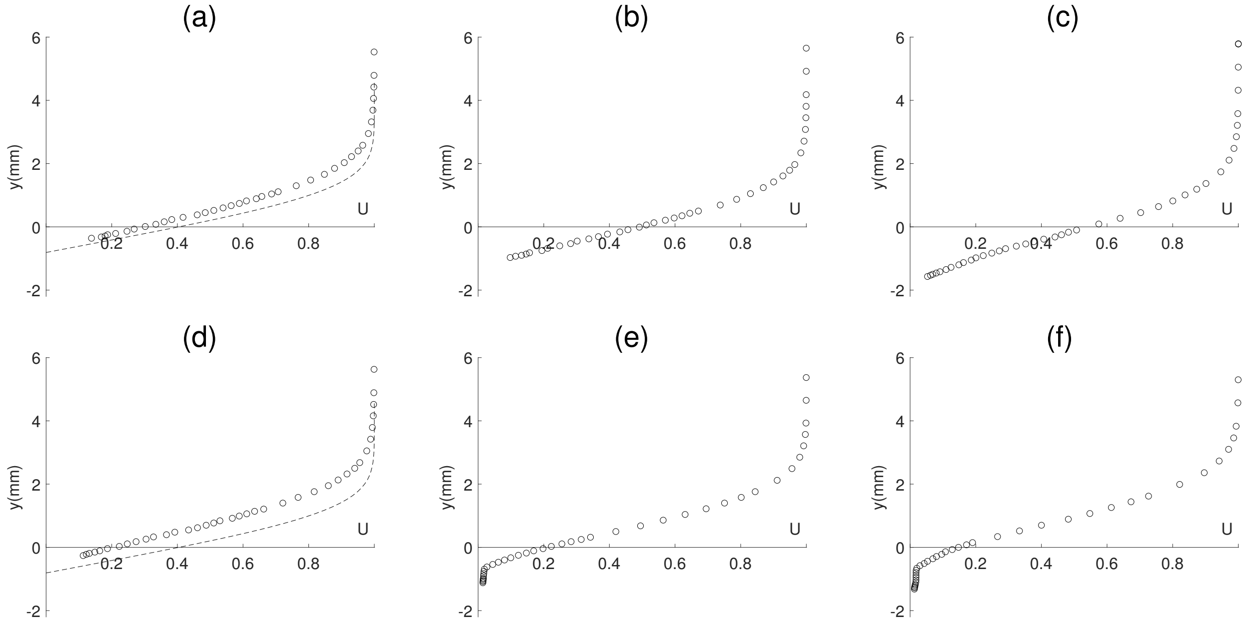

Figure 2 – The velocity profiles at the minimum point of the depression, $x = 650mm$ along the centreplane for the case of $\lambda = 162mm$ on the top and $\lambda = 81mm$ at the bottom, where (a) and (d), (b) and (e), and (c) and (f) corresponds to the indentation heights of $h = 0.81mm$, $h = 1.62mm$ and $h = 2.17mm$ respectively. The dashed lines in (a) and (d) corresponds to the Blasius profile with the origin of the vertical axis shifted by 0.81mm.

From figure 2 (a) and (d), when compared with the Blasius velocity profile (with the wall normal position shifted down by $0.81mm$, the depth of the depression) even the shallowest indentation proved to be sufficient in modifying the mean flow. The less full velocity profile in the case of the depression is due to the adverse pressure gradient which is expected to start at the beginning of the depression and persist until $x = 650mm$, the position of the minimum in the depression. From figure 3 the effect of the adverse pressure gradient on the artificially excited T-S wave is more evident on the evolution of the local maximum in the fluctuating streamwise velocity components, u'_{max}, amplitude along the centreplane. For the case of $\lambda = 162mm$, in figure 3 (a) the growth rate increases with increasing depth of the indentation. When the wavelength is further reduced the growth rate steepens even for the shallowest indentations of $h = 0.81mm$ as a result of the increase in the local radius of curvature of the indentation. Downstream of the minima in the indentations the growth rates are reduced due to the damping effect provided by favourable pressure for the shallowest indentations, $h = 0.81mm$, this helps the TS waves to recover to the levels similar the smooth case in the absence of an indentation. For the rest of the indentations the destabilisation is significant enough to cause a TS wave to carry-on amplifying beyond the recovery region, with the case of $\lambda = 162mm$ and $h = 2.17mm$, $\lambda = 81mm$ and $h = 1.62mm$, and $h = 2.17mm$ attaining levels of about 1% of the freestream velocity which would indicate the beginning of non-linearity and eventual break-down. The non-linear behaviour can be confirmed from figure 5 which shows the development of the harmonics of the fundamental 172Hz artificially forced mode for the cases of $\lambda = 81mm$ and $h = 1.62mm$, and $h = 2.17mm$.

The steep growth rate in the cases of $\lambda = 81mm$ and $h = 1.62mm$, and $h = 2.17mm$, considered as the most severe indentations is due to the formation of a laminar separation bubble within the indentation and this can be confirmed from the inflectional nature of the velocity profiles in figure 2 (e) and (f) and also for the case of $\lambda = 162mm$ and $h = 2.17mm$ in figure 2 (c) which is not as clearly defined as the former cases as a results of the limitations of the hot wire technique. This arises due to (1) directional insensitivity in the recirculation region not allowing negative velocity components to be resolvable; (2) calibration curve ranges between $1-20m/s$ and departs from King's linear law for velocities less

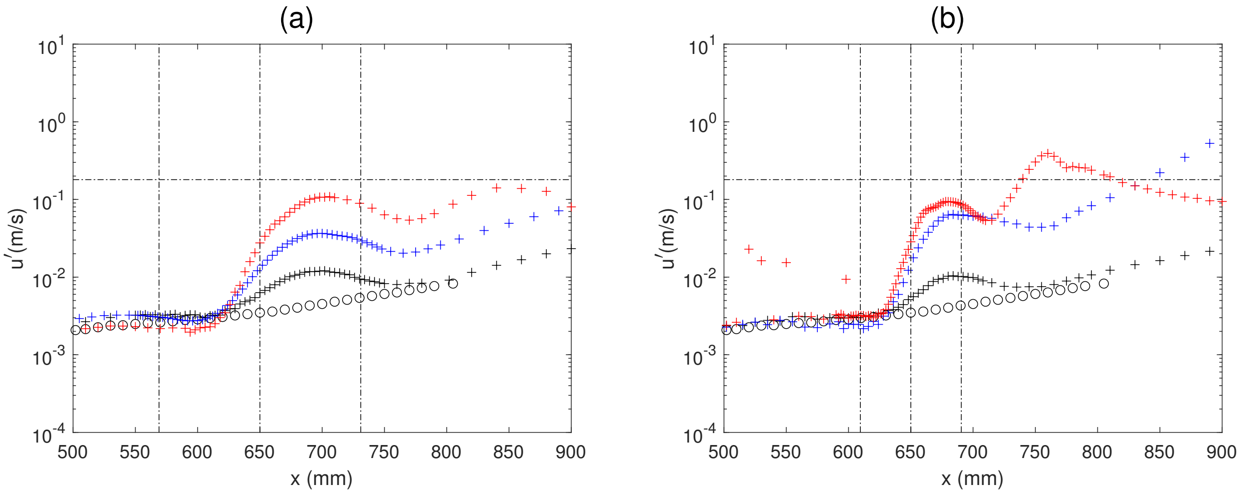

Figure 3 – Streamwise evolution of the local maximum in u' corresponding to the 172Hz artificially excited mode, (a) $h = 1.62mm$ and (b) $h = 2.17$ for the case of $\lambda = 81mm$.

than $1m/s$; and (3) due to heat transfer between probe and surface, measurements were restricted to approximately 0:3 mm from the surface. However, the very low velocity within the inflectional part of the profile qualitatively confirms the presence of the recirculating bubble. The approximate spatial extent of the separation bubble which clearer in the most severe cases is presented in figure 4. Here, a qualitative agreement was found with our previous DNS simulations [17] which also showed that the recirculation bubble extended the vertical bounds of the indentation for the case of $\lambda = 81mm$ and $h = 2.17mm$ and break down was incipient at $x \geq 750mm$ as seen shown in figure 4. For this case, in figure 5(d), in fact the non-linear process already started at the minimum point of the depression, at $x = 650mm$ due to the presence of superharmonics which where still low in energy at the same position for the case $h = 1.62mm$ and $\lambda = 81mm$ in figure 5(a). These superharmonics became more pronounced further downstream in the case of $h = 2.17mm$ but remained relatively weak for the case of $h = 1.62mm$ when compared in figure 5(e) and (f), and (b) and (c) respectively. For the case $\lambda = 162mm$ and $h = 2.17mm$, and $\lambda = 81mm$ and $h = 2.17mm$ the non-linear effects started to intensify at $x > 800mm$ but break down did not take place within the measurement domain which was limited to $x = 1000mm$.

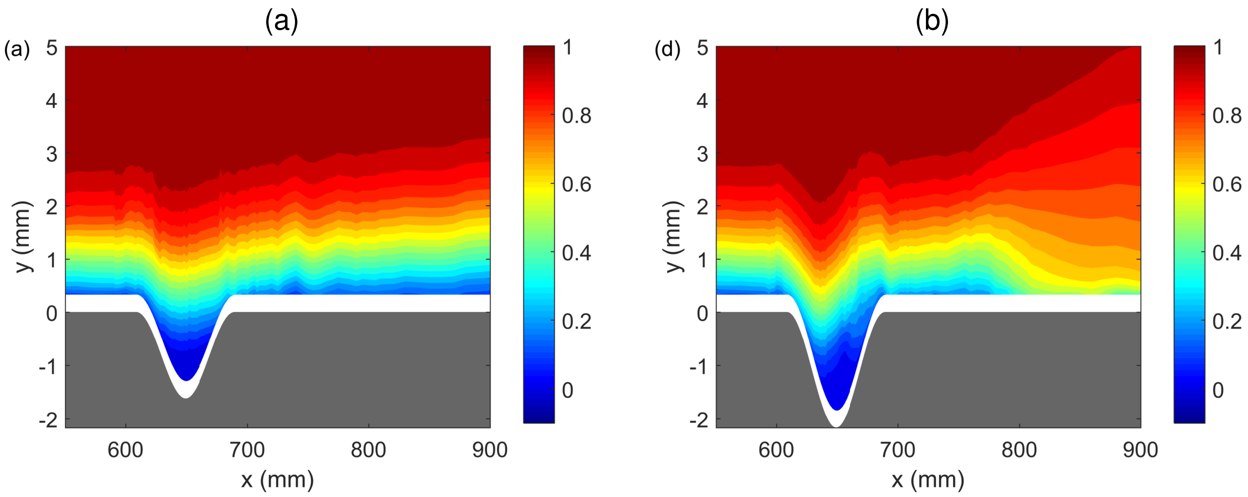

Figure 4 – The contours of the normalise mean streamwise velocity, U, for the artificially excited case with high initial forcing, (a) $h = 1.62mm$ and (b) $h = 2.17$ for the case of $\lambda = 81mm$.

While developing through a 2D laminar separation bubble an artificially excited convective mode is known to grow very rapidly where the Kelvin-Helmholtz instability kicks-in when the shear layer is sufficiently lifted from the wall. The non-linear instability and break down process is first through three-dimensionalisation in the bubble through the formation of oblique modes which is followed by the formation of $\Lambda - structures$ leading to streaks formation and break down. Here, the severe 3D indentations impose a three-dimensionalisation of the laminar separation bubble and therefore a non-uniform spanwise distribution of the maximum amplitude of the velocity fluctuation of the artificial forced mode. From figure 6 the spanwise distortion was still present downstream of the depressions at $x = 700mm$. The maximum in the amplitude occurred along the centreplane, $z = 0mm$ and when comparing figure 6 (c) and (f) in the case of $\lambda = 81mm$ and $h = 2.17mm$ the energy started to be more focussed closer to the centreplane. As shown in from our DNS simulations [17] the three-dimensional nature of the separation bubble trapped in the indentation enhanced the formation of a $\Lambda - structure$ immediately downstream of the depression leading to the strong non-linearity confirmed by the superharmonics in the spectra in figure 5(e) and (f) very rapid breakdown straight after.

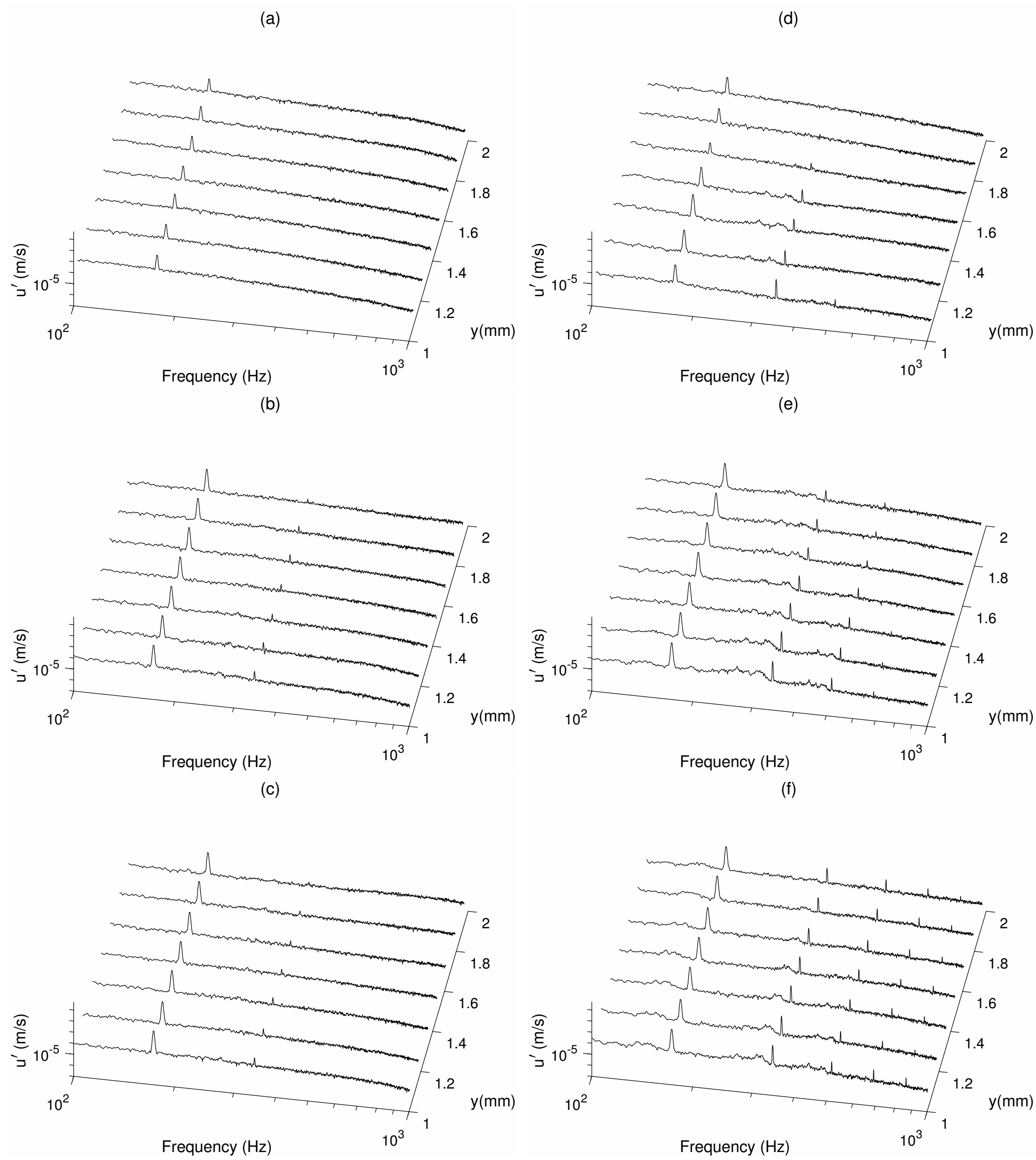

Figure 5 – The fast Fourier transform of the hot wire signal at various position in the boundary layer developing over the indentation with $h = 1.62mm$ on the left and $h = 2.17$ on the right, where (a) and (d), (b) and (e), and (c) and (f).

In addition to the superhamonics for the case $\lambda = 81mm$ and $h = 2.17mm$ from figure 5(e) and (f) some energy was also observed around a narrow frequency band centred at approximately 315Hz and this was not observed in the case of $\lambda = 81mm$ and $h = 1.62mm$ on the left of figure 5 and in the case of $\lambda = 182mm$ and $h = 2.17mm$ (not shown here) which also experienced the formation of a non-negligible laminar separation bubble which promoted rapid linear and non-linear growth. The mode centred at

315Hz was in fact excited naturally and will be addressed in the following section.

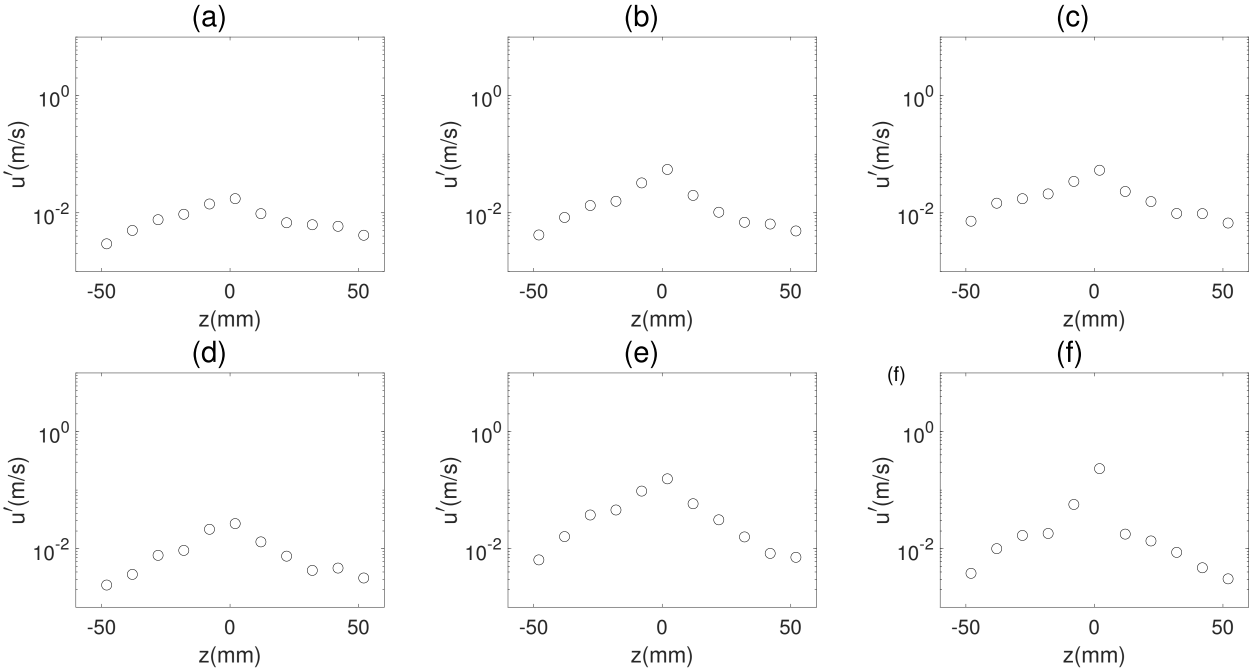

Figure 6 – Spanwise evolution of the local maximum in u' corresponding to the 172Hz artificially excited mode for the case of $h = 1.62mm$ on the top and $h = 2.17mm$ for $\lambda = 81mm$, where (a) and (d), (b) and (e), and (c) and (f) corresponds to $x = 650mm$, $x = 680mm$ and $x = 700mm$ respectively.

3.2 Naturally Excited

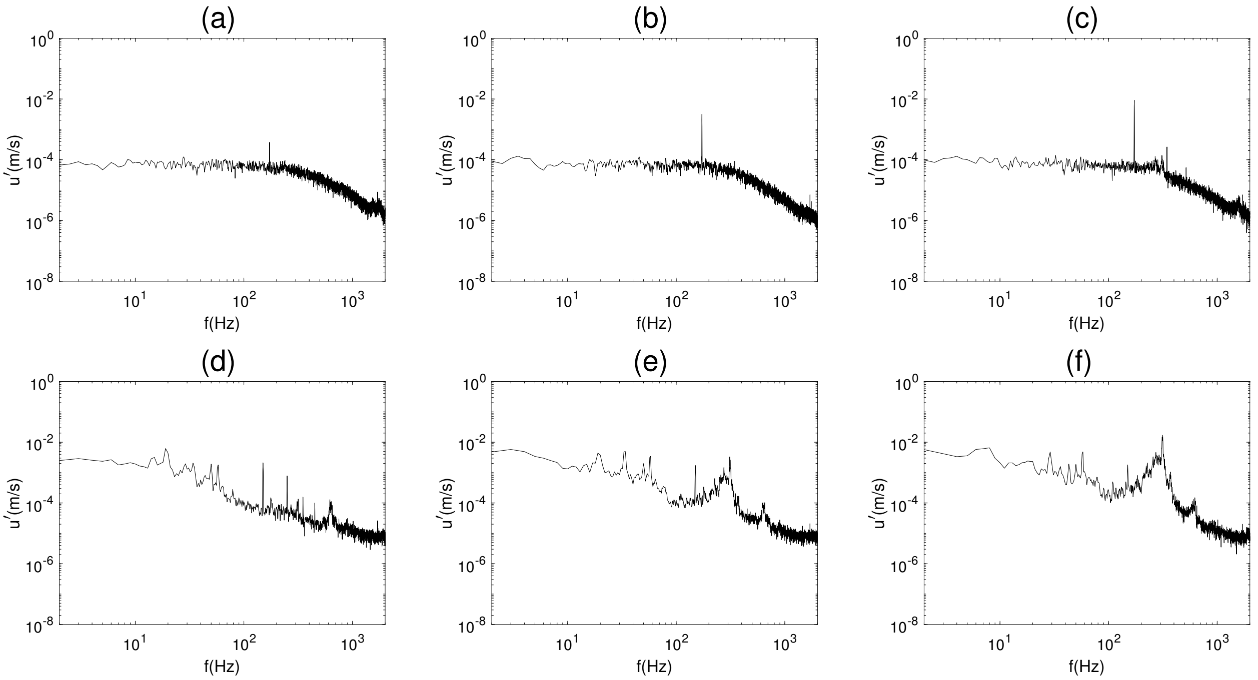

Figure 7 – Temporal Fourier transform of the hot wire signal within depression at $x = 630mm$, $x = 650mm$ (mininum point) and $x = 670mm$ along the centreplane. The wall normal position y corresponds to the position were u' is maximum.

The frequency spectra at three streamwise poistions within the most severe indentation, $h = 2.17mm$ and $L = 81mm$ are shown in figure 7 for the artificially and naturally excited case (no artificial excitation). For the artificially excited case, in figure 7 (a) to (c) the 172Hz forcing is clearly depicted and the peak increases in amplitude while developing downstream of the indentation, showing the rapid growth and the emergence of the harmonics, which also increase in amplitude once again confirming the presence of non-linearity within the indentation itself. In the purely naturally excited case, in 7 (d) to (f) the mode centred at approximately 315Hz is more clearly depicted as opposed to case of the artificial excitation where it started to be identifiable at $x = 670mm$ in figure 7 (c). The naturally excited mode increased in energy as it developed downstream of the minima in the indentation. The sharp peaks 50Hz and the odd harmonics are due to contamination from the mains. Since the flow was naturally excited the initial amplitudes were very low and therefore the signal had to be amplified by a gain of thousand resulting in amplification of the contaminations too.

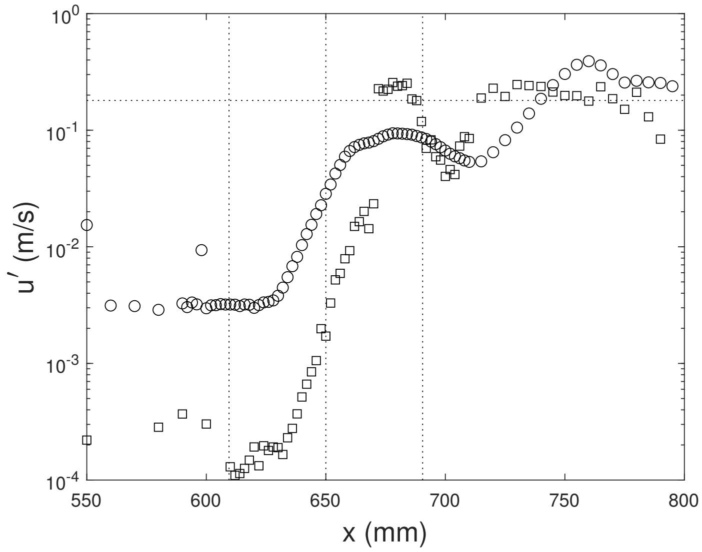

Figure 8 – Streamwise evolution of the local maximum in u' corresponding to the 172Hz artificially excited mode (circle) and the naturally excited 315Hz mode (square) for the case of $h = 2.17$ and $\lambda = 81mm$.

The growth of the $315Hz$ naturally excited mode and of the $172Hz$ artificially excited mode has been presented in figure 8. In both cases the rapid growth takes place at similar streamwise position, however the naturally excited mode has a steeper growth rate. It also attained the amplitude levels equivalent to 1% of the freestream flow much earlier within the indentation itself indicating that the non-linear effects are more significant in this case and would result in earlier breakdown. In the linear growth regime, the naturally excited mode appears to grow spatially similar to the artificially excited mode thus showing convective instability characteristics. However, at $x \geq 670mm$ a jump in the amplitude is observed and is followed by a saturation and drop in amplitude in the non-linear growth stage which can be confirmed by the increase in energy of the superhamonics in figure 7. This jump in the amplitude of the mode is similar to the the bifurcation process that takes place in non-linear dynamic systems which are self-excited, this kind of instability grows in time and are considered as absolute instability. From the temporal evolution of the hot wire signal shown in figure 9 the naturally excited modes also shows temporal growth behaviour which potentially shows the absolute instability characteristics.

However, the naturally excited case also shows spatial growth behaviour in figure 8 and this cast some doubts on whether the instability is purely through a self-excited mode, thus whether this flow is globally unstable in the presence of a severe depression. A very quick check based on the length of the separation bubble, λ_b, and convective velocity here based on the freestream velocity, U_∞, shows that the dominant frequency of the natural mode, $f = U_\infty/\lambda_b \approx 315Hz$. This suggest that the naturally excited mode is a result of the oscillation of the confined laminar separation bubble. Since such kind of behaviour was not observed in the case of $\lambda = 81mm$ and $h = 1.62mm$, it could be considered as a sub-critical case and was therefore naturally stable. Although not shown here in the absence of an artificial excitation no significant mode was identified from the spectra for the case of $\lambda = 81mm$ and

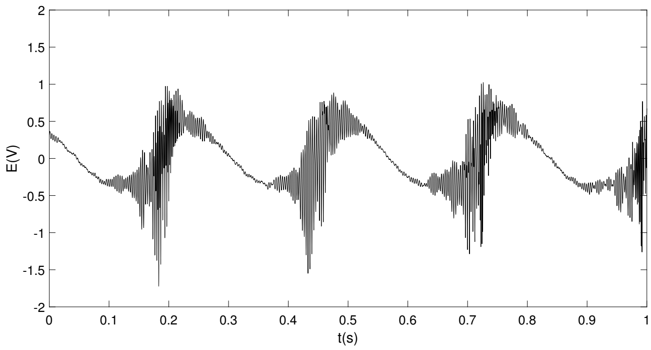

Figure 9 – Amplitude of the 172Hz artificially excited and the

$h = 1.62mm$ and therefore considering the relatively low turbulence intensity of the facility, backqround noise of wind tunnel was not considered as a source of contamination which could potentially excite convective instability modes. The convective instability behaviour depicted by the naturally excited mode in the case of $\lambda = 81mm$ and $h = 2.17mm$ could be due to the noise generated by the oscillation of the bubble which propagates upstream and excited in the receptivity zones. This would thus make the pure absolute instability characteristic of the naturally excited mode difficult to be distinguished and to confirm the globally unstable nature of the flow.

4. Conclusion

All the indentations studied here appeared to increase the growth rate of an artificially excited convective mode (TS wave). In the presence of locally separated flow in the severe indentation a rapid growth of the artificially excited mode is observed within the laminar separation bubble and could be described by linear modal stability. In the absence of artificial excitation, the case of $\lambda = 81mm$ and $h = 2.17mm$ was observed to be a critical case as it was able to naturally excite instability modes which showed both spatial and temporal growth characteristics. The spatial growth was at a higher rate than the artificially excited mode and very quickly jumped to higher values which corresponds to the onset of non-linearity, indicative of a bifurcation process observed in inherently non-linear self-excited dynamic systems. The non-negligible extent of spatial growth observed by the naturally excited case which also shows the temporal growth behaviour cast some doubts on whether this was a globally unstable flow.

From an application perspective the current findings suggest that the stability and transition of a laminar boundary layer in the presence of an indentation is not so trivial and the linear modal stability theory for estimating transition location should be employed very cautiously. Especially in the case where the magnitude of the reverse flow is sufficiently high to accommodate a global instability.

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6. Copyright Statement

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