33RD CONGRESS
OF THE INTERNATIONAL COUNCIL
OF THE AERONAUTICAL SCIENCES
STOCKHOLM, SWEDEN, 4-9 SEPTEMBER, 2022

OPTIMAL DESIGN OF STIFFENED PANELS UNDER BUCKLING CONSTRAINTS: A DESIGN METHODOLOGY CONSIDERING CAD-BASED PARAMETERIZATION WITH SIMULTANEOUS LAYOUT AND SIZING OPTIMIZATION

Hammad Rahman* [1], Roeland De Breuker† [1], Saullo G. P. Castro‡ [1]

[1]Delft University of Technology, 2629 HS, The Netherlands

Abstract

An innovative design methodology is proposed for the simultaneous layout and sizing optimization of stiffened panels. A computer-aided design (CAD) modelling tool is used to generate the panel geometry and parameterize the stiffeners layout. The same geometric parameterization is used in the gradient-based optimization. Therefore no additional constraints are applied on grid points coordinates in order to conform with the geometric requirements. The proposed method also eliminates the extra step of converting the optimal design back to the CAD model. An effective parameterization scheme is used that defines the location and orientation of a stiffener with only two layout design variables. The sensitivities of the structural responses with respect to both layout and sizing design variables are computed through the semi-analytical method of Nastran. The proposed methodology is used to re-design a benchmark metallic stiffened panel from literature. The obtained results are used to demonstrate the effectiveness of the proposed method.

Keywords: Simultaneous layout and sizing optimization, gradient-based optimization, CAD-based parameterization, metallic stiffened panel, buckling

1. Introduction

Stiffened panels are widely used in aerospace and marine applications owing to their higher specific stiffness and strength. A stiffened panel structure comprise of a thin skin and stiffeners which are attached for the reinforcement of the skin as shown in Fig. 1. In an aircraft structure, many of these stiffened panels are assembled together to create the skins of wings and fuselage. Since these panels are thin-walled structures, they are susceptible to fail due to buckling, besides strength constraints. In order to design a stiffened panel with minimum mass, optimization techniques are generally employed to tailor the stiffness of the panel in order to ensure its enough resistance against stress and buckling instability. The key factors that influence the stiffness include the panel material, stiffeners layout, skin and stiffeners sizing and the type of stiffener.

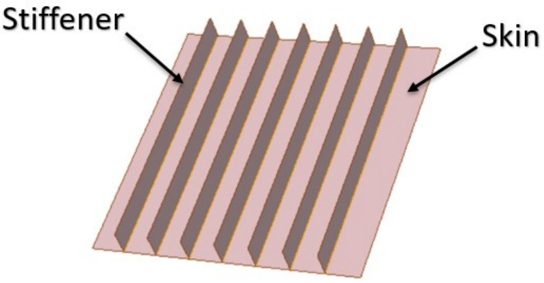


Figure 1 - Stiffened panel structure

---

[*] PhD Candidate, Aerospace Structures and Computational Mechanics, Faculty of Aerospace Engineering.
[†] Associate Professor, Aerospace Structures and Computational Mechanics, Faculty of Aerospace Engineering.
[‡] Assistant Professor, Aerospace Structures and Computational Mechanics, Faculty of Aerospace Engineering.

The layout defines the placement of the stiffeners on the skin including their location and orientation. Since the stiffeners are typically connected with the skin through fastening, the design decision for their layout is customarily driven by manufacturing constraints, resulting in a traditional layout design with uniform distribution of the stiffeners across the skin. If the layout configuration is predetermined and only the thicknesses of the skin and stiffeners are optimized, then the mass saving through the optimization becomes limited. Further mass saving can be achieved by optimizing the panel structural layout and sizing simultaneously. Moreover, the recent progress in the manufacturing technologies, such as Electron Beam Freeform fabrication (EBF3) [1] pave the way for the development of more intricate layouts. Therefore, various research studies in the last decade have focused on conceiving nonconventional complex layouts. In some of these studies, the optimal design of the stiffened panel was achieved through simultaneous layout and material optimization in the case of composite panels [2], [3] and through combined sizing and layout optimization in the case of metallic panels [4]–[8]. The common aspect in all these studies is the use of gradient-free optimization methods, such as genetic algorithm and particle swarm optimization. Although the gradient-free methods can be more easily implemented, they usually require a higher number of evaluations when compared to gradient-based methods, being therefore more applicable with models that require a small computational cost, or with a limited number of design variables. These limitations of gradient-free methods limits the amount of design variables or the model capability to capture the structural behavior more accurately, therefore limiting the possible improvement in the structural performance that is brought by the optimization. In some recent studies [9]–[11], a design methodology was presented for the simultaneous sizing, layout and topology optimization of a metallic stiffened panel by employing a gradient-based method. A mesh deformation strategy based on the Free-Form Deformation (FFD) method was used to manipulate the finite element (FE) mesh in order to accommodate the perturbed layouts generated during the optimization. Although a relatively large design space was explored, the use of FFD created additional constraints on the grid coordinates to ensure that the adjacent stiffeners do not overlap or intersect with each other in the deformed meshes. The sensitivities of such constraints were computed by means of finite difference schemes, which also significantly increase the computational cost. Although FFD is widely used, it does not allow large changes in the layout because of the mesh distortion issues and also pose a limitation while working directly with the CAD models of the configurations.

The research herein presented tries to overcome these limitations by developing an innovative design methodology that employs a CAD modeling tool to parameterize the stiffened panel geometry and generate the FE mesh, while generating the sensitivities of the grid coordinates with respect to the layout design variables. The use of this CAD-based parameterization offers two advantages. Firstly, the geometric constraints to avoid overlapping and crossing of stiffeners are implicitly applied through the CAD model parameterization, therefore eliminating the requirement of applying additional constraints on the grid points coordinates. Secondly, a link with the CAD model is maintained throughout the optimization process given that the same design variables are controlled by the optimizer. Therefore, the additional step of converting the optimal design back to the CAD model is not required, which is crucial for wider adoption of the proposed methodology in industry. A re-meshing strategy is used instead of FFD. The FE mesh is used within Nastran to calculate the sensitivities of the structural responses with respect to both layout and sizing design variables. These are supplied to the external optimizer. The proposed methodology is used to re-design a stiffened panel benchmark case already published by Chu et al. [9]. The main novelty of the present work lies in establishing a framework for gradient-based simultaneous layout and sizing design and optimization of stiffened panels using the CAD-based parameterization.

The remainder of this paper is organized as follows. The design methodology is introduced in Section 2. The results of a standard shape optimization example are also presented in the same section for verification. The stiffened panel design problem is presented in Section 3. The comparison of results with the benchmark problem is shown and discussed in Section 4. The conclusions are given in section 5.

2. Design Methodology

A gradient-based optimization problem is setup in OpenMDAO [12]. A parameterized CAD model is used to create surface-based geometry and mesh using in the Engineering Sketch Pad (ESP) software [13]. The CAD design parameters are the layout design variables, and could also be shape design variables controlling the shape of any geometric features. ESP is a feature-based solid modeling tool built on OpenCSM, OpenCASCADE geometry kernel and EGADS geometry generation system [14]. All of these software are open-source. ESP has the capability to model a common CAD configuration for different analysis disciplines of varying fidelities [15]. The Python module pyCAPS [16], which is associated with ESP, is used to link the CAD model and its mesh with Nastran. The structural responses, and sensitivity calculation of the structural responses with respect to both layout and sizing design variables are performed in Nastran. These response sensitivities are required to evaluate the gradients of objective and constraints, which are passed to the OpenMDAO optimizer for solving the optimization problem. In Nastran, the response sensitivities are calculated using a semi-analytical method explained next.

For the case of the benchmark stiffened panel herein investigated, only a linear buckling constraint is considered. The governing equation to calculate the buckling load factor in Nastran [17], is:

$$[K]\{\emptyset_n\} + \lambda_n[K_d]\{\emptyset_n\} = \{0\}$$

where λ_n and $\{\emptyset_n\}$ are the n^{th} eigenvalue and eigenvector, respectively. $[K]$ is the structural stiffness matrix and $[K_d]$ is the differential stiffness matrix. After differentiating the governing equation with respect to the i^{th} design variable x_i and solving for the buckling load factor sensitives results in:

$$\frac{\partial \lambda_n}{\partial x_i} = \frac{\{\emptyset_n\}^T \left(\frac{\partial [K]}{\partial x_i} + \lambda_n \frac{\partial [K_d]}{\partial x_i}\right)\{\emptyset_n\}}{\{\emptyset_n\}^T [K_d]\{\emptyset_n\}}$$

In Nastran's semi-analytical method, the partial derivatives of the stiffness, $(\partial[K]/\partial x_i)$ and $(\partial[K_d]/\partial x_i)$, are approximated through finite difference schemes. The process for the sensitivity calculation of the structural stiffness is briefly explained next. Initially, the stiffness matrix $[K]$ is computed for the baseline design and afterwards a new stiffness matrix $[K]_{new}$ for the perturbed design variable $(x_i + \Delta x_i)$ is computed. Then, using the forward difference method the stiffness derivative is approximated as:

$$\frac{\partial [K]}{\partial x_i} \cong \frac{[K]_{new} - [K]}{\Delta x_i}$$

In order to obtain $[K]_{new}$ for the layout design variables, additional information is required to establish a relationship between the perturbed grid points coordinates and the layout design variables. This information includes the derivatives of the grid points coordinates with respect to the layout design variables and is obtained from ESP and provided to Nastran. The layout changes in Nastran is achieved by computing the change in grid point location as:

$$\{G\}_{new} = \{G\}_o + (\{x\}_{new} - \{x\}_o)[T]$$

where, $\{G\}_{new}$ and $\{G\}_o$ are the new and the baseline coordinates of the grid point, respectively. $\{x\}_{new}$ and $\{x\}_o$ are the new and the baseline layout design variables, respectively. Matrix $[T]$ contains the shape-basis vectors. The geometric sensitivities are entered in Nastran as shape basis vectors through the bulk data card DVGRID. A workflow is developed to generate these sensitivities for each layout design variable and grid point pair using ESP. These sensitivities are calculated analytically in ESP by differentiating the process through which the geometry was built. The details on the CAD sensitivity calculation is provided by Dannenhoffer and Haimes [18]. The response sensitivities are used to calculate the gradients of objective and constraints. The values of objective, constraints and their respective gradients are transferred to the external gradient-based optimizer that solves the optimization problem. The flow chart of the proposed method is shown in Fig. 2. In every design iteration, a new CAD model is created in ESP. The geometric sensitivities of every layout design variable are generated through ESP and using this data the DVGRID cards are printed for every layout design variable and grid point pair. These cards are included in the Nastran input file for SOL 200. The input data for mesh, boundary conditions, loads, objective function, constraints and design variables are stored in the same file. Nastran is restricted to execute only the initial FE analysis and the design sensitivity analysis. The latter provides the required input data for the external optimizer. The above mentioned tasks are repeated in every design iteration till convergence.

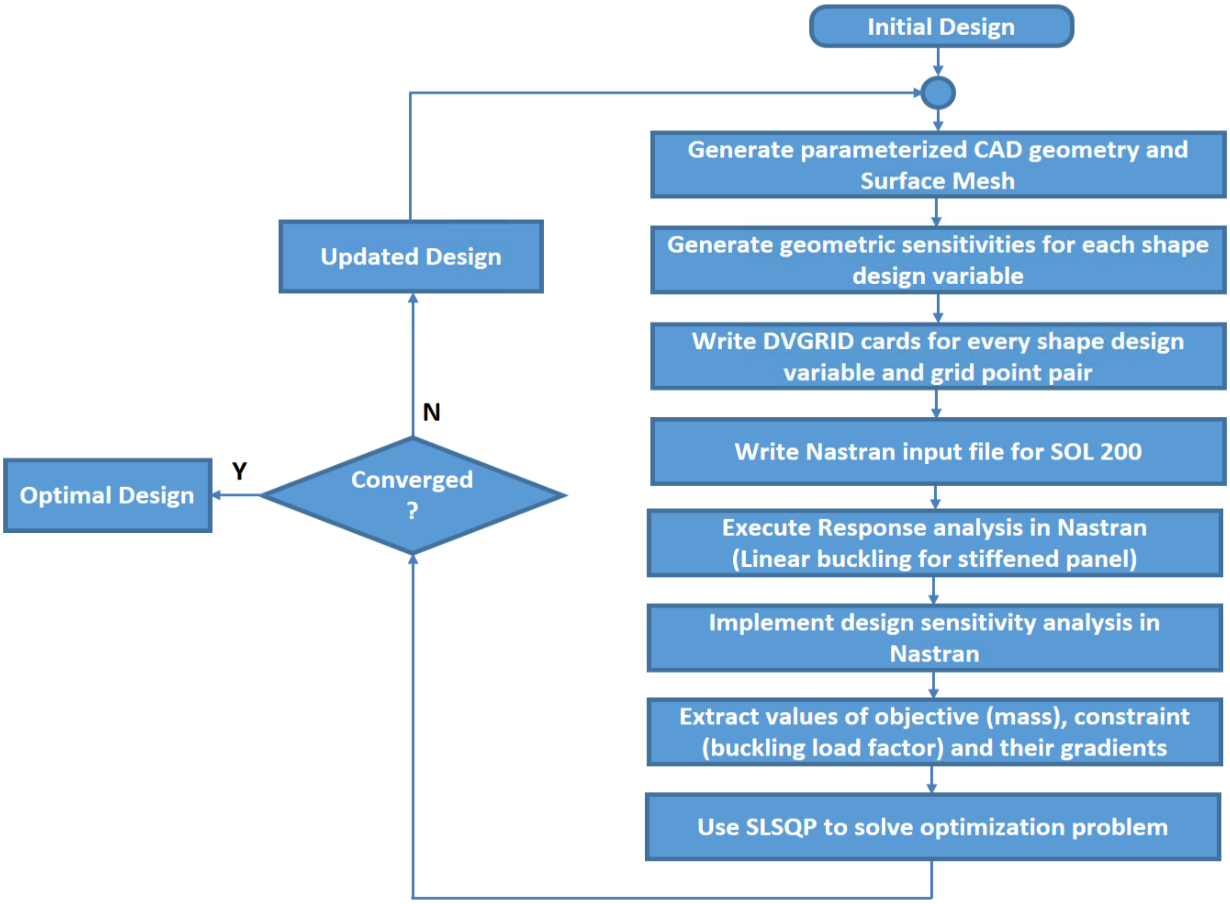


Figure 2 - Flow chart of the proposed method

The proposed methodology is employed to investigate the standard example of shape optimization. It is an aluminum rectangular panel 6m x 4m with an elliptic hole at the center. The loading and boundary conditions are shown in Fig. 3. The objective is to minimize the mass of the panel under Von-Mises stress constraint. The design variables included the shape design variables “rx” and “ry”, and sizing design variable “t” that is the thickness of the panel. Nastran can generate additional responses including the stress response and its sensitivities with respect to the design variables. The optimization results are shown in Table 1. The shape of the hole in the optimum design is modified. It has aligned itself with the loading direction for reducing the stress in the panel. The intuitive shape changes are successfully demonstrated in the optimization results which verifies the proposed methodology.

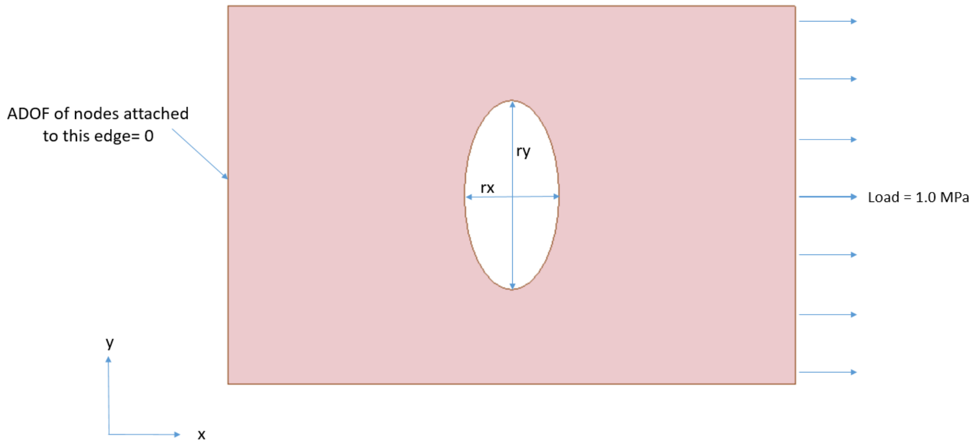
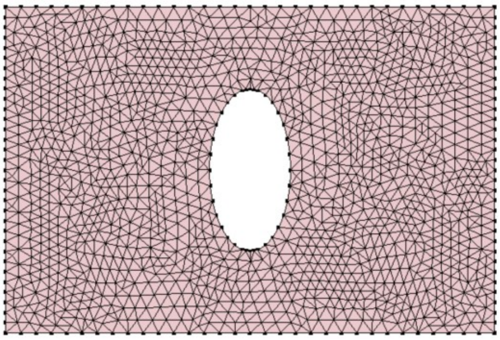
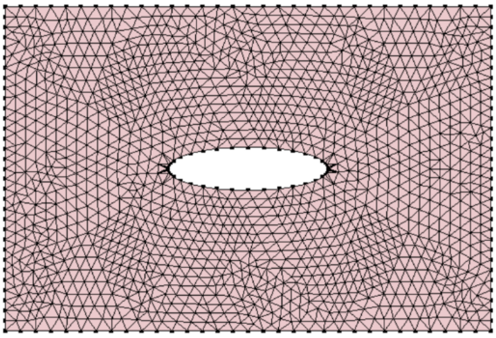


Figure 3 - Rectangular panel with elliptic hole

Table 1 – Optimization results of the rectangular panel with elliptic hole

	Initial Design	Optimal Design
Objective (kg)	3141.4	2970
Design Variables (m)	rx = 0.5, ry = 1.0, t = 0.05	rx = 0.999, ry = 0.269, t = 0.046
Bounds (m)	[0.01, 1.0], [0.01, 1.5], [0.001, 0.5]	
Max. Stress (MPa)	981.56	300
Constraint (MPa)	LB = -300, UB = 300	
Meshed Model in ESP		

3. Design Problem Formulation

3.1 Stiffened Panel Geometry

The stiffened panel geometry of the benchmark case [9] is modelled in ESP. The model and the FE mesh are shown in the Fig. 4. The panel comprises of a 0.3m x 0.3m skin and seven blade stiffeners each with a fixed height of 0.03m. The top edge of the skin is loaded under shear. The skin and each stiffener are discretized with 80 x 80 and 8 x 80 CQUAD4 shell elements, respectively. The isotropic material properties of the panel include Young's modulus = 73GPa, Poisson's ratio = 0.33 and density = 2795kg/m^3. In order to hold a comparison of results with the referenced literature, the same discretization, boundary conditions, loading and material properties are used.

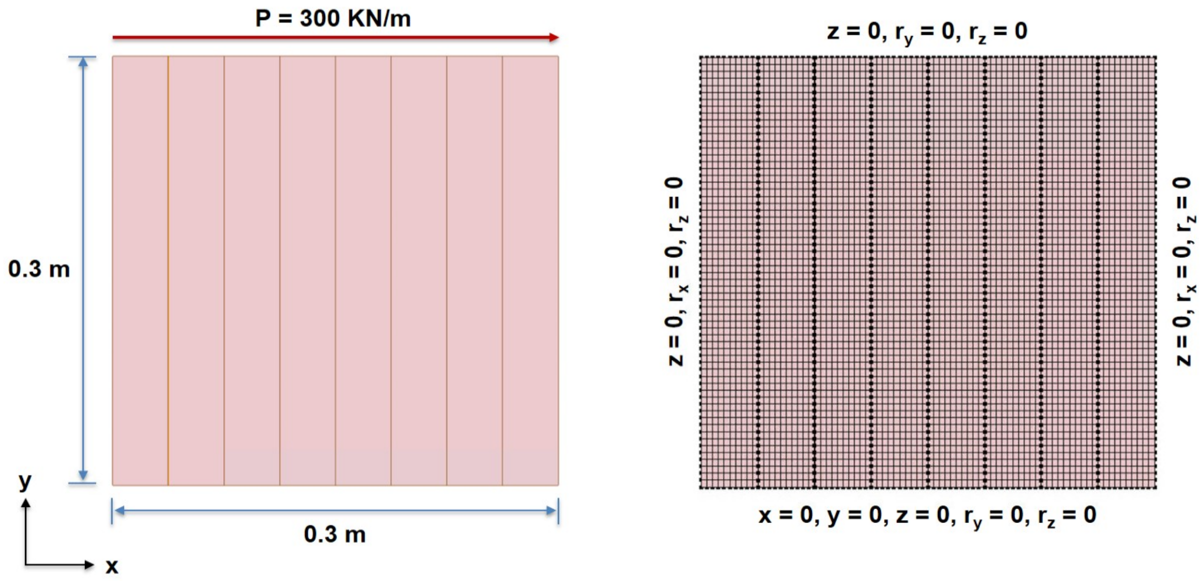
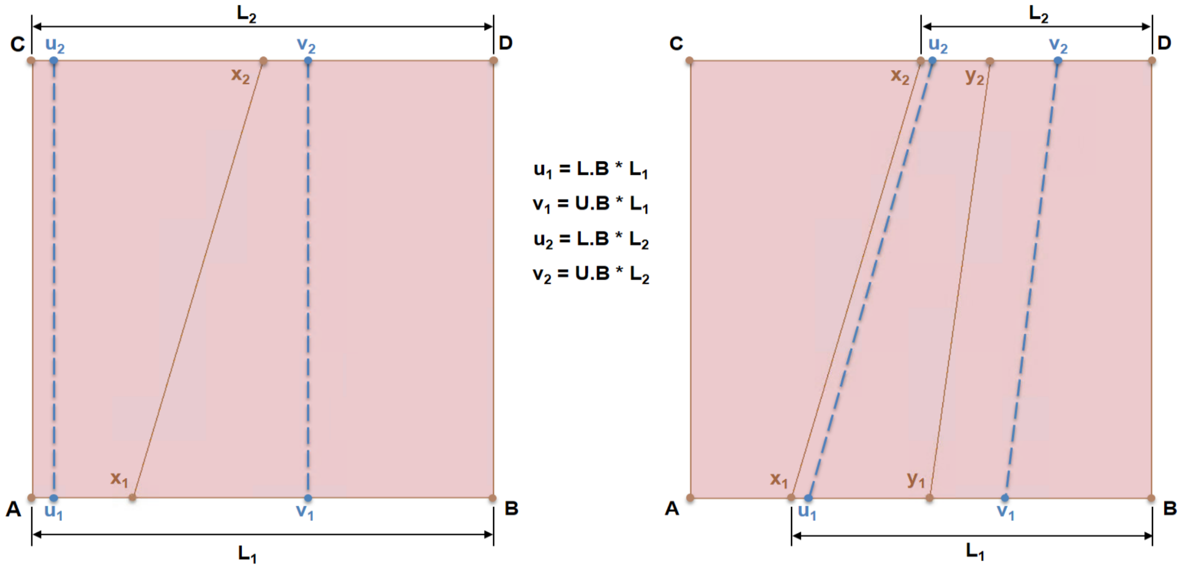


Figure 4 - Stiffened panel model and FE mesh details

3.2 Layout Parameterization Scheme

The stiffened panel model is parameterized in such a way that the location and the orientation of each stiffener is controlled by only two design variables. The illustration of the layout parameterization scheme is shown in the Figs. 5(a) and 5(b). The panel skin is represented by the square ABCD. The stiffeners 1 and 2 are represented by the lines connecting the points x_1 and x_2, and y_1 and y_2, respectively. The model is defined in a way that the points x_1 and y_1 are always located on the line AB. Similarly, the points x_2 and y_2 are always located on the line CD. The stiffeners are always positioned between their respective blue dotted lines u_1u_2 and v_1v_2. The blues lines set the lower and the upper limits for the placement of the stiffener. The location of points u_1 and v_1 depends on the lower bound (L.B) and the upper bound (U.B) values of the layout design variable x_1 and the distance L_1. Similarly, the location of points u_2 and v_2 depends on the L.B and the U.B values of the layout design variable x_2 and the distance L_2. For the stiffener 1, L_1 is the distance between points A and B, and L_2 is the distance between points C and D as shown in the Fig 5(a). After the placement of stiffener 1, L_1 and L_2 are adjusted as show in the Fig 5(b). The stiffener 2 can be positioned in the region marked by x_1Bx_2D with in its bounds shown with blue dotted lines. This modification is made in order to avoid the stiffeners from overlapping and crossing each other. Similarly, the stiffener 3 can be positioned in the region marked by y_1By_2D. This parameterization scheme is very effective because the location and the orientation of seven stiffeners is controlled by only fourteen design variables. Hence, a greater design freedom is achieved with a very limited number of design variables.



(a) (b)

Figure 5 - Illustration of the Layout Parameterization scheme

3.3 Optimization Setup and the Initial Design

The optimization problem is setup in the OpenMDAO environment using the SciPy SLSQP optimizer. A total of 22 design variable are defined which included fourteen layout variables defining the stiffeners layout, one sizing variable for the skin thickness and seven sizing variables for the thickness of each stiffener. The L.B and U.B of all the layout variables are 0.05 and 0.6 respectively, while the same for all the sizing variables are 0.001 and 0.003, respectively. The value of all the sizing variables for the initial design is 0.002. The objective function is to minimize the mass of the stiffened panel while satisfying the linear buckling constraint. In order to handle the mode switching, the first 10 buckling modes are constrained. The lower bound for all the buckling constraints is set to 1.0.

The linear buckling analysis results of the initial design is shown in the Fig. 6. It is evident from the mode shapes that the right bottom side of the skin can undergo buckling under the applied shear loading due to the in-plane bending. The comparison between the analysis results and the reference data [9] are tabulated in Table 2. The relative difference between the results is less than 4% and it can be attributed to the use of different element types. Since the mode shapes are also similar to the reference mode shapes, it can be concluded that the generated parameterized model resembles well with the reference model in the literature.

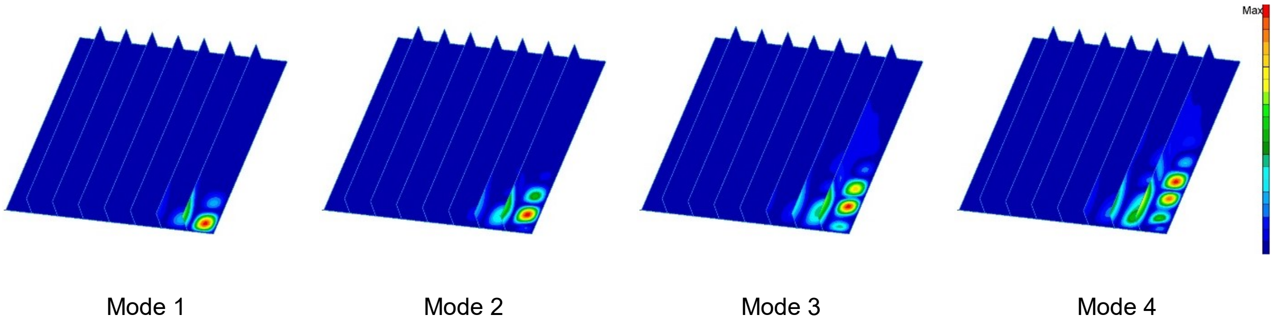


Figure 6 - Buckling results of the initial design

Table 2 - Initial design results comparison with reference data

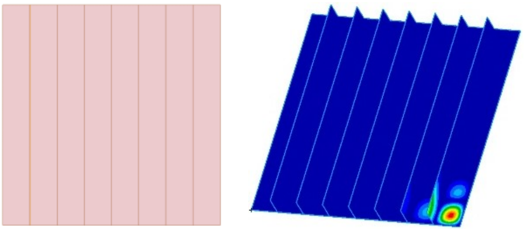
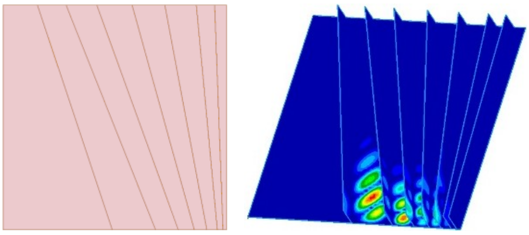
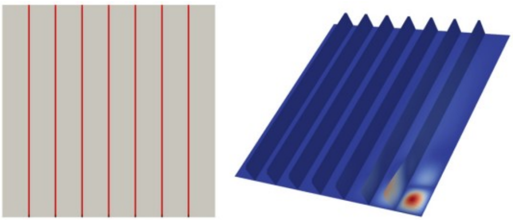
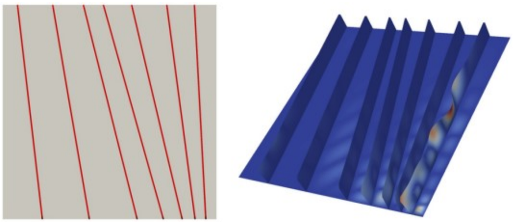
	Analysis Results	Reference Data	Relative Difference (%)
Mass (kg)	0.85527	0.855	0.032
Mode 1	1.107	1.141	2.98
Mode 2	1.413	1.459	3.15
Mode 3	1.647	1.702	3.23
Mode 4	1.839	1.903	3.36

4. Results and Discussion

The optimal designs achieved through the sizing optimization and the simultaneous layout and sizing optimization are compared and the results are presented in the Table 3. The buckling constraints are satisfied in both the cases. The simultaneous layout and sizing optimization has produced about 33% lighter design in comparison with the design achieved through the sizing optimization. The stiffness of the panel, in this case, is tailored predominantly by the movement of its stiffeners towards the right side of the skin at the bottom edge. A similar behavior is observed in the benchmark panel, where a mass reduction of about 30% is achieved through the simultaneous layout and sizing optimization. It can be seen that the optimal mass achieved through the sizing optimization is within 1% of the benchmark panel value. The mode shapes are also similar. However, a lower optimal mass is achieved when compared to the benchmark panel through the simultaneous layout and sizing optimization. The mode shapes are also different. The reason for this can be attributed to the difference in the parameterization scheme used to manipulate the stiffener layout in both the cases.

In the CAD-based parameterization herein employed, every stiffener can be positioned within 60% of the available space on the skin and the use of re-meshing avoided the mesh distortion issues. A restricted movement of stiffeners is observed in case of benchmark panel, possibly to avoid the mesh distortion issues given that the authors [9] used the Free-Form Deformation (FFD) method, which is more restrictive than the re-meshing scheme herein adopted. It is evident from the mode shape results that the global buckling mode do not cross the stiffeners and remains localized between the stiffeners.

Table 3 - Optimal design results comparison with reference data

Optimization	**Optimal Design [Mode 1]**	**Buckling Load Factor**	**Mass (kg)**	**Mass Relative Difference (%)**
Sizing		1.0	0.675	32.74
Layout + Sizing		1.0	0.454	
Sizing [ref]		1.0	0.670	29.55
Layout + Sizing [Ref]		1.0	0.472	

The optimal design variables values achieved through sizing optimization and simultaneous layout and sizing optimization are tabulated in Table 4. It can be observed that during the sizing optimization the thickness of the skin, represented as DV 1, is slightly reduced while the thicknesses of all the stiffeners are reduced to their lower bounds. This is explained due to the fact that the stiffeners are not directly loaded and the in-plane bending load is transferred to them through the skin. The authors kept this original loading condition to make the results comparable with the benchmark study [9]. The trend herein observed is that the thickness of the skin is adjusted by the optimizer to meet the buckling constraint while maintaining a minimum thickness for all the stiffeners. A different trend is observed in the simultaneous layout and sizing optimization, where the thicknesses of the skin and the 6 stiffeners are reduced to their lower bounds. The thickness of the stiffener at the extreme right end is slightly reduced whereas the stiffeners are shifted towards the right end of the skin at the bottom edge in order to meet the buckling constraint. It can be concluded from this that the buckling response of the current stiffened panel is more sensitive to its layout design variables than the sizing design variables, when optimized simultaneously.

Table 4 - Design variable results

Design Variable	Design Variable Type	Initial Design	Sizing Optimal	Layout and Sizing Optimal
DV 1	**Sizing** [L.B = 0.001, U.B = 0.003]	0.002	0.0019	0.001
DV 2		0.002	0.001	0.001
DV 3		0.002	0.001	0.001
DV 4		0.002	0.001	0.001
DV 5		0.002	0.001	0.001
DV 6		0.002	0.001	0.001
DV 7		0.002	0.001	0.001
DV 8		0.002	0.001	0.0016
DV 9	**Layout** [L.B = 0.05, U.B = 0.6]	0.125	-	0.599
DV 10		0.125	-	0.155
DV 11		0.143	-	0.537
DV 12		0.143	-	0.151
DV 13		0.167	-	0.482
DV 14		0.167	-	0.191
DV 15		0.200	-	0.449
DV 16		0.200	-	0.272
DV 17		0.250	-	0.489
DV 18		0.250	-	0.345
DV 19		0.333	-	0.584
DV 20		0.333	-	0.508
DV 21		0.500	-	0.595
DV 22		0.500	-	0.600

5. Conclusions

A design methodology for simultaneous layout and sizing optimization of stiffened panels was proposed. The layout variables from CAD-based parameterization were used as design variables in the gradient-based optimization, enabled by using the geometric sensitivities. The effectiveness of the proposed method was successfully demonstrated in a re-design investigation of a metallic stiffened panel taken from literature. The better optimum was achieved in the case of simultaneous layout and sizing optimization because greater layout changes were allowed with the presented methodology that is based on re-meshing in place of mesh morphing, thus avoiding mesh distortion issues for more extreme layout changes throughout the optimization. The proposed methodology is not limited to the stiffened panels, and can be used for the design of complex configurations such as wings or other thin-walled structures. The use of semi-analytical response sensitivities in the proposed gradient-based optimization scheme makes the methodology especially suitable for preliminary design.

6. Contact Author Email Address

The contact author email address: h.rahman@tudelft.nl

7. Copyright Statement

References

[1] K. Taminger and R. Hafley, “Electron beam freeform fabrication: a rapid metal deposition process,” *Proc. 3rd Annu. Automot. Compos. Conf.*, pp. 9–10, 2003, [Online]. Available: http://www.ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20040042496_2004036110.pdf.

[2] S. Arranz, A. Sohouli, and A. Suleman, “Buckling optimization of variable stiffness composite panels for curvilinear fibers and grid stiffeners,” *Journal of Composites Science*, vol. 5, no. 12. 2021, doi: 10.3390/jcs5120324.

[3] H. An, S. Chen, and H. Huang, “Concurrent optimization of stacking sequence and stiffener layout of a composite stiffened panel,” *Engineering Optimization*, vol. 51, no. 4. pp. 608–626, 2019, doi: 10.1080/0305215X.2018.1492570.

[4] B. Colson, M. Bruyneel, S. Grihon, C. Raick, and A. Remouchamps, “Optimization methods for advanced design of aircraft panels: A comparison,” *Optim. Eng.*, vol. 11, no. 4, pp. 583–596, 2010, doi: 10.1007/s11081-008-9077-8.

[5] R. K. Kapania, J. Li, and H. Kapoor, “Optimal design of unitized panels with curvilinear stiffeners,” *Collect. Tech. Pap. - AIAA 5th ATIO AIAA 16th Light. Syst. Technol. Conf. Balloon Syst. Conf.*, vol. 3, no. September, pp. 1708–1737, 2005, doi: 10.2514/6.2005-7482.

[6] S. B. Mulani, W. C. H. Slemp, and R. K. Kapania, “EBF3PanelOpt: A framework for curvilinear stiffened panels optimization under multiple load cases,” *13th AIAA/ISSMO Multidiscip. Anal. Optim. Conf. 2010*, no. September, 2010, doi: 10.2514/6.2010-9238.

[7] M. Bhatia, R. K. Kapania, and D. Evans, “Comparative study on optimal stiffener placement for curvilinearly stiffened panels,” *J. Aircr.*, vol. 48, no. 1, pp. 77–91, 2011, doi: 10.2514/1.C000234.

[8] H. Chen, Y. Xu, J. Hu, and X. Wang, “Optimization of lightweight sub-stiffened panels with buckling analysis and imperfection sensitivity analysis,” *Proc. Inst. Mech. Eng. Part G J. Aerosp. Eng.*, vol. 233, no. 15, pp. 5507–5521, 2019, doi: 10.1177/0954410019856782.

[9] S. Chu, S. Townsend, C. Featherston, and H. A. Kim, “Simultaneous size, layout and topology optimization of stiffened panels under buckling constraints,” *AIAA Scitech 2021 Forum*, no. January, pp. 1–19, 2021, doi: 10.2514/6.2021-1894.

[10] S. Chu, C. Featherston, and H. A. Kim, “Design of stiffened panels for stress and buckling via topology optimization,” *Struct. Multidiscip. Optim.*, vol. 64, no. 5, pp. 3123–3146, 2021, doi: 10.1007/s00158-021-03062-3.

[11] S. Chu, S. Townsend, C. Featherston, and H. A. Kim, “Simultaneous layout and topology optimization of curved stiffened panels,” *AIAA J.*, vol. 59, no. 7, pp. 2768–2783, 2021, doi: 10.2514/1.J060015.

[12] J. S. Gray, J. T. Hwang, J. R. R. A. Martins, K. T. Moore, and B. A. Naylor, “OpenMDAO: an open-source framework for multidisciplinary design, analysis, and optimization,” *Struct. Multidiscip. Optim.*, vol. 59, no. 4, pp. 1075–1104, 2019, doi: 10.1007/s00158-019-02211-z.

[13] R. Haimes and J. F. Dannenhoffer, “The engineering sketch pad: A solid-modeling, feature-based, web-enabled system for building parametric geometry,” *21st AIAA Comput. Fluid Dyn. Conf.*, pp. 1–21, 2013, doi: 10.2514/6.2013-3073.

[14] J. Joe, V. Gandhi, J. F. Dannenhoffer, and H. Dalir, “Rapid generation of parametric aircraft structural models,” *AIAA Scitech 2019 Forum*, no. January, pp. 1–10, 2019, doi: 10.2514/6.2019-2229.

[15] R. A. Canfield, S. Alnaqbi, R. J. Durscher, D. E. Bryson, and R. M. Kolonay, “Shape continuum sensitivity analysis using astros and caps,” *AIAA Scitech 2019 Forum*, no. January, pp. 1–12, 2019, doi: 10.2514/6.2019-2228.

[16] R. Durscher and D. Reedy, “Pycaps: A python interface to the computational aircraft prototype syntheses,” *AIAA Scitech 2019 Forum*, no. January, 2019, doi: 10.2514/6.2019-2226.

[17] “MSC Nastran Design Sensitivity and Optimization User ’ s Guide,” 2018.

[18] J. F. Dannenhoffer and R. Haimes, “Design sensitivity calculations directly on CAD-based geometry,” *53rd AIAA Aerosp. Sci. Meet.*, pp. 1–25, 2015, doi: 10.2514/6.2015-1370.