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A GLOBAL/LOCAL ELEMENT-WISE APPROACH FOR THE BUCKLING AND POST-BUCKLING ANALYSIS OF THIN MULTILAYERED STRUCTURES

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ABSTRACT

An accurate description of the three-dimensional stress field is mandatory for a reliable analysis and design of structures of aerospace interest. Although this, the computational effort for simulating the behavior of always more sophisticated structures is high. A global/local approach can be employed to cut down the computational time required while keeping an high level of accuracy. This research activity proposes a global/local technique for the buckling and post-buckling analysis of thin multilayered structures. It consists in a multi-step procedure. As a first step, a global analysis is conducted with commercial software Nastran to evaluate both the displacements and the most critical areas. Subsequently, those domain are analyzed locally using the previous displacements as boundary constraints, opportunely manipulated, of a refined higher-order local model of the selected elements. In addition, the conducted analyses regard the large displacement and rotation field, so the geometrical nonlinear analysis is performed at both global and local levels. The results demonstrate the capability and reliability of the proposed approach and assess the importance of using a reliable global/local technique for an accurate design of aerospace components.

Keywords: Global/local; Large displacements and rotations; Geometrical nonlinearity; Higher-order local models; Virtual testing.

1 INTRODUCTION

The finite element approach represents on the most adopted tool for the structural analysis in many engineering fields, including the aeronautical one. When dealing with the design of an aircraft structure, its mathematical model is usually made by combining 1D and 2D elements, which opportunely approximate mathematical regions of stringers, panels, ribs, and other components

Of course, this approximation represents a simplification of the real structures. In fact, in most of the application, it is necessary to accurately evaluate the 3D stress fields in particular domains of the model. To accurately describe these localized 3D effects, solid models or high-order theories are often employed. However, in order to balance computational cost and results accuracy, i.e. to make the model more efficient, a global/local approach is often employed.

There are two main global/local approaches possible: (1) refining the mesh or the describing functions only in correspondence of critical areas; (2) formulating multi-model methods, in which different subregions of the structure are analysed with different mathematical models and in different steps. For the first technique, the *h*-adaption method [1] is used when the mesh size is different in the structure subdomains, whereas the *p*-adaption method [2] can be applied when the subregions differ in the polynomial order of the approximating shape functions. Among the second approach, i.e. multiple-model methods, the so-called “Multisteps methods” are popular. Basically, the analysis of the critical local region requires the boundary conditions at the interface level that are extracted by the analysis on the global structure. For instance, in the global/local method proposed by Mao et al. [3], a coarse mesh was used to analyse the entire structure to obtain the nodal displacements which were subsequently used as boundary conditions for the refined local analysis. According to [3], the application of the boundary conditions in the local region unavoidably introduces errors. To minimize the effect of such errors, the local analysis generally requires a region larger than the critical region where accurate stress fields are to be evaluated. Ransom and Knight [4] presented a method for performing a global/local stress analysis using a two-step analysis.

This work proposes a global/local methodology that consists of a two-step procedure for the evaluation of accurate stress fields in critical domains of structures subjected to large displacements and rotations. In the proposed method, the first step is the global analysis, performed with the Nastran commercial software, whereas the second is conducted employing a refined two-dimensional higher-order mathematical model based on CUF. A criterion is established to identify the most critical region from the global analysis, which is subsequently analysed in the second step by using high-order models, to obtain accurate stress fields. The refined theories used in the detailed analysis are implemented in the CUF framework. Recently, this approach was used for the global/local analysis of structures in the linear analysis [5, 6, 7], here the formulation is further extended to deal with large displacements and rotations (see [8, 9]) for the application of CUF in the geometrical nonlinear field).

2 GLOBAL/LOCAL APPROACH

The proposed methodology consists of two-steps. The first step involves the analysis of the global model to identify the critical region using a criterion that is established by the analyst and to extract a proper set of BCs to be applied to the local model which is analysed with CUF. The static analysis on the entire structure is done by the commercial software Nastran, and the displacements and rotations at the interface nodes are known. Then, the most critical regions to be locally analysed are evaluated 8for instance, by looking at maximum stresses). A linear interpolation function is used to maintain conformity with the kinematics of the global model. Furthermore, such interpolation procedures allow the use of the global and local meshes, which are incompatible at the interface. As far as the local element is considered, the Reissner - Mindlin displacement field is used, in order to compute the translational displacements for each node at the interface of the CUF local model. The Reissner - Mindlin displacement field is reported in Eq. (1).

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_y(x,y) \\ v(x,y,z) &= v_0(x,y) - z\theta_x(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u, v are the in-plane directions, w is the transverse direction and u_0, v_0, w_0, θ_y and θ_x are evaluated from the global model. The local model is built using higher-order 2D plate theory based on CUF, according to which, the 3D local displacement field $\boldsymbol{u}(x, y, z)$ can be expressed as a 1D through-the-thickness (z) expansion function of the primary unknowns of the mid-surface (x, y) evaluated with the finite element method. This approach is expressed by Eq. (2).

$$u(x, y, z) = F_\tau(z)\, N_i(x, y)\, \mathrm{q}_{\tau i} \tag{2}$$

where F_τ is the expansion function, N_i is the finite element shape function, $\mathrm{q}_{\tau i}$ is the vector of the nodal unknowns and the indices τ and i represent the number of the terms of the thickness expansion and the number of the finite element nodes, respectively. In this work, cubic interpolation functions based on Lagrange polynomials are used as F_τ and N_i. Each layer of the composite local model has its own expansion function, so it has its kinematic described independently in a Layer-Wise (LW) sense. It must be stated that Lagrange polynomials represent a displacement-based formulation, so the unknowns of the local problem are pure displacements.

Once u, v, w are evaluated within the local model, the $\boldsymbol{K_S}$ secant stiffness matrix can be evaluated. The nonlinear system of equation coming from the principle of virtual work $dLint = dLext$ is then solved with a displacement control method, using the displacement from global model as boundaries.

3 NUMERICAL RESULTS

To verify the accuracy of the proposed nonlinear global/local approach, it is applied to two a composite hinged panel subjected to uniform pressure and a reinforced composite panel subjected to tension force. Finally, the application of a compressed stiffened panel is proposed.

3.1 Composite hinged panel subjected to uniform pressure

In order to show the capability of the proposed nonlinear global/local approach, a composite plate is considered. The structure has a rectangular shape with 0.6 m and 0.8 m edges, and 0.0024 m for the thickness. The material properties are E1 = 165 GPa, E2= 9 GPa, E3 = 9 GPa, ν12= 0.4, G12 = G13 = 5.6 GPa and G23 = 2.8 GPa and the stacking sequence involves 12 layers [0°/90°/45°/-45°/0°/0°]sym. Finally, a uniform transverse pressure is acting on the bottom surface and the plate is hinged on its edges. The static results are reported in Fig. 1, for both linear and nonlinear analyses using the global model and Nastran software, employing 4800 CQUAD4 elements.

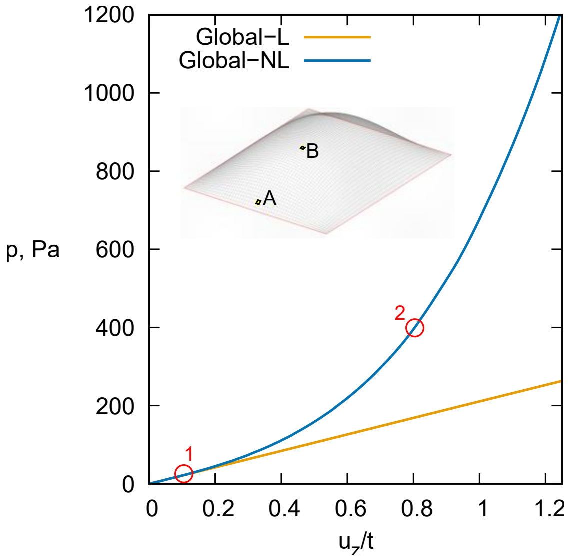

Figure 1 - Linear and nonlinear static results for the composite hinged panel subjected to transverse uniform pressure.

As far as the local analysis is concerned, the elements A and B are selected as local elements, at the equilibrium states 1 and 2 of the nonlinear equilibrium path (A, B, 1 and 2 are reported in Fig. 1). Clearly, the equilibrium state 1 lays in the linear range. For this reason, no difference between linear and nonlinear local analyses are expected. The results are shown in Figs. 2 and 3 and indeed both linear and nonlinear in-plane stress distributions match with those evaluated with the Nastran model.

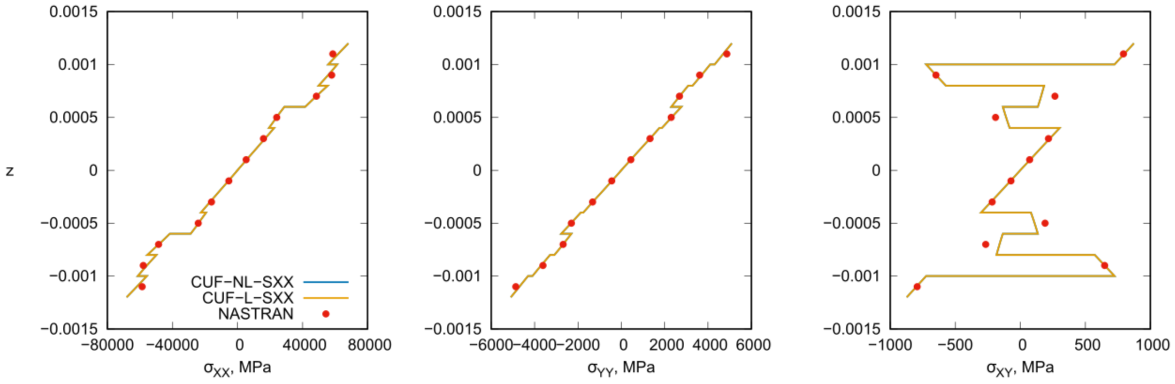

Figure 2 - Local analysis of element A at equilibrium state 1 (see Fig. 1)

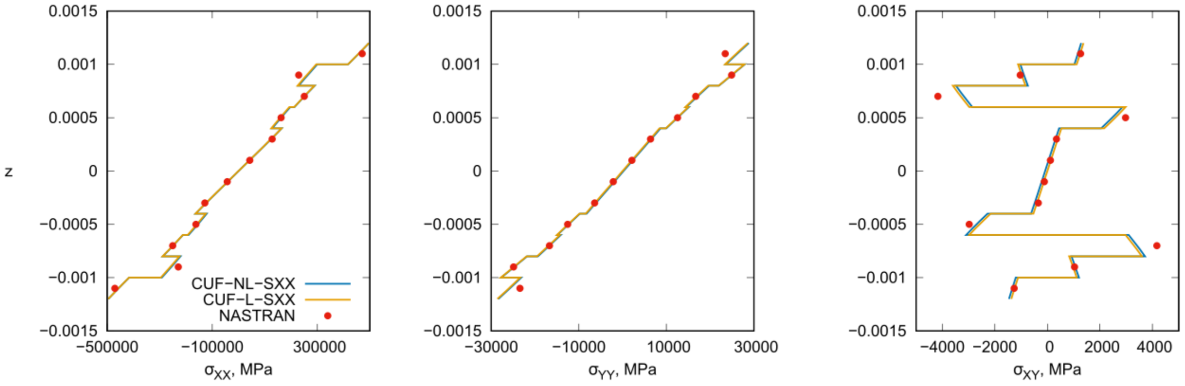

Figure 3 - Local analysis of element B at equilibrium state 1 (see Fig. 1)

On the contrary, the equilibrium state 2 (see Fig. 1) lays on the far nonlinear regime. The correspondent nonlinear results of elements A and B are reported in Figs. 4 and 5.

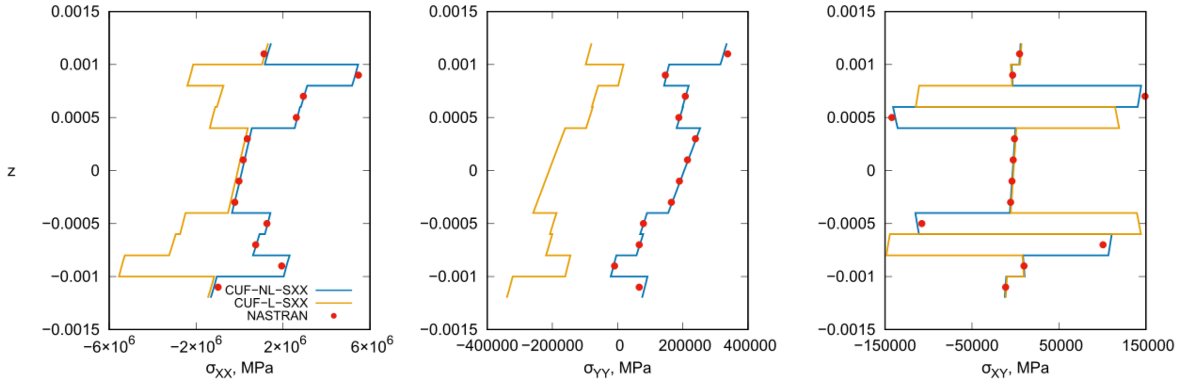

Figure 4 - Local analysis of element A at equilibrium state 2 (see Fig. 1)

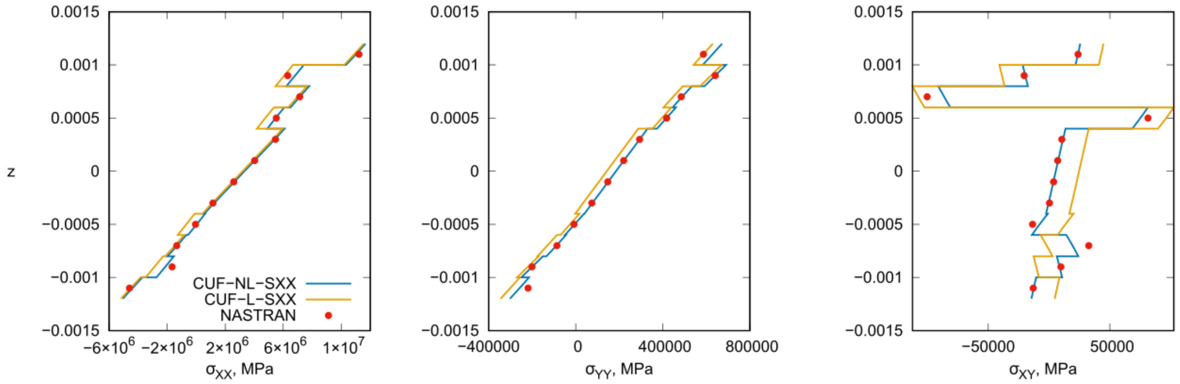

Figure 5- Local analysis of element B at equilibrium state 2 (see Fig. 1)

In these figures, the need of adopting an opportune nonlinear global/local technique is highlighted. The yellow line, correspondent to an equivalent linear solution, shows completely wrong results compared to the red dots from nonlinear Nastran solution, whereas the blue line can accurately evaluate the nonlinear stress distributions.

3.2 Reinforced composite panel subjected to tension force

Three longitudinal reinforcements are added to the previous plate, in order to show the capability of the proposed approach to deal with stiffened structures. The webs of the reinforcements are 0.05 m as height and 0.1 m as width. The structure is clamped at one side and subjected to a traction force, as depicted in Fig. 6.

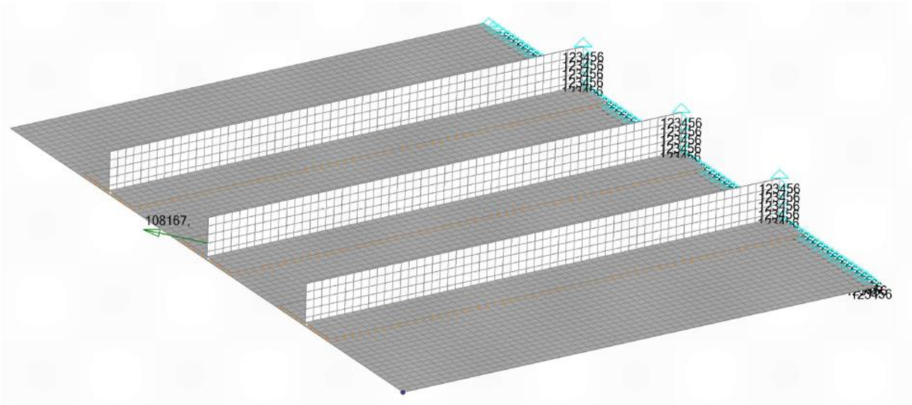

Figure 6 - Geometry of the reinforce composite panel subjected to a traction force.

The correspondent linear and nonlinear static solutions are reported in Fig. 7.

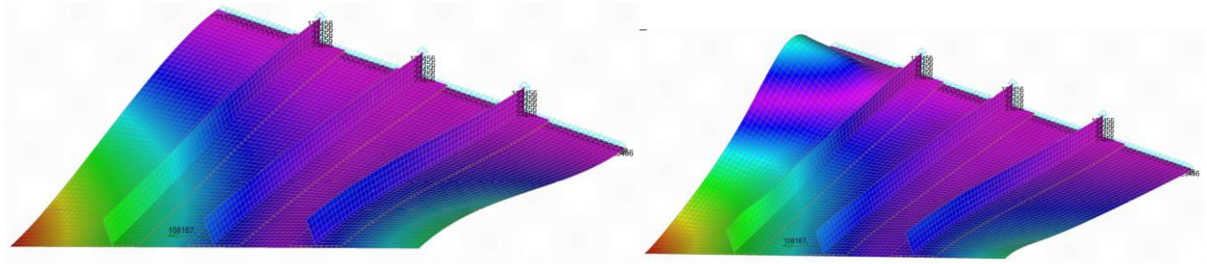

Figure 7 - Linear and nonlinear solution of the reinforced composite panel subjected to a traction force.

The selected elements for the local analysis are those highlighted in Fig. 8

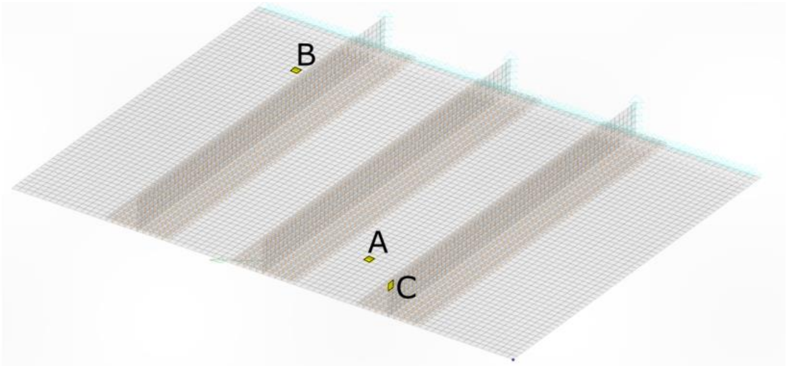

Figure 8 - A, B and C elements selected for the local analysis.

The correspondent nonlinear local in-plane stress distributions are reported in Figs 9, 10 and 11. The same conclusions as in the previous example can be drawn, as the proposed nonlinear global/local procedure can accurately evaluate the stress distribution.

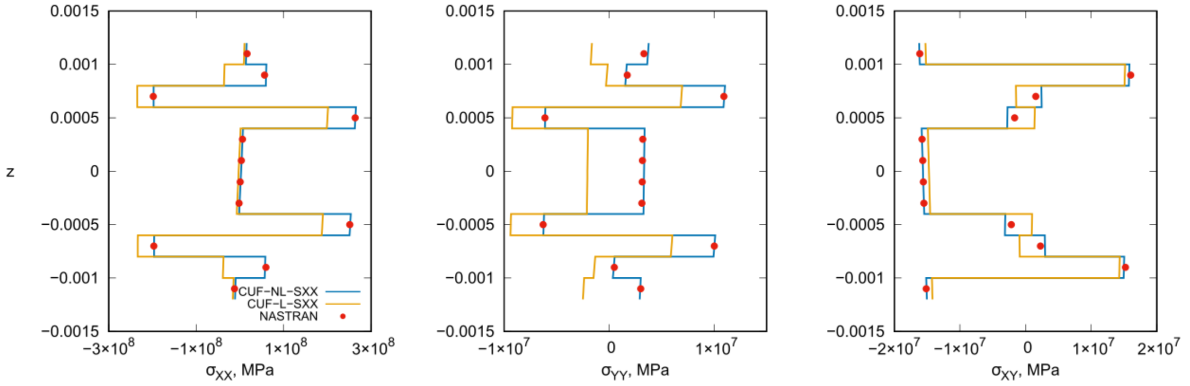

Figure 9 - Nonlinear local analysis of the element A (see Fig. 8)

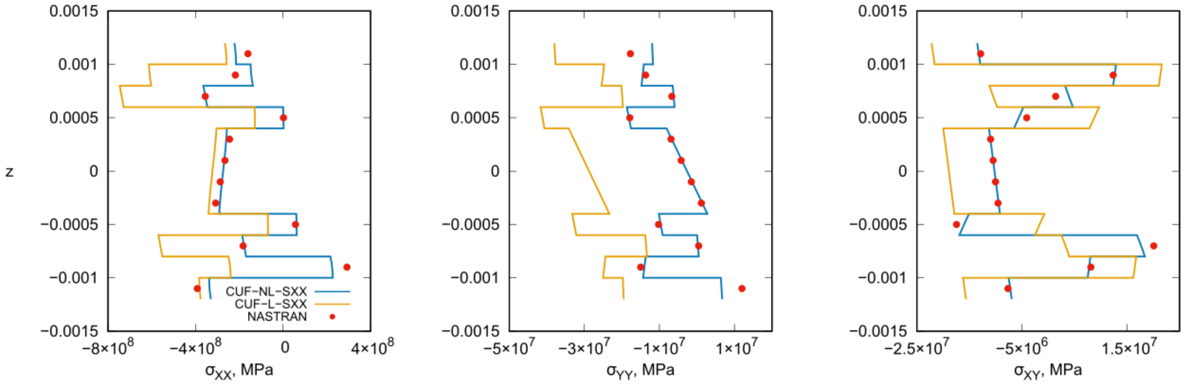

Figure 10 - Nonlinear local analysis of the element B (see Fig. 8)

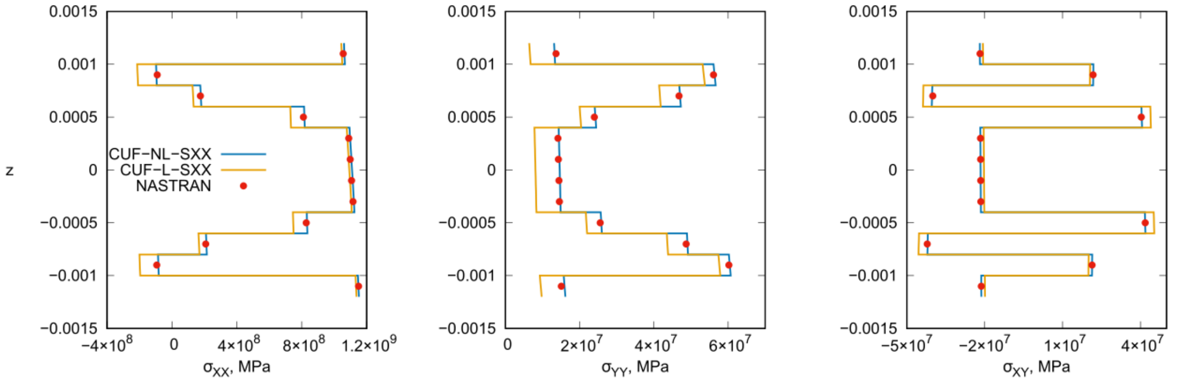

Figure 11 - Nonlinear local analysis of the element C (see Fig. 8)

The main advantage of the proposed technique is to build an equivalent 3D local model with nonlinear capabilities. This leads to the possibility of evaluating the out-of-plane stress components in both centroid and free-edge of an element. This is shown in Fig. 12, where the element C (see Fig. 8) at the tip of the stringer is considered and its σ_{zz} through-the-thickness distribution is depicted.

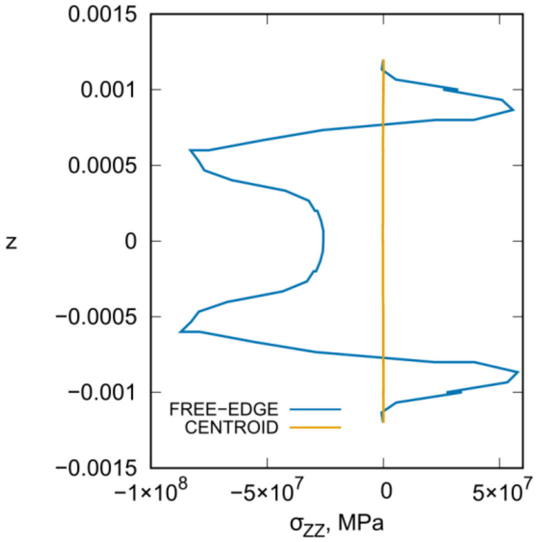

Figure 12 - Difference between free-edge and centroid out-of-plane stress evaluation

3.3 Compressed stiffened panel

Figure 1 shows the geometry and the stacking sequence of the reinforced panel, where h1 = 9.52 mm, h3 = 39.3 mm, h3 = 3.66 mm, t = 7.3 mm, b = 270 mm, b1 = 70 mm, l1 = 50 mm and l = 690 mm. The material properties are E1 = 119 GPa, E2= 119 GPa, E3 = 119 GPa, ν12= 0.316, ν 13= 0.26, ν 23= 0.33, G12= 4.7 GPa, G13 = G23 = 1.76 GPa and ρ = 1580 kg/m3 and the skin of the reinforced panel has 32 layers [90°/45°/0°/0°/- 45°/0°/0°/45°/90°/90°/45°/0°/-45°/0°/0°/-45°]sym, the stringer base has 20 layers [45°/0°/0°/- 45°/0°/0°/-45°/0°/45°/90°]sym, and the stringer web has 40 layers [45°/0°/0°/-45°/0°/0°/- 45°/0°/45°/90°/90°/45°/0°/-45°/0°/0°/-45°/0°/0°/45°]sym.

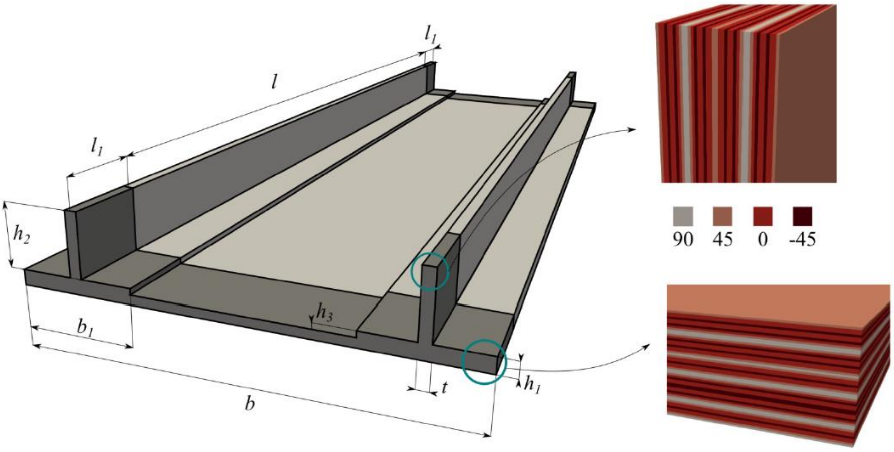

Figure 13 - Geometry and material stacking sequence of the reinforced panel.

The structure is clamped at one side and subjected at the other to an external compressive load equals to 780kN, which has been demonstrated to be near the first critical buckling load of the panel (see [10]). The global Nastran model consists in 4984 CQUA4 elements. An element laying on the free-edge of the web base is considered, to validate the capability of the proposed global/local approach to deal with free-edge effects. The considered is highlighted in Fig. 14.

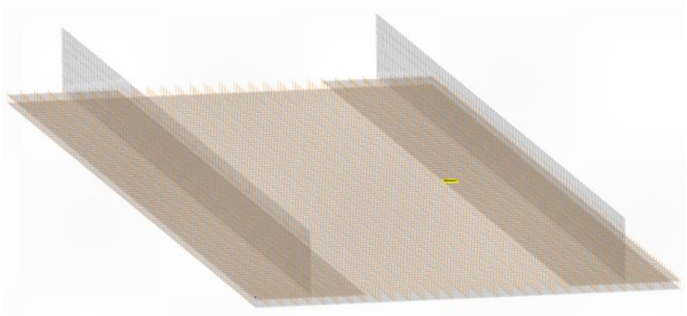

Figure 14 - Considered element at the free-edge of the stringer base.

Both centroid and free-edge stress distributions are evaluated within the local model. If we only considers the σ33, the results are reported in Figure 15.

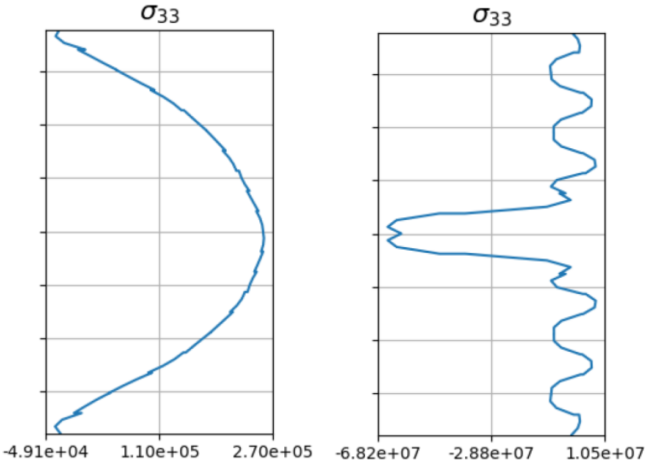

Figure 15 - Transverse stress component evaluated at the centroid and at the free-edge, respectively, of the element on the base of the stringer (see Fig. 15)

The peaks of the free-edge stress distribution is more than 100 times bigger than the one evaluated at the centroid of the element, this means that a proper evaluation of the 3D stress distribution is mandatory. This is evident looking at the correspondent failure indeces (evaluated using Hashin 3D).

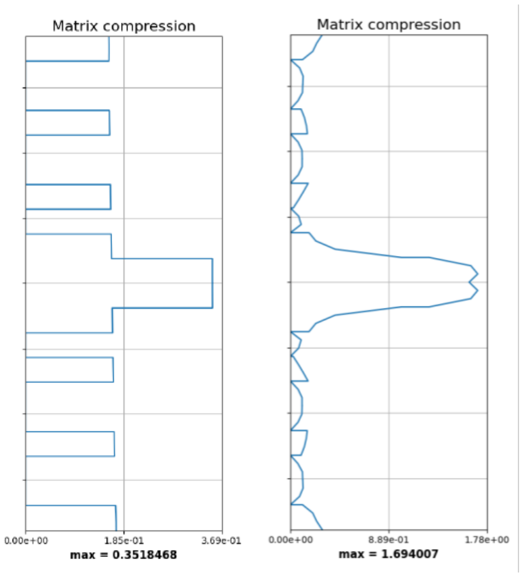

Figure 16 - Failure index evaluated at the centroid and at the free-edge, respectively, of the element at the base of the stringer (see Fig. 14).

4 CONCLUDING REMARKS

The present work proposed a nonlinear global/local approach for the analysis of panels and stiffened panel of aeronautic interest. The approach is based on a two-step procedure, where the global analysis is carried out with commercial software Nastran with a coarse mathematical model, whereas a refined mathematical model based on the Carrera Unified Formulation is adopted for the local elements. The refined model is able to capture 3D effects and stress fields on the nonlinear regime, thanks to a layer-wise description of the composite. The results show that the proposed mathematical model is able to accurately describe the in-plane components of the stress field both in the linear and nonlinear regime compared to the global results. Moreover, it is able to evaluate the σ33 distribution, which is a fundamental design step when dealing with domains with free-edge. The results show that, without a 3D description of the problem, the results could be misleading and eventually lead to a wrong design of the structure. These phenomena are way more evident when dealing with large displacements.

Copyright Statement

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