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VIBRO-ACOUSTIC ANALYSIS OF MULTI-LAYER CYLINDRICAL SHELL-CAVITY SYSTEMS VIA CUF FINITE ELEMENTS

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Abstract

The understanding of the vibro-acoustic behaviour of the aircraft structure is an important step in order to reduce the noise and vibration (N&V) of aircraft itself. A powerful tool is the well-known Finite Element Method (FEM), which allows to study the vibro-acoustic problem in the low frequency range. The FEM applied to vibro-acoustic has a limitation: to increase the frequency range, it is necessary to decrease the size of the elements. Therefore, in order to study complex model as an aircraft structure, the maximum frequency is often too low to have useful results. Moreover, the exploitation in the aeronautical field of multi-layer and sandwich materials with visco-elastic core, leads to the decrease of accuracy of classical shell elements and the need to often adopt solid elements, further increasing the size of the FEM problem. In this work we exploit a powerful notation for shells, the Carrera's unified formulation (CUF), to reduce the size of the vibro-acoustic problem. Curvilinear shell elements are applied to a cylindrical structure, similar to a fuselage, to study different materials, without increasing the size of the problem or losing accuracy in the solution. In this way it will be possible to reach higher frequency in the vibro-acoustic analysis, still describing the behaviour of sandwich and composite structures with the same accuracy of the solid elements.

Keywords: vibro-acoustics, noise & vibrations, Carrera's Unified Formulation, cabin noise

1. Introduction

The acoustic footprint of a commercial passenger aircraft depends on several parameters and conditions, from the flight phase to the most specific characteristics of each component. Although, there are no doubts about the importance of a correct understanding of the aircraft's acoustic behaviour and sure enough several works study this field [1, 2]. The aim is to obtain reliable methods and tools in order to improve the aircraft's acoustic efficiency, thus, to reduce noise and vibrations (N&V) generated by the aircraft. If external noise is considered an environmental problem, which affects the near airport areas, internal noise is related to the passenger's comfort. In this work we will focus on the internal noise since comfort is becoming an important driving factor in commercial aircraft design. Moreover, high level of vibrations inside the aircraft can damage the structures and reduce their operating life and efficiency.

Internal noise is generated by several components in an aircraft [3]. Although, it usually follows two paths from the source to the receiver (e.g., the passenger or the crew): an airborne path or/and a structural one. The former is described by the Wave equation and Helmholtz equation in the time and frequency domain respectively. Therefore, the noise, as an acoustic pressure, spreads through the acoustic cavity (e.g., the passenger cabin or other aircraft cavities filled with air). The latter stands for the interaction of the acoustic waves with solid materials (e.g., the aircraft primary and secondary structures). The sound travels through vibrations, which are displacements in a rigid body. These variables are described by the structural models depending on the structures itself and on the other parameters, as the computational cost and, the needed accuracy. In order to have a complete acoustic model, the interactions between the two paths must be considered, integrating vibro-acoustic theories to describe the fluid-structure coupling. As a matter of fact, numerical methods are the strongest tools to solve the vibro-acoustic equations and thus, to study complex systems in a reliable and flexible way. The Finite Element Method (FEM) is commonly exploited for vibro-acoustic problems in the low and medium frequency range. Classical materials and structures (e.g., single layer plate or beam) can be modelled with classical structural theories and numerical models based on the Equivalent Single Layer (ESL) approach, where the structure is supposed to be an integral equivalent layer. Although, new advanced materials and multi-layer plates have taken hold in the aircraft structures since several decades: not only composite material in aircraft primary structures, but also sandwich plates with honeycomb core play an important role in the aircraft secondary structure and in the acoustic behaviour of the fuselage. Moreover, new advanced solution are understudying by researchers; some of this solution can not be studied with ESL method because involve multi-layer visco-elastic materials, as acoustic metamaterials , or layer-embedded piezo-electric systems. In order to have a reliable numerical model able to understand the complex kinematic behaviour of these structures, we can choose between solid models, which use three-dimensional elements, and refined two-dimensional models, the so called Layer Wise (LW) approach. This numerical approach supposes the structure composed by independent layers. In this way it is possible to have a three-dimensional description of stress–strain state in the material.

Vibro-acoustic FEM model accuracy strongly depends on the elements size, which are inversely proportional to the maximum frequency of the problem. Moreover, a fully coupled vibro-acoustic model we need to model both the structure and the fluid cavity. These considerations strongly increase the number of degree of freedom (DoF) of the problem, limiting the FEM analysis for complex structure, as aircraft, to the very low frequency range, usually for a fuselage FEM model it is difficult to overcome the 200-300 Hz. Moreover, if we exploit a solid model for particular multi-layer structures and acoustic solutions, we obtain a further increase in the DoF of the problem. This increase is partially avoidable with LW model. To summarize, FEM model need high computational power to solve the vibro-acoustic problem of coupled complex system also in the low-medium frequency range, where other method, as the Statistical Energy Analysis (SEA), are not still reliable.

In this work we exploit the Carrera's Unified Formulation (CUF), which enhance a new class of beam and shell models [4]. The advantages of this formulation were already demonstrated in other works, both for the structural and the vibro-acoustic part [5, 6, 7]. The formulation theory will be described in Section 2.2 In the vibro-acoustic field, we limit the CUF to simple plane problem, due to the difficulty to use the through the thickness interpolation on complex three-dimensional structures, as an aircraft fuselage. In particular, in this work [6] we discovered an important difference between ESL model and LW one, also for single layer plate made of an orthotropic material.

In the following sections, an enhancing of this formulation in curvilinear coordinates is developed. The formulation of the shell finite elements used in this work is based on the merging of FEM shape functions and CUF approximating functions in unique 3D approximating functions employed for both displacements and geometry of the shell element, following the philosophy of non-conventional 2D elements presented in [8]. The resulting 3D approximation is then exploited for the derivation of 3D elements in general curvilinear coordinates, as explained in [9], which the shell elements are particular case of. Therefore, a study case is solved: the study of a multi-layer cylindrical shell (a quasi-fuselage structure) coupled to an internal cavity. This example want to be on of the first steps in the exploits of CUF for studying the vibro-acoustic behaviour of complex structure, following the work by Cinefra [10].

Several studies solve the vibro-acoustic response of a cylindrical structure, of a cylindrical cavity or to the coupled problem. The results can be applied to several real problems, as the reduction of vibration of fuselage, studying the both the modal and the frequency response of the structure. Moreover, this geometry has an quasi-analytical solution of the free-vibration problem, both the shell and the cavity, based on the Bessel function. In particular Rona [11] calculated the eigen-frequencies of a cylindrical cavity with free surfaces. Cylindrical shell and cavities are widely exploited as simplified model of fuselage, as in the work by da Rocha et al. [12] and by Missaoui et al. [13], or to study unconventional materials, as the visco-elastic or porous materials, in the work by Boily et al. [14].

2. Governing equation and FEM approximation

2.1 A brief overview of the vibro-acoustic problem

The vibro-acoustic problem can be briefly described as the coupling between the structure and a fluid cavity. Therefore the well-known structural and acoustic systems are exploited to mathematically represent this problem. Moreover, in order to take in account the coupling, two coupling equations, one for each system, are added to connect the two systems. The unknown of the acoustic problem is the fluid pressure p, while for the structure system it is the displacement vector u_i, where the Einstein's notation is applied (u_i represents the components of the vector $\boldsymbol{u} = \{u_1\ u_2\ u_3\}^T$ in which the components are the displacements in the three directions). For sake of simplicity, we report the variational formulation for the coupled system obtained in terms of virtual displacement δu_i and virtual pressure δp:

$$\begin{cases} \int_{\Omega_s} \delta u_i \sigma_{ij} dV + \int_{\Omega_s} \delta u_i \rho_s \ddot{u}_i dV = \int_{\Gamma_N^s} \delta u_i f_i ds + \int_{\Gamma_{fs}} \delta u_i p n_i ds \\ \int_{\Omega_f} \delta p_{,i} p_{,i} dV + \frac{1}{c_f^2} \int_{\Omega_f} \delta p \ddot{p} dV = -\rho_f \int_{\Gamma_{fs}} \delta p \ddot{u} n_i ds \end{cases} \tag{1}$$

Therefore, the vibro-acoustic system is built by three parts:

- the fluid cavity (subscript f) is defined by the domain Ω_f. The cavity is filled by a fluid, whom properties are the density ρ_f and speed of sound c_f. The cavity is closed by rigid walls applied on the domain Γ_N^f;
- the plate, so the elastic structure, is defined by the domain Ω_s and made by a material defined by the stress tensor σ_{ij} and so by the matrix of the elasticity coefficient $\boldsymbol{C}$. On the plate there are regions where can be applied loads, as f_i, Γ_N^s and set displacements Γ_D^s;
- the two regions have a common interface defined by Γ_{fs}. On this interface the fluid-structure coupling happens.

We neglected the body force as assumption.

2.2 Carrera's Unified Formulation

Considering a local curvilinear reference system as represented in Figure 1, a generic shell lays on the local plane $\alpha^1 - \alpha^1$ plane and the local perpendicular axis to its midsurface is α^3. Along the axis α^3, the thickness h (from $-h/2$ to $h/2$) develops on different layers, each pointed by the index k, so the thickness of each layer is h_k. According to the CUF the three-dimensional field of the displacement of a shell can be split in a two-dimensional field on the shell plane and an expansion on the thickness. The in-plane field is described by the chosen shell model $\boldsymbol{u}_\tau(\alpha^1,\, \alpha^2)$, while the expansion on the thickness by the function $F_\tau(\alpha^3)$, called the thickness function:

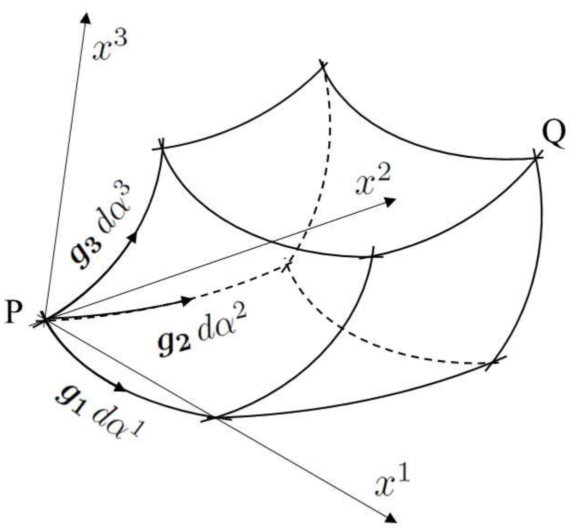

Figure 1 – Local curvilinear reference system and rectangular Cartesian reference system.

$$\boldsymbol{u}\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3}\right)=F_{\tau}\left(\alpha^{3}\right)\boldsymbol{u}_{\tau}\left(\alpha^{1},\,\alpha^{2}\right) \tag{2}$$

In Equivalent Single Layer (ESL) model the thickness functions are expressed on the through-the-thickness domain of the multilayered shells. These theories are usually accurate to estimate the global laminate response, but they became unsuitable if stresses at ply level are required. Moreover they can be inaccurate when there are localized loads or in case of high anisotropy. These inaccuracies are amplified for sandwich structure, where the central layer is much thicker than the faces. In order to correctly estimate the vibro-acoustic response of a vibrating shell, the thickness functions change according to the layer:

$$\boldsymbol{u}^{k}\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3}\right)=F_{\tau}^{k}\left(\alpha^{3}\right)\boldsymbol{u}_{\tau}^{k}\left(\alpha^{1},\,\alpha^{2}\right) \tag{3}$$

where $\alpha_k^3 \in h_k$. The displacement continuity conditions have to be enforced at the layer interfaces in this case.
In a ESL model the thickness functions are based on Taylor expansion $F_\tau = (\alpha^3)^\tau$ where in a finite expansion the highest order is N and so $\tau = 0,...,N$ with $N+1$ functions.
In a LW model the thickness functions are expressed by Lagrange e interpolation polynomials through the thickness of layer k:

$$F_{\tau}^{k}\left(\alpha_{k}^{3}\right)=\prod_{i=0,i\neq\tau}^{N}\frac{\alpha_{k}^{3}-\alpha_{k_i}^{3}}{\alpha_{k_\tau}^{3}-\alpha_{k_i}^{3}} \tag{4}$$

in which α_k^3 is the adimensional thickness coordinate within layer k (the bottom for $\alpha_k^3=-1$ and the top for $\alpha_k^3=1$). The interpolation points are usually equally spaced in the layer k. In order to guarantee the displacement continuity at the interfaces between the layers, the following condition is imposed:

$$\boldsymbol{u}_{t}^{k}=\boldsymbol{u}_{b}^{k+1},\; k=1,...,N_l-1 \tag{5}$$

so the displacement at the top of a layer must equal to that at bottom of the following layer, for the total number of layers N_l. The stress continuity can be obtained if enough Lagrange expansion term are used.
In a vibration analysis, the displacement field is time-dependent and Eq. 3 has the following formulation:

$$\boldsymbol{u}^{k}\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3},\,t\right)=F_{\tau}^{k}\left(\alpha_{k}^{3}\right)\boldsymbol{u}_{\tau}^{k}\left(\alpha^{1},\,\alpha^{2},\,t\right) \tag{6}$$

2.3 FEM approximation

In the FEM approximation the physical domain is divided in elements defined by nodes. In order to build the continuous field for the unknowns from the discrete nodal unknowns, the shape functions N_i are introduced. For the shell elements, the displacement field obtained in Eq. 3 is two-dimensional. The continuous variable of the vibro-acoustic problem, displacement $\boldsymbol{u}_\tau^k$ and pressure p, are calculated from the nodal displacement $\boldsymbol{U}_{\tau i}^k$ and pressure P_i:

$$\begin{aligned}&\boldsymbol{u}_{\tau}^{k}\left(\alpha^{1},\,\alpha^{2},\,t\right)=N_{i}\left(\alpha^{1},\,\alpha^{2}\right)\boldsymbol{U}_{\tau i}^{k}(t)\\&i=1,...,m_u\;\tau=1,...,n_i^k\end{aligned} \tag{7}$$

$$\begin{aligned}&p\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3},\,t\right)=N_{i}\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3}\right)P_{i}(t)\\&i=1,...,m_p\end{aligned} \tag{8}$$

in which i is nodal index inside the element, m_u and m_p the number of structural and fluid nodes respectively, n_i^k is the number of adopted LW expansions in layer k on the corresponding node i. Note that a classical 3D approximation based on 3D Lagrange polynomials is adopted for the acoustic pressure.
On Eq. 7, we apply the CUF approximation:

$$\boldsymbol{u}^{k}\left(\alpha^{1},\,\alpha^{2},\,\alpha^{3},\,t\right)=N_{i}\left(\alpha^{1},\,\alpha^{2}\right)F_{\tau}^{k}\left(\alpha_{k}^{3}\right)\boldsymbol{u}_{\tau i}^{k}(t) \tag{9}$$

In a similar way is possible to obtain $\delta \boldsymbol{u}^k$ and δp:

$$\delta \boldsymbol{u}^k \left(\alpha^1,\ \alpha^2,\ \alpha^3,\ t\right) = N_i \left(\alpha^1,\ \alpha^2\right) F_\tau^k \left(\alpha_k^3\right) \delta \boldsymbol{u}_{\tau i}^k(t) \tag{10}$$

$$\delta p \left(\alpha^1,\ \alpha^2,\ \alpha^3,\ t\right) = N_j \left(\alpha^1,\ \alpha^2,\ \alpha^3\right) \delta P_j(t) \tag{11}$$

where indices j and s have the same meaning of i and τ, respectively.

2.4 A new class of shell finite elements

The formulation of the shell finite elements used in this work is based on the merging of FEM shape functions and CUF approximating functions in non-conventional 3D approximating functions employed for both displacements and geometry of shell element, following the philosophy of non-conventional 2D elements presented in [8]. The resulting 3D approximation is then exploited for the derivation of 3D elements in general curvilinear coordinates, as explained in [9], which the shell elements are particular case of. In the following, the authors try to summarize the main steps of this derivation but the readers are invited to refer to the works [8, 9] for more details.

2.4.1 Non-conventional 3D modelling

In the framework of FEM, it is possible to extend the models of Carrera Unified Formulation to the modelling of generic curvilinear geometries by incorporating the CUF kinematic assumption and the FEM discretization in a unique 3D approximation, as follows:

$$\boldsymbol{u}^k(\alpha^1,\alpha^2,\alpha^3) = (F_\tau^k N_i)\boldsymbol{U}_{\tau i}^k = L_{\tau i}^k(\alpha^1,\alpha^2,\alpha^3)\boldsymbol{U}_{\tau i}^k \tag{12}$$

where $L_{\tau i}^k(\alpha^1,\alpha^2,\alpha^3) = (F_\tau N_i)$. In this expression, $L_{\tau i}$ represents a non-conventional 3D shape function in which the order of expansion can be different along one of the spatial directions. Similarly, the virtual variation δ of displacements, that will be used in the derivation of governing equations below, can be approximated by:

$$\delta \boldsymbol{u}^k = (F_s^k N_j)\delta \boldsymbol{U}_{sj}^k = L_{sj}^k(\alpha^1,\alpha^2,\alpha^3)\delta \boldsymbol{U}_{sj}^k \tag{13}$$

Considering this formalism, the volume integrals involved in the governing equations will be not split in 1D and 2D integrals as usual, but the functions $(N_i F_\tau)$ and $(N_j F_s)$ will be handled as regular 3D shape functions; consequently, the Jacobian matrix relative to the transformation from natural coordinates $\alpha^1,\alpha^2,\alpha^3$ to global coordinates x,y,z will be computed in 3D form. However, a formal separation of the coordinates as in Equation (3) is still possible and the choice CUF FE approximations follows the same principles as in conventional shell elements based on CUF, that is functions with higher polynomial order are employed along the larger directions.
The results will demonstrate that this particular approach allow us to save degrees of freedom with respect to the use of meshes based on classical 3D finite elements. Indeed, conventional 3D elements employ the same order of expansion in the three spatial directions and this usually implies some limitations on the choice of the aspect ratio of the element by leading to a detrimental increase of the number of elements used.

2.4.2 Interpolation of geometry

The interpolation of solid geometry is easily accomplished using an isoparametric procedure. The position vector of the generic point P in the discretized domain is given by:

$$\boldsymbol{x}^k(\alpha^1,\alpha^2,\alpha^3) = L_{i\tau}^k(\alpha^1,\alpha^2,\alpha^3)\boldsymbol{x}_{i\tau}^k \tag{14}$$

where the Einstein notation has been adopted and a summation on the repeated index $i = 1,...,n$ is implicit; n is the number of the nodes of the solid element. Hexahedral elements are considered in this work and L_i is the Lagrangian shape function corresponding to node i. $\boldsymbol{x}_{i\tau} = \{x_{i\tau}^1, x_{i\tau}^2, x_{i\tau}^3\}^T$ is the position vector for the node $i\tau$, corresponding to the nodal displacement $\boldsymbol{U}_{i\tau}$ in Eq.7. The basis of the Cartesian coordinate system, in which $\boldsymbol{x}_{i\tau}$ is defined, is given by the usual unit base vectors $(\boldsymbol{i_1}, \boldsymbol{i_2}, \boldsymbol{i_3})$. From this point on, the formulation of present shell elements follows the same derivation presented in [9] for 3D finite elements in curvilinear coordinates.
Note that, the number of degrees of freedom in these elements is not increased with respect to classical shell elements formulated in the framework of CUF [15, 16].

2.5 Fundamental matrices

Applying the finite element approximation and the CUF, defined in Eq. 7 and Eq. 8, in the system 1, we obtain the following matricial system:

$$\begin{bmatrix} \boldsymbol{M} & 0 \\ -\rho_f \boldsymbol{S}^T & \boldsymbol{Q} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K} & \boldsymbol{S} \\ 0 & \boldsymbol{H} \end{bmatrix} \cdot \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_s \\ \boldsymbol{F}_p \end{Bmatrix} \tag{15}$$

This system has as unknowns the displacements vector $\boldsymbol{U}$ and the pressure vector $\boldsymbol{P}$, the first term represents the mass matrix of the system, while the second the stiffness matrix. The mass matrix is not diagonal due to the coupling term, although for low density fluid can be neglected, simplifying the resolution of the system. The mass and stiffness matrix of the structure $\boldsymbol{M}$ and $\boldsymbol{K}$, and those of the fluid $\boldsymbol{Q}$ and $\boldsymbol{H}$, the coupling matrix $\boldsymbol{S}$ and the structural and acoustic external loads $\boldsymbol{F}_s$ and $\boldsymbol{F}_p$ (this last term is not included in Eq. 1, but it can be added depending on the chosen boundary conditions). This matrices are written using the fundamental nuclei formulation. A fundamental nucleus does not depend on the problem unknown, but only on the thickness and shape functions, the materials properties and on the differential operators. The formulation of these nuclei is referred to those reported in several works [4, 6, 10].
The system can be transformed in the frequency domain through the Fourier transform:

$$\begin{bmatrix} -\omega^2 \boldsymbol{M} + \boldsymbol{K} & \boldsymbol{S} \\ -\rho_f \omega^2 \boldsymbol{S} & -\omega^2 \boldsymbol{Q} + \boldsymbol{H} \end{bmatrix} \cdot \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_s \\ \boldsymbol{F}_p \end{Bmatrix} \tag{16}$$

The damping terms is absent, but it can be included as the imaginary part of the stiffness matrix, multiplied by ω, the pulsation directly linked to the frequency $f = 2\pi\omega$.

3. Method validation

3.1 Fluid cavity

The fluid cavity is a cylinder, Figure 2(b), with a radius equal to 0.5 m and an eight to 2 m. It is filled with air (speed of sound $c = 346\ m/s$ and density $\rho = 1.225\ kg/m^3$). The cavity is modeled by 6000 Hexa27 elements. A rigid wall on the cylinder's bases and on the lateral surface is automatically imposed. On a rigid wall the normal velocity is zero, and so the acceleration, we obtain a the following boundary condition $p_{,i} = 0$ (Neumann boundary condition). The first 6 modes are compared with those obtained by an ESL method in Table 1. The results for this simple case show the high similarity between the frequencies and the shapes do not present any difference.

Table 1 – The first six eigen-frequencies for the cylindrical cavity calculated exploiting the CUF theory (MUL2) and an ESL one.

Mode	CUF-LW3	ESL	Difference [%]
1	86.55	86.5	0.058
2	173.10	173	0.058
3	220.59	220.46	0.057
4	220.59	220.46	0.057
5	259.66	259.5	0.061
6	266.70	266.55	0.058

3.2 Cylindrical shell

In order to validate the shell in Figure 2(a), we compare the first 6 modes obtained by a LW approach to those obtained by an ESL approach. We use a cylinder with the following characteristics:

- radius 1 m, height 2 m, thickness 1 cm;
- material Aluminum (Young's modulus $E = 71\ GPa$, Poisson's ratio $\nu = 0.33$ and density $\rho = 2770\ kg/m^3$);

- clamped on the two edges.

The numerical model is made of 2480 Quad9 shell elements. The first 6 modes are compared in Table 2. The results for this case show the high similarity between the frequencies and the shapes do not present any difference. We choose a LW3 model for the thickness expansion, according to [6]. The differences are low and due to the difference kind of elements, from the plane elements exploited by the ESL approach to the curvilinear one of MUL2.

Table 2 – The first six eigen-frequencies for the cylindrical shell calculated exploiting the CUF-LW3 theory (MUL2) and an ESL one.

Mode	CUF-LW3	ESL	Difference [%]
1	133.28	129.91	2.596
2	133.28	129.91	2.596
3	137.18	135.3	1.393
4	137.18	135.3	1.393
5	149.43	145.94	2.388
6	149.43	145.94	2.388

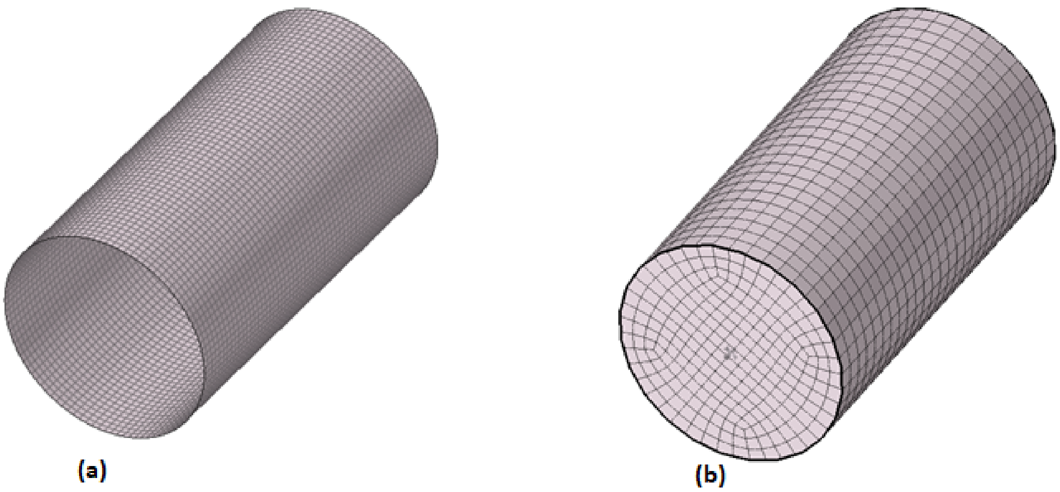

Figure 2 – The FEM geometry on which the natural mode are extracted. (a) Shell structure. (b) Cylindrical fluid cavity.

3.3 Multi-layer shell

In order to exploit the LW model, we consider a similar cylindrical shell validated in Section 3.2 but with a different material and radius, equal to 0.5 m, in order to wrap the cavity in the next analysis. We choose a sandwich material composed by three layer:

- two thin faces made by aluminum and a thickness equal to 1 mm each one;
- a core made by an orthotropic honeycomb material and thickness equal to 8 mm and the damping terms is neglected ($E_1 = E_2 = 1.0\ MPa$, $E_3 = 0.255\ GPa$, $\nu_{12} = 0.49$, $\nu_{13} = \nu_{23} = 0.001$, $G_{12} = 0.1$ MPa, $G_{13} = 37\ MPa$ and $G_{23} = 10\ MPa$).

These materials are chosen to show the limit of an ESL theory with a multi-layer material, where the thicknesses of the layer are very different and the materials properties too. Traditionally these materials are studied through three-dimensional elements in the core [17]. In order to validate the model (and the elements) we use a plate made by 3D elements in the core and 2D (with ESL approach) on the faces.

Table 3 – The first six eigen-frequencies for the cylindrical multi-layer shell calculated exploiting the CUF theory (MUL2) and an ESL one.

Mode	CUF-LW	ESL -3D	Difference [%]
1	8.15E+01	7.91E+01	3.048
2	8.15E+01	8.20E+01	0.572
3	8.58E+01	8.30E+01	3.344
4	8.58E+01	8.80E+01	2.469
5	8.89E+01	8.81E+01	0.965
6	8.89E+01	8.82E+01	0.841

Firstly, the modal solutions of the multi-layer shell is compared in terms of frequencies to those obtained by an ESL-3D model in Table 3.

4. Frequency response

In order to study the frequency response of the coupled model (cavity and shell in Figure 2), we use the same shell and cavity previously validated and coupled. The same boundary conditions are applied and we add an external load: a local force ($F = -1\ N$ on the y direction) on the cylindrical shell placed a the centre ($x = 0\ m$, $y = 0.5\ m$ and $z = 1\ m$). The frequency range starts from 1 Hz and ends to 500 Hz with a step equals to 5 Hz. The frequency response is calculated on two points in the cylindrical cavity ($A = (0.10,\ 0.11,\ 0.65)\ m$ and $B = (0.30,\ 0.13,\ 1.5)\ m$). The material of the shell is the core of multi-layer sandwich described in Section 3.3 while the cavity is filled with air. We compare the results to those obtained for an aluminum shell.

4.1 Results

The results are reported in Figure 3 for Point A and Figure 4 for Point B, they show the pressure in a logarithmic scale as a function of the frequency for the coupled model. The main goal of the work is to reduce the DoF of the problem, for the structure, with respect to a 3D shell model. In our analysis we have 80000 DoF (3 for each structural nodes, the three displacements, and 1 for each fluid nodes, the acoustic pressure). Almost 50000 DoF are used to model the fluid. Therefore, we obtain the results with 30000 DoF in the structure compared to the 100000 DoF necessary to build a 3D shell for the same structure, this number is obtained assuming the 3D elements of a size similar to the thickness of the shell. Therefore there is a reduction of almost of the 70% of the DoF and on the coupled model around the 45%. The exploitation of CUF increases its importance in multi-layer structures and honeycomb materials, because according to [4, 6, 18, 7] . Compared to the aluminum shell, as expected, the honeycomb material has an increase in the vibration due to the lower mechanical properties and therefore an increase in the acoustic pressure in the low frequency range. Moreover, we neglect the damping, the stiffness matrix is real. The differences between the two materials decrease, rising the frequency.

5. Conclusions

In this work we exploit a new class of curvilinear shell elements for a Layer Wise approach. In this way it is possible to study cylindrical structures, as an aircraft's fuselage, in the boundaries of the Carrera's Unified Formulation. The elements and the model are validated through a modal analysis.
The results show the frequency response of the structure, obtained with a decreasing in the number of DoF compared to a 3D shell model. Advantages of this formulation increase if we consider the size of a aircraft fuselage, our model has an height equal to 2 m and a diameter of 1 m. In particular we refer to the multi-layer material in passenger cabin lining panel, usually made of composite material and Nomex, which requires a 3D formulation of the core. This component plays an important role in the N&V absorption.
The next step will be the study of several configurations of multi-layer materials with visco-elastic core and therefore a complex stiffness matrix, in order to validate the formulation and to study new acoustic solutions. Finally, the creation of a FEM model, based on CUF, of an aircraft fuselage will be

the last step of this work, in order to expand the frequency range, without a decrease in the accuracy or an increase in the computational cost of the problem.

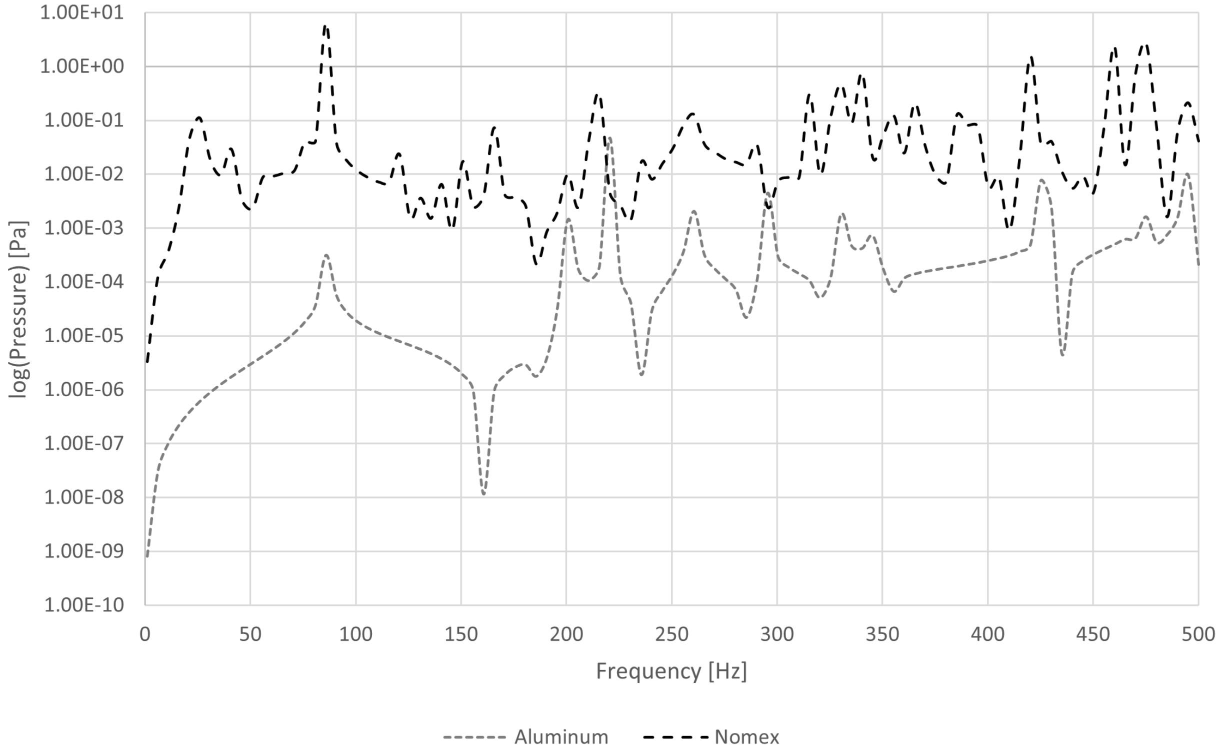

Figure 3 – The frequency response of the cylindrical cavity coupled to a cylindrical shell for Point A, the results show the comparison between an honeycomb material, similar to Nomex, and aluminum.

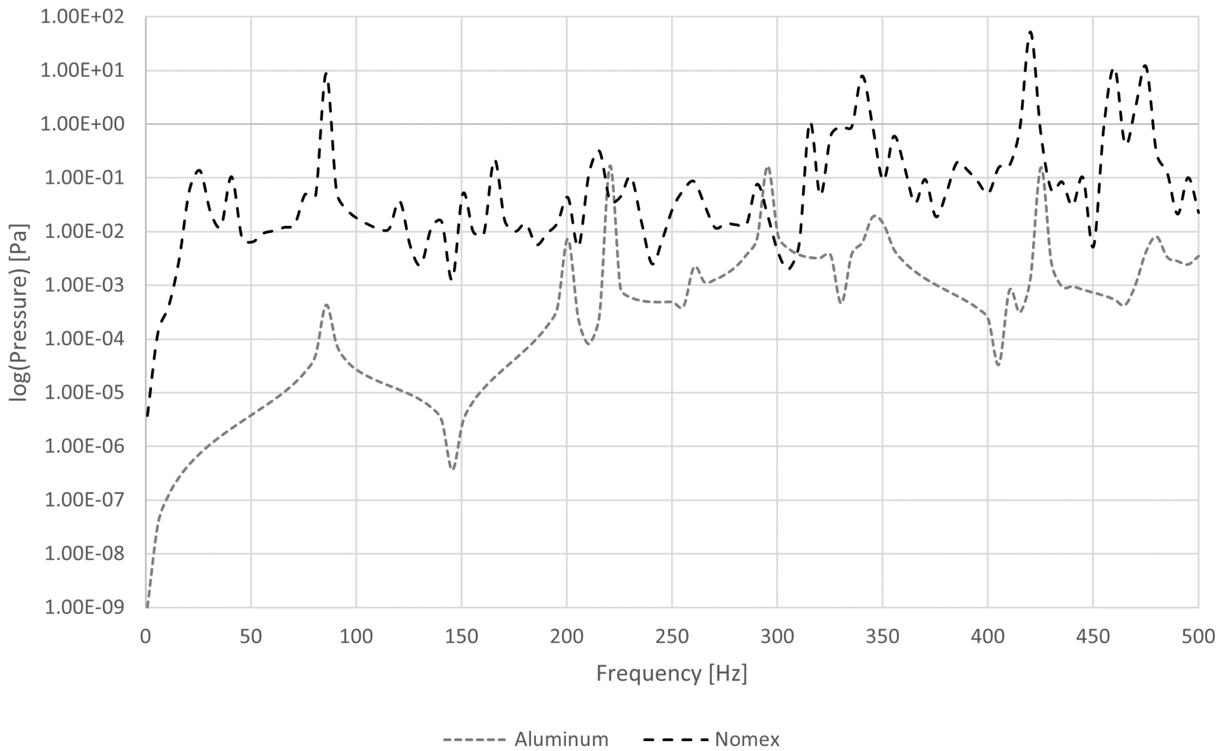

Figure 4 – The frequency response of the cylindrical cavity coupled to a cylindrical shell for Point B, the results show the comparison between an honeycomb material, similar to Nomex, and aluminum

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