33RD CONGRESS
OF THE INTERNATIONAL COUNCIL
OF THE AERONAUTICAL SCIENCES
STOCKHOLM, SWEDEN, 4-9 SEPTEMBER, 2022

ICAS 2022 SWEDEN

ESTIMATION OF STEPWISE VARYING WIND BASED ON DERIVATIVE ESTIMATION OF AIRCRAFT USING SLIDING WINDOW RECURSIVE LEAST SQUARES METHOD

Luu HuySon[1] & Ochi Yoshimasa[2]

[1] National Defense Academy, Yokosuka, Kanagawa, 239-8686, Japan; email: em59047@nda.ac.jp
[2] National Defense Academy, Yokosuka, Kanagawa, 239-8686, Japan; email: ochi@nda.ac.jp

Abstract

This paper considers estimating wind using only the data of an inertial measurement unit without any information from an air-data sensor by estimating stability derivatives of a lateral-directional motion model of an aircraft. Assuming the wind that changes stepwise, we derive the relation between wind and stability derivatives, which allows us to calculate wind. An advantage of this method is that estimated wind is relatively insensitive to the estimation error of the derivatives. We illustrate the effectiveness of the proposed method by simulation using NASA's Generic Transport Model.

Keywords: Aircraft flight dynamics, Parameter estimation, Sliding-window least squares method

1. Introduction

In system identification for linear systems, once the order of the model is determined, derivatives are estimated from measurement data using an appropriate method such as least-squares method. Although estimation methods take account of measurement and/or process noise, which is usually considered as a stochastic signal such as Gaussian noise, when deterministic disturbances exist, we need to incorporate them into the postulated model and estimate or measure. In addition, in order to define state variables of a linear model, we need to know equilibrium states. Although equilibrium states may be known or measured by bringing the system to a steady state, this is not always possible, depending on experiment environment or disturbance. Furthermore, equilibrium states can vary due to steady disturbance, which leads to variation of derivatives. Thus, an estimation method is desired that can simultaneously estimate derivatives, equilibrium states, and disturbances using only state measurement data.

Aircraft usually fly more or less in the presence of wind, and their performance of guidance and control is affected by wind, especially in small or lightweight aircraft. Air data sensors are necessary to measure the wind around an aircraft directly, but many small or unmanned aircraft are not equipped with such sensors. Although wind speed and direction can be obtained from meteorological data, wind varies with time, location, and altitude, and it is not easy to accurately determine the wind in the actual flight environment.

There are many studies on wind estimation, which can be classified into two main categories. One is to estimate the wind using known parameters of a static or dynamic model [1]-[4], and the other is to estimate the wind using known airspeeds (vectors) but without using the parameters [5]-[8]. The previous studies applied estimation methods such as the least-squares method (LSM), (extended) Kalman filter, or nonlinear observer to static aerodynamic model, kinematic model, point-mass motion model, or six-degree-of-freedom motion model, assuming that either model parameters are known, or air data are available. However, to the best of our knowledge, there has been no study of wind estimation in which both airspeed and model parameters are unknown.

This study proposes a method to estimate wind using only data from an inertial measurement unit (IMU), without using information from air data sensors. This study derives a linear motion model of an aircraft that takes wind into account. In this model, some of the derivatives are represented as a linear function of wind. The wind estimation is performed in two steps. First, the derivatives of the wind-aware linear model are estimated from IMU observation data and aileron and rudder angle inputs. Next, the wind is calculated from the estimated values of the derivatives using the relation between the wind and derivatives. Thus, this method uses only IMU data to simultaneously estimate both the derivatives and the wind.

The wind can also be calculated as the difference between ground speed and airspeed if these speeds are measurable. The ground speed can be known using an IMU with global navigation satellite system (GNSS) receivers, a relatively inexpensive measuring device. On the other hand, the airspeed vector is measured and calculated by an air data system with a porous probe, but it requires calibration by a wind tunnel test, making it an expensive system to be mounted on a small unmanned air vehicle [1]. Therefore, the airspeed is measured using an inexpensive Pitot static pressure tube, and the measured airspeed is used together with the measurement data of IMU and GNSS to obtain the wind and airspeed vector (speed magnitude, angle of attack, and sideslip angle). Many studies have been reported on wind estimation and airspeed correction (i.e., estimation of bias) ([9] and the references therein). The methods proposed in those studies so far require parameters of an aerodynamic model or the measurement of airspeed. In ICAS2021, we proposed a method for estimating (constant) wind using only IMU and GNSS measurements (attitude angles, attitude angular velocities, and ground speeds) without using the information of airspeed or parameters of the lateral-directional motion models [9]. This study applies the estimation method to the wind that changes stepwise using a sliding-window recursive least squares method (SW-RLSM) [10].

Although the proposed method can be applied to the longitudinal motion model, estimating the wind in the direction of flight requires data for two flight directions to estimate the two wind components in the horizontal plane [11]. In this study, we apply the method to the lateral-directional motion model, which uses only a single set of flight data. In order to verify the effectiveness of this method, we conducted a simulation using a nonlinear motion model (NASA's Generic Transport Model, GTM (tail number T2)) of a 5.5%-scale twin-engine jet transport aircraft [12]-[13].

This paper is organized as follows. Section 2 gives a general description of the problem to present how additive disturbances to states appear in parameters of a linear system under the assumption that the disturbance is a function of the states. In Sections 3 and 4, we apply this formulation to a lateral-directional motion model of an aircraft, showing how to obtain steady wind from estimated stability derivatives. Section 6 shows numerical analysis results. Finally, Section 7 gives conclusions.

2. Linearized systems with additive disturbances to state variables

Consider the nonlinear system described by the following equation:

$$\dot{x} = f(x + d(x), \delta) \tag{1}$$

where $x \in \mathrm{R}^n$ is the state vector, $d \in \mathrm{R}^n$ is the disturbance vector being a known function of x with unknown parameters, and $\delta \in \mathrm{R}^m$ is the input vector. For a trim input δ^*, let x_0^* be the equilibrium state in the absence of disturbance. Let x^* be the trim state when adding a disturbance d and the disturbance at x^* be $d^* := d(x^*)$.

Applying Taylor series expansion to the right-hand side of (1) around the equilibrium point (x^*, δ^*), we have

$$\dot{x} = f(x^* + d(x^*), \delta^*) + A(\Delta x + \Delta d(x)) + B\Delta\delta + o(\Delta x, \Delta\delta) \tag{2}$$

where $\Delta x = x - x^*$, $\Delta\delta = \delta - \delta^*$,

$$A := \left.\frac{\partial f(x + d(x), \delta)}{\partial \left(x + d(x)\right)^T}\right|_{\substack{x = x^* \\ \delta = \delta^*}}, \quad B := \left.\frac{\partial f(x + d(x), \delta)}{\partial \delta^T}\right|_{\substack{x = x^* \\ \delta = \delta^*}} \tag{3}$$

$$\Delta d(x) := \left.\frac{\partial d(x)}{\partial x^T}\right|_{\substack{x = x^* \\ \delta = \delta^*}} \Delta x \tag{4}$$

and $o(\Delta x, \Delta\delta)$ denotes the higher-order terms. Substituting (3) and (4) into (2) and neglecting the higher-order terms than the first order, (2) becomes

$$\dot{x} = A\left(I_n + \left.\frac{\partial d(x)}{\partial x^T}\right|_{\substack{x=x^* \\ \delta=\delta^*}}\right)\Delta x + B\Delta\delta =: A_d x + B\Delta\delta + x_{Ad}^* \tag{5}$$

where I_n is an *n*-by-*n* identity matrix and x_{Ad}^* is defined as

$$x_{Ad}^* = -A\left(I_n + \left.\frac{\partial d(x)}{\partial x^T}\right|_{\substack{x=x^* \\ \delta=\delta^*}}\right)x^* = -A_d x^* \tag{6}$$

Equation (5) is the estimation model used to estimate A_d, B, and x^*. Note, however, that x^* cannot be estimated directly. Instead, A_d, B, and x_{Ad}^* can be estimated, and then x^* is calculated as $x^* = -A_d^{-1}x_{Ad}^*$.

3. Lateral-directional motion model affected by wind

This section first represents wind defined in the inertial coordinates $\sum_I = (X_I, Y_I, Z_I)$ in the body-fixed frame $\sum_B = (X_B, Y_B, Z_B)$ and then applies the representation to a lateral-directional model with a wind disturbance.

Let the wind in $\sum_I$ be $V_{wI} = [u_{wI}\ v_{wI}\ w_{wI}]^T$, and we can express V_{wI} in $\sum_B$ as

$$V_{wB} = T_{BI}V_{wI} \tag{7}$$

where T_{BI} is the coordinate transformation matrix from $\sum_I$ to $\sum_B$ given by

$$T_{BI}(\Phi,\Theta,\Psi) = \begin{bmatrix} \cos\Theta\cos\Psi & \cos\Theta\sin\Psi & -\sin\Theta \\ \sin\Phi\sin\Theta\cos\Psi - \cos\Phi\sin\Psi & \sin\Phi\sin\Theta\sin\Psi + \cos\Phi\cos\Psi & \sin\Phi\cos\Theta \\ \cos\Phi\sin\Theta\cos\Psi + \sin\Phi\sin\Psi & \cos\Phi\sin\Theta\sin\Psi - \sin\Phi\cos\Psi & \cos\Phi\cos\Theta \end{bmatrix} \tag{8}$$

where Φ, Θ, and Ψ are the roll, pitch, and yaw angles (rad), respectively. From (1) and (2) v_{wB} is given by

$$v_{wB} = u_{wI}\left(\Phi\sin\Theta^*\cos\Psi - \sin\Psi\right) + v_{wI}\left(\Phi\sin\Theta^*\sin\Psi + \cos\Psi\right) + w_{wI}\Phi\cos\Theta^* \tag{9}$$

where Θ^* is a trim pitch angle, and the approximations of $\sin\Phi \cong \Phi$ and $\cos\Phi \cong 1$ are used, assuming small perturbation of aircraft motion from wing-level flight.

The lateral-directional motion in the presence of wind is described by

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\Phi} \end{bmatrix} = \begin{bmatrix} Y_v & W^* + Y_p & -U^* + Y_r & g\cos\Theta^* \\ L_v' & L_p' & L_r' & 0 \\ N_v' & N_p' & N_r' & 0 \\ 0 & 1 & \tan\Theta^* & 0 \end{bmatrix} \begin{bmatrix} v - v_{wB} \\ p \\ r \\ \Phi \end{bmatrix} + \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a}' & L_{\delta_r}' \\ N_{\delta_a}' & N_{\delta_r}' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{10}$$

Where v is the sideward velocity; p and r are the roll and yaw angular velocities (rad/s), respectively; δ_a and δ_r are the deflection angles (deg) of the aileron and rudder, respectively. Substituting (3) into (4) yields

$$\begin{aligned} \begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\Phi} \end{bmatrix} &= \begin{bmatrix} Y_v & W^* + Y_p & -U^* + Y_r & (g - Y_v w_{wI})\cos\Theta^* \\ L_v' & L_p' & L_r' & -L_v' w_{wI}\cos\Theta^* \\ N_v' & N_p' & N_r' & -N_v' w_{wI}\cos\Theta^* \\ 0 & 1 & \tan\Theta^* & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \Phi \end{bmatrix} + \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L_{\delta_a}' & L_{\delta_r}' \\ N_{\delta_a}' & N_{\delta_r}' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \\ &+ \begin{bmatrix} Y_v u_{wI} & Y_v v_{wI} \\ L_v' u_{wI} & L_v' v_{wI} \\ N_v' u_{wI} & N_v' v_{wI} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\sin\Theta^*\Phi\cos\Psi + \sin\Psi \\ -\sin\Theta^*\Phi\sin\Psi - \cos\Psi \end{bmatrix} =: A_{dlat}\begin{bmatrix} v \\ p \\ r \\ \Phi \end{bmatrix} + B_{lat}\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} + D_{lat}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \end{aligned} \tag{11}$$

Ψ is governed by

$$\dot{\Psi} = r\cos\Phi\sec\Theta^* \tag{12}$$

Note that (6) is nonlinear with respect to the yaw angle, which is not a state variable in this linear system, and that the trim states for the lateral-directional motion are zero except for v.

4. Sliding-window recursive least squares method and wind estimation

First, let us define $\theta = [A_{dlat} \quad B_{lat} \quad D_{lat}] \in \mathrm{R}^{4\times 8}$ and $\xi = [v \quad p \quad r \quad \Phi \quad \delta_a \quad \delta_r \quad f_1 \quad f_2]^T \in \mathrm{R}^8$. We can then write (5) as

$$y = \theta\xi \tag{13}$$

where $y = [\dot{v} \quad \dot{p} \quad \dot{r} \quad \dot{\Phi}]^T \in \mathrm{R}^4$. The measurements of the variables are sampled every Δt at t_k ($:= k\Delta t$, $k = 0, 1, 2, \ldots$) and filtered with a second-order Butterworth filter. The derivatives in y are replaced with the filtered output. Given N observations from time t_{k-N+1} to t_k, $Y_k := [y_{k-N+1}\ y_{k-N+2} \ldots y_k] \in \mathrm{R}^{4\times N}$ and $\Xi_k = [\xi_{k-N+1}\ \xi_{k-N+2} \ldots \xi_k] \in \mathrm{R}^{8\times N}$ for $k \geq N$-1 and using the SW-RLSM, we have the estimated parameters:

$$\hat{\theta}(t_k) =: Y_k \Xi_k^T \left(\Xi_k \Xi_k^T\right)^{-1} \tag{14}$$

According to the definition of θ, we have the following equation for the first-row of θ

$$[\theta_{14} \quad \theta_{17} \quad \theta_{18}] = \left[(g - Y_v w_{wI})\cos\Theta^* \quad Y_v u_{wI} \quad Y_v v_{wI}\right] \tag{15}$$

Replacing the parameters with their estimates and solving each equation for the wind elements, we obtain the estimated wind:

$$\hat{V}_{wI}(t_k) = \left[\frac{\hat{\theta}_{17}(t_k)}{\hat{\theta}_{11}(t_k)} \quad \frac{\hat{\theta}_{18}(t_k)}{\hat{\theta}_{11}(t_k)} \quad \frac{-\hat{\theta}_{14}(t_k) + g\cos\Theta^*}{\hat{\theta}_{11}(t_k)\cos\Theta^*}\right]^T \tag{16}$$

where '^' denotes the estimated parameters and $\hat{Y}_v(t_k)$ is expressed as $\hat{\theta}_{11}(t_k)$. Similarly, from the definition of the second and third rows of θ, we can calculate the wind estimates as

$$\hat{V}_{wI}(t_k) = \left[\frac{\hat{\theta}_{27}(t_k)}{\hat{\theta}_{21}(t_k)} \quad \frac{\hat{\theta}_{28}(t_k)}{\hat{\theta}_{21}(t_k)} \quad \frac{-\hat{\theta}_{24}(t_k)}{\hat{\theta}_{21}(t_k)\cos\Theta^*}\right]^T \tag{17}$$

and

$$\hat{V}_{wI}(t_k) = \left[\frac{\hat{\theta}_{37}(t_k)}{\hat{\theta}_{31}(t_k)} \quad \frac{\hat{\theta}_{38}(t_k)}{\hat{\theta}_{31}(t_k)} \quad \frac{-\hat{\theta}_{34}(t_k)}{\hat{\theta}_{31}(t_k)\cos\Theta^*}\right]^T \tag{18}$$

respectively using the estimated derivatives.

5. Simulation results

We generate measurement data for estimation by simulation using NASA's GTM (Generic Transport Model [12]) in the flight condition of the ground speed of 198 ft/s and the angle of attack (AoA) of 2.6 (deg). We assume the wind changing stepwise and add the noise with the standard deviation of $[0.5\ 0.01\ 0.01\ 0.02\ 0.02]^T$ to the variables, v, p, r, Φ, and Ψ computed in the simulation (see [4] and [6]). The sampling period of data is Δt = 0.02s. No turbulence is assumed.

5.1 M-sequence input

Figure 1 shows the input's time history of the aileron and rudder angles. GTM generates the time response of the state variables for the input, as shown in Fig. 2. From the measurements of the variables and inputs, we can estimate the derivative matrices A_{dlat}, B_{lat}, and D_{lat} in (11) by the SW-RLSM. The length of the sliding window in Eq. (14) is N =1000. In the estimation by the SW-RLSM, a second-order Butterworth filter, $1/(s^2+1.41s+1)$, is applied in the form of a digital filter.

Figure 1 – Time history of inputs (aileron and rudder)

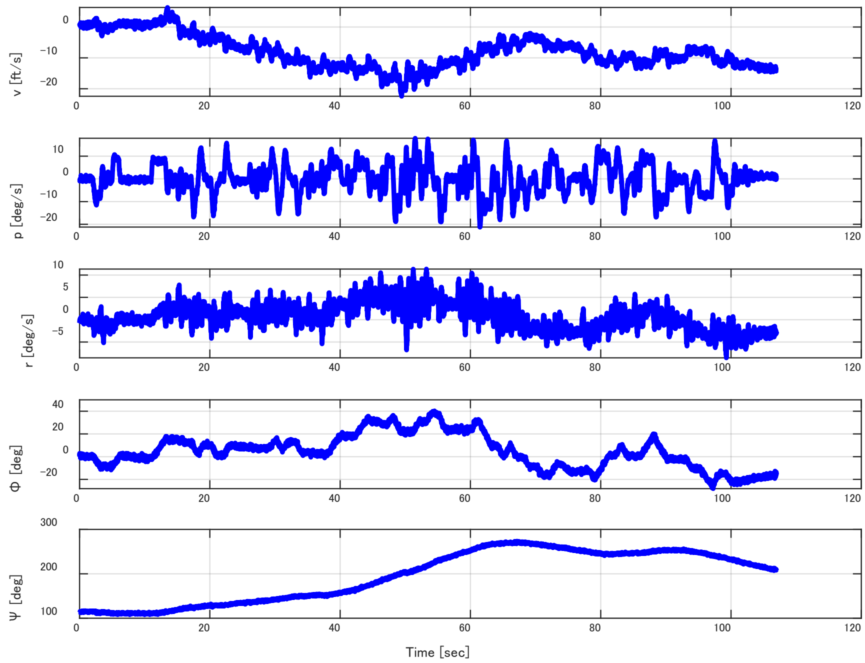

Figure 2 – Time history of measured outputs

Figures 3, 4, and 5 show the estimates of the wind computed using (16), (17), and (18), respectively. According to Fig. 3, a significant estimation error occurs. Figs. 4 and 5 indicate that the wind estimation results were obtained with a minor error, especially estimation error of the wind components in the horizontal plane.

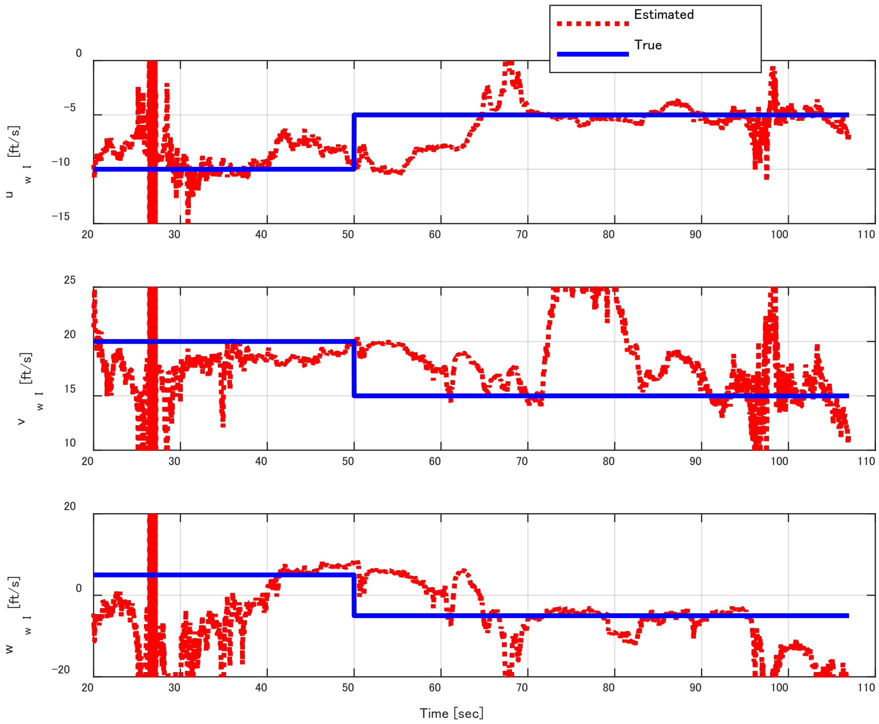

Figure 3 – Estimated wind using (16)

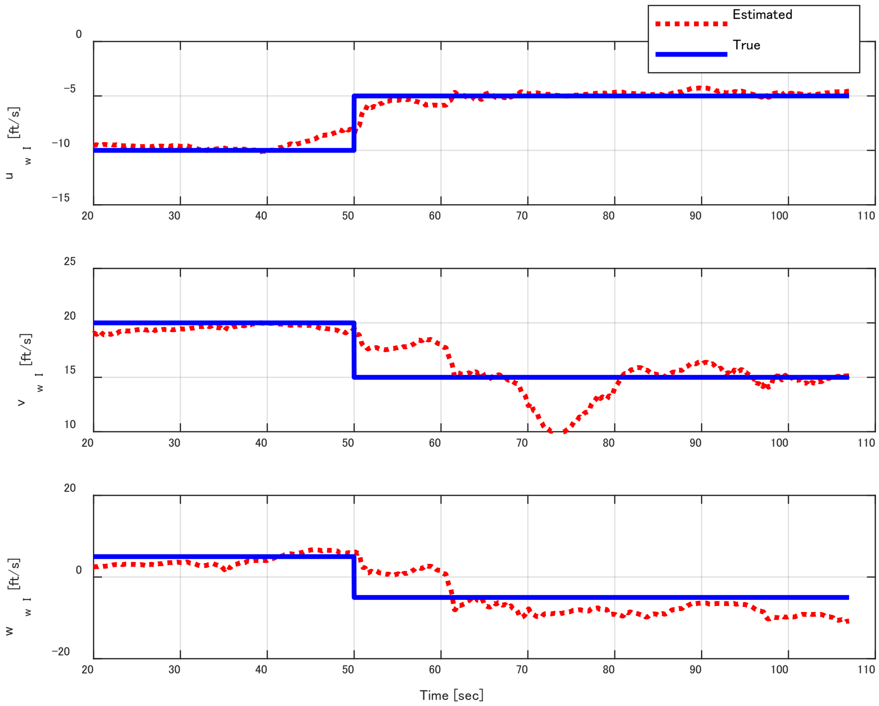

Figure 4 – Estimated wind using (17)

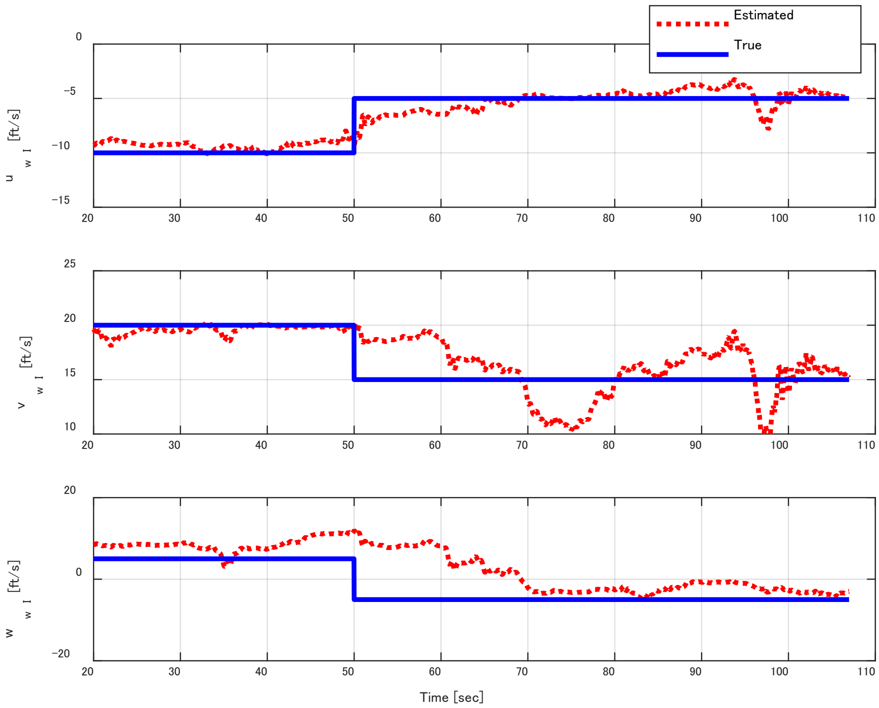

Figure 5 – Estimated wind using (18)

5.2 Damping M-sequence input

Figure 6 shows the input's time history of the aileron and rudder angles. GTM generates the time response of the state variables for the input, as shown in Fig. 7. From the measurements of the variables and inputs, we can estimate the derivative matrices A_{dlat}, B_{lat}, and D_{lat} in (11) by the SW-RLSM. The parameters were estimated with N=1000 in (14).

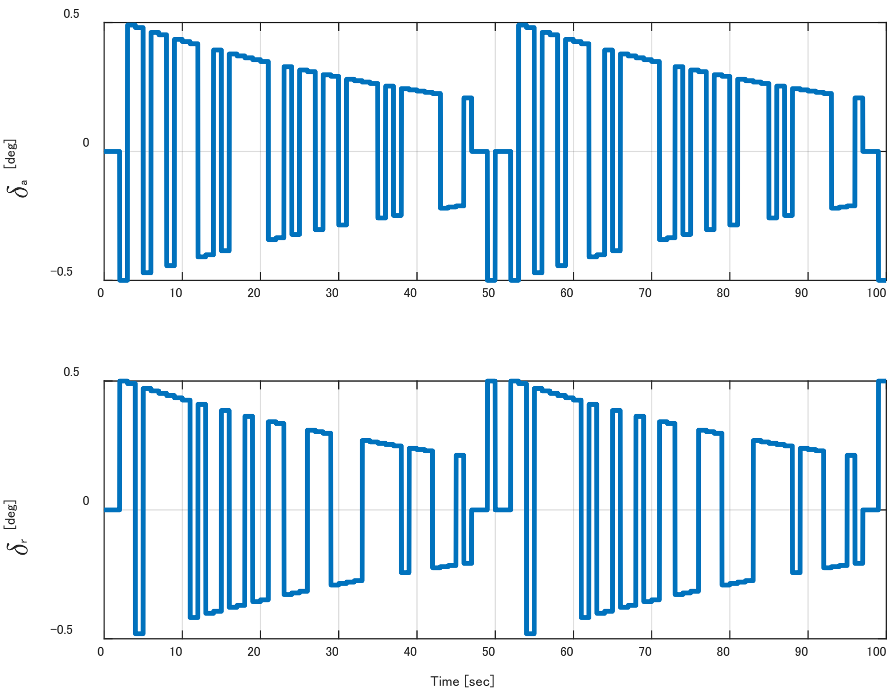

Figure 6 – Time history of inputs (aileron and rudder)

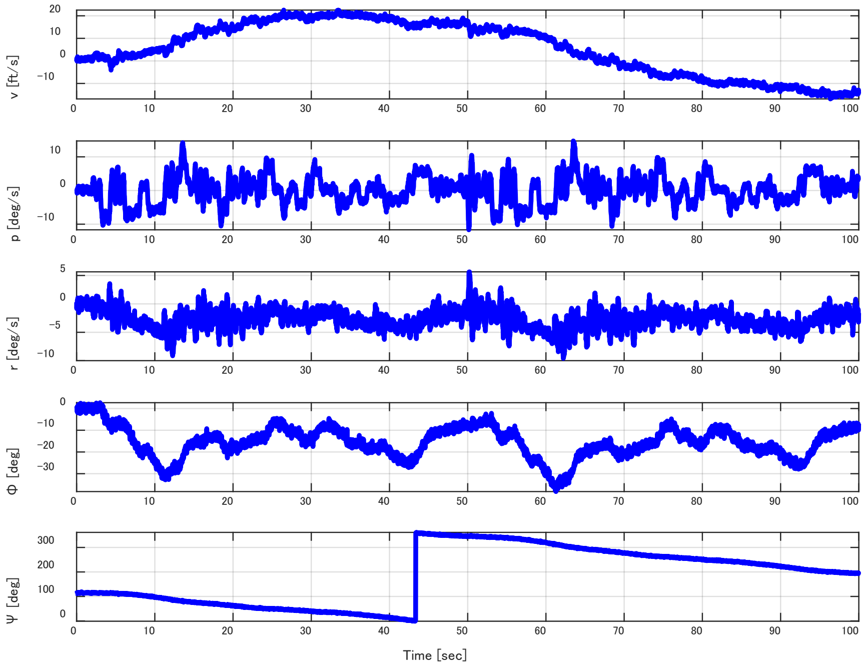

Figure 7 – Time history of measured outputs

Figures 8, 9, and 10 show the estimates of the wind computed using (16), (17), and (18), respectively. According to Fig. 8 shown a significant estimation error occurs using (16). Although Fig. 8 shows a large estimation error, Figs. 9 and 10 indicate the wind estimation results with a relatively minor error. Therefore, we can conclude that the proposed method provides reasonable estimates of the wind changing stepwise, especially using (17) or (18).

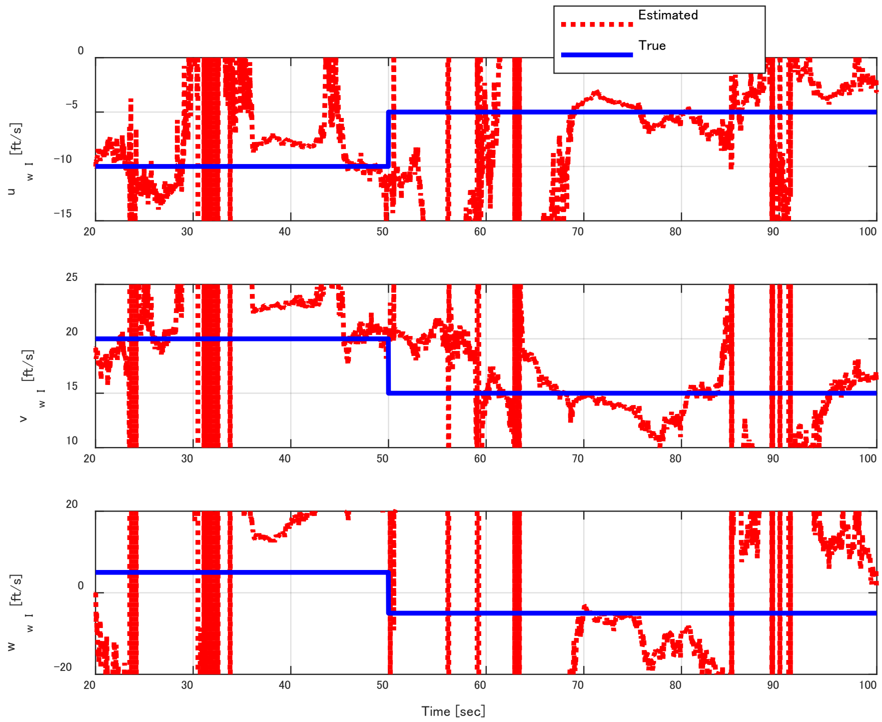

Figure 8 – Estimated wind using (16)

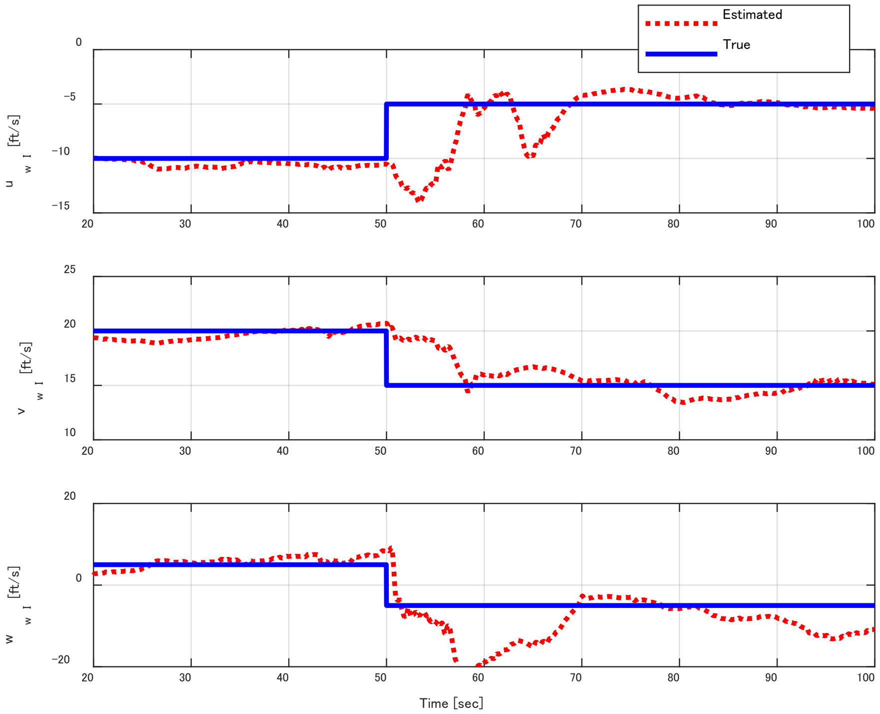

Figure 9 – Estimated wind using (17)

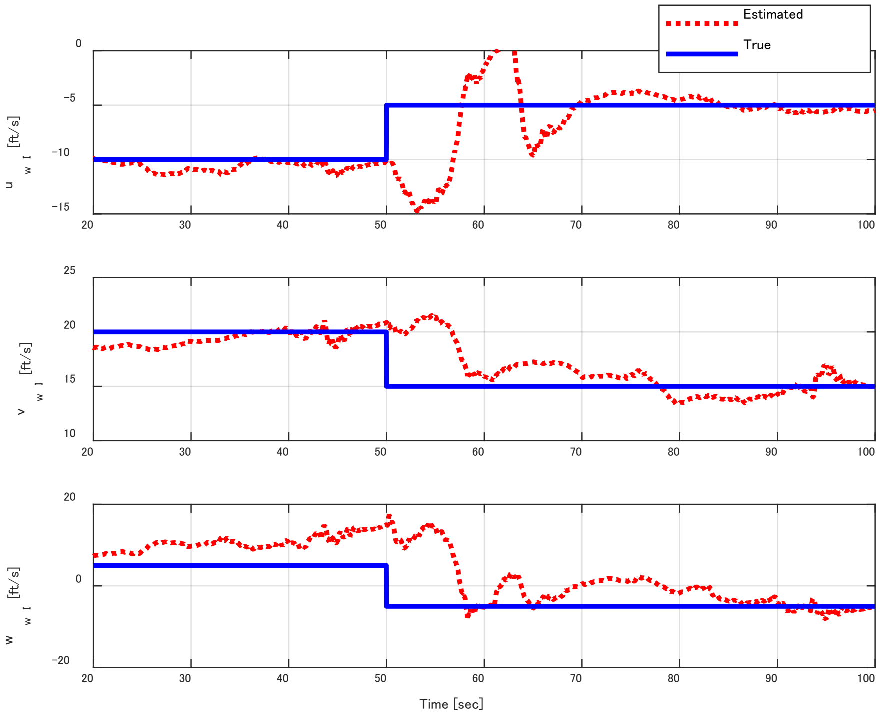

Figure 10 – Estimated wind using (18)

6. Conclusions

This paper has shown that the wind is modeled as an additive disturbance to the states of an aircraft's dynamic model, making the wind appear as part of stability derivatives in the linearized model, which allows us to estimate the wind by estimating the stability derivatives of a lateral-directional motion model of an aircraft. We applied the proposed method to flight data generated by NASA's Generic Transport Model in the presence of the stepwise-changing wind. The simulation results show that the estimation error of the wind was relatively small by the sliding-window recursive least squares method. Especially, we obtained a more accurate estimate of the wind using the roll-rate equation of motion. In future work, we should investigate estimation performance by giving various noises to obtain statistics and other ways of wind variation.

7. Copyright Statement

References

[1] Borup K T, Stovner B N, Fossen T L and Johansen T A. Kalman filters for air data systems bias correction for a fixed-wing UAV, *IEEE Trans. Control Systems Technology*, Vol. 28, No. 6, pp 2164-2176, 2020.

[2] Uemura T and Kato K. A method of estimating down draft from digital flight recorded data, *Trans. JSASS Aerospace Tech. Japan*, Vol. 41, No. 474, pp 32-38, 1993 (in Japanese).

[3] Kitaki Y, Matayoshi N, Suzuki S and Uemura T. Wind vector estimation of a flying airplane, *Trans. JSASS Aerospace Tech. Japan*, Vol. 45, No. 524, pp 58-60, 1997 (in Japanese).

[4] Lie F A P and Gebre-Egziabher D. Synthetic air data system, *Journal of Aircraft*, Vol. 50, No. 4, pp 1234-1249, 2013.

[5] Borup K T, Fossen T I and Johansen T A. A nonlinear model-based wind velocity observer for unmanned aerial vehicles, *IFAC-PapersOnLine*, Vol. 49, No. 18, pp 276-283, 2016.

[6] Langelaan J W, Alley N and Neidhoefer J. Wind field estimation for small unmanned aerial vehicles, *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 34, No. 4, pp 1016-1030, 2011.

[7] Petrich J and Subbarao K. On-board wind speed estimation for UAVs, *Proc. AIAA Guidance, Navigation, and Control Conference and Exhibit*, Portland Oregon, AIAA 2011-6223, 2011.

[8] Johansen T A, Cristofaro A, Sorensen K, Hansen J M and Fossen T I. Estimation of wind velocity, angle-of-attack and sideslip angle of small UAVs using standard sensors, *Proc. ICUAS International Conference on Unmanned Aircraft Systems*, Denver CO, IEEE 2015, 2015.

[9] Luu H S and Ochi Y. “Estimation of wind by estimating derivatives of a lateral-directional motion model”, *32nd Congress of the International Council of the Aeronautical Sciences*, September, 2021.

[10]Mouche, K and Slock, D T M. “The generalized sliding window recursive least-squares (GSWRLS) algorithm,” Institut EURECOM, Research Report No. 95-021,1995.

[11]Luu H S and Ochi Y. Estimation of wind by estimating derivatives of longitudinal equation of motion of aircraft, *Trans. JSASS Aerospace Tech. Japan*, Vol. 70, No. 3, 2022 (in Japanese). (to appear)

[12]The Generic Transport Model, https://github.com/nasa/GTM_DesignSim (accessed January 28, 2021).

[13]Cunningham K, Cox D E, Murri D G and Riddick S E. A piloted evaluation of damage accommodating flight control using a remotely piloted vehicle, *Proc. AIAA Guidance, Navigation, and Control Conference and Exhibit*, Portland Oregon, AIAA 2011-6451, pp 1-20, 2011.