

RESEARCH ON UNSTEADY AERODYNAMIC MODELS AT HIGH ANGLES OF ATTACK BASED ON NONLINEAR REGRESSION

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Abstract

The prosperous development of the aviation industry has raised new requirements for fighter maneuverability at a high angle of attack, namely, post-stall maneuverability. Aiming at the specific requirements and forms of the unsteady aerodynamic model of the aircraft, this paper uses the method of nonlinear regression analysis to improve the nonlinear characteristics of the modeling process, we established a more accurate mathematical models and can reduce the error. In view of the strong nonlinear characteristics of the traditional state space model and the difficulty of parameter identification, the state space model was improved to reduce the complex differential operation, and a more accurate unsteady aerodynamic model was established by using the method of nonlinear regression analysis combined with genetic algorithm. As for the universality problem of aerodynamic modeling, the modern intelligent neural network algorithm was used to conduct nonlinear regression analysis, and the unsteady aerodynamic model of aircraft at large angles of attack was constructed. The results show that the unsteady aerodynamic model at high angles of attack based on the nonlinear regression method has high precision and good applicability. The work of this paper provides new multidisciplinary improvement ideas for the unsteady aerodynamic modeling at high angles of attack, and has a reference significance for the aerodynamic design of a new generation of high maneuvering aircraft.

Keywords: nonlinear regression, unsteady aerodynamic modeling, state space model, neural network

1. General Introduction

Compared with the obvious linear relationship between aerodynamic forces and the angle of attack for an aircraft at small angle of attack, the aerodynamic forces and moments show highly nonlinear and unsteady characteristics due to the flow separation and vortex breakdown caused by aircraft post-stall maneuver at large angles of attack. At present, the wind tunnel test is still the main approach to study the unsteady aerodynamics of aircraft during post-stall maneuvering. However, on account of some technical limitations, the wind tunnel test cannot completely simulate the motion state of the aircraft. Therefore, it has become one of the focuses for unsteady aerodynamics to establish a mathematical model to predict the aerodynamic performance of the whole aircraft through a small amount of existing aerodynamic data.

Unsteady aerodynamic modeling at high angles of attack mainly develops in two directions [1]: (a). the traditional modeling methods based on linear superposition principle, such as algebraic models, step response models, integral equation models, state-space models, differential equation models, and difference equation models; (b). the intelligent modeling methods based on modern computer technology, such as neural network models and fuzzy logic models.

Traditional modeling approaches can be regarded as "white box" problems. The "white box" problem is mechanism analysis modeling, that is, combining the physical background and mechanism analysis of the problem to establish the mathematical relationship between aerodynamic forces and flight states. The early developed aerodynamics models were approximately expressed by linear terms in the Taylor expansion of the motion variable and its time derivative at several instants. This way is suitable for describing the flowing state of attached flow at small angles of attack instead of

high angles of attack. In the 1970s, Tobak [2] adopted the step response method to give the general form of the nonlinear mathematical model of unsteady aerodynamic force, and established the integral form of unsteady aerodynamic model. Although this method was effective, the motion equation of aircraft was in differential form, so it was difficult to connect the integral model of unsteady nonlinear aerodynamic forces with the differential equations of aircraft motion. Therefore, this integral model had not been widely used. On this basis, Goman [3] rewrote the above ordinary differential equations into an input-state-output dynamic system by introducing the internal state variable of the flow field, and established the state space model of unsteady aerodynamics. Studies showed that the unsteady aerodynamic state space model can well reflect the unsteady nonlinear aerodynamic properties of aircraft at large angles of attack.

In contrast to traditional modeling methods, modern intelligent modeling approaches are "black box" problems. The "black box" model itself, as a model of the system, does not require explicit mathematical expressions like the state space model. Modern intelligent models have strong autonomous learning ability and no limits on the number of parameters, so it is suitable for unsteady aerodynamic modeling [4][5].

In view of the numerical calculation results of typical NACA0012 airfoil [6], this paper aims at and establishing an improved unsteady aerodynamic state space model at high angles of attack by using nonlinear regression analysis and genetic algorithm. Meanwhile, the modern intelligent neural network is applied into nonlinear regression analysis, and a more applicable unsteady aerodynamic neural network model at high angles of attack is developed.

2. Numerical Calculation Method and Verification

2.1 Computational Models and Grids

Unsteady aerodynamic modeling of aircraft at large angles of attack requires a large amount of aerodynamic data. In this paper, a typical NACA0012 airfoil is used for Computational Fluid Dynamics (CFD) method to obtain the required data for modeling. Computational simulations are performed by ANSYS FLUENT software [7], which solves the 2D unsteady, incompressible Navier-Stokes equations based on a finite volume method, to examine the aerodynamic features of slotted wings and to visualize the flow distribution over wings. The accuracy of this package has been extensively validated against several experimental and numerical studies in flapping wing aerodynamics [8]. The relative thickness of the airfoil is 12.0%, the maximum thickness is at 30.9% of the chord length, and the chord length of the airfoil is $c = 1000\text{mm}$. Figure 1 and Figure 2 show the structured mesh used in the numerical calculation. The total mesh amount is about 200,000. We adopt the overset moving grid method to realize the motion of the airfoil. The calculation region of the background grid is composed of a semicircular region and a rectangular region. The $k-\omega$ SST turbulence model was chosen for this CFD study. The radius of the semicircular region is $30c$, and the right-most point of the rectangular region from the trailing edge of the airfoil is also $30c$. The inner mesh calculation area is similar to the background mesh, with a semi-circular area with a radius of $5c$. The mesh height of the first layer in the boundary layer is about $5 \times 10^{-3}\text{mm}$, and the y plus value was taken to be 1 when we calculate the mesh height of the first layer, which is consistent with the $k-\omega$ SST turbulence model.

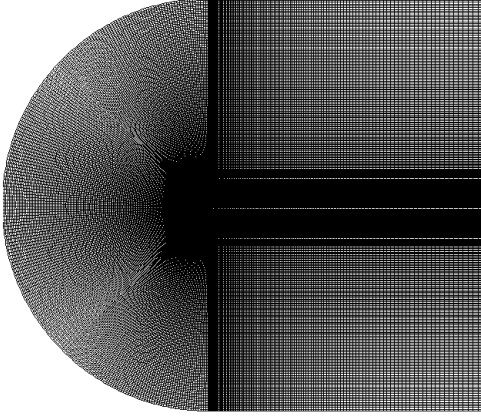


Figure 1 - The overset mesh system for numerical simulation: background mesh and component mesh.

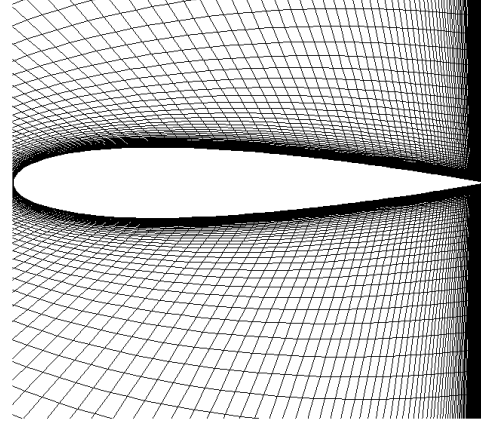


Figure 2 – The enlarged view of the component mesh around the NACA0012 airfoil.

2.2 Calculation Method and Verification

The numerical simulation method used in this paper is solving the two-dimensional Reynolds average Navier-Stokes equation, whose integral form is:

$$\frac{\partial}{\partial t} \iint_{\Omega} Q d\Omega + \int_{\partial\Omega} F_C \cdot ndS = \int_{\partial\Omega} F_V \cdot ndS \quad (1)$$

Wherein, Q is the flow variable, F_C and F_V are respectively the non-viscous flux and viscous flux, and n is the outer normal vector of the control surface. Adopt the $k - \omega$ SST (Shear Stress Transport) turbulence model. The model overcomes the shortcoming of the standard $k - \omega$ member turbulence model which is very sensitive to the variation of free flow parameters, and takes full advantage of the high accuracy of the $k - \omega$ layer turbulence model for adverse pressure gradient flow. The governing equations are discretized by finite volume method, and the spatial discretization scheme is the second order upwind scheme. The far field boundary is the velocity inlet and pressure outlet, and the wall boundary adopts the no-slip wall condition.

In order to verify the correctness of the numerical calculation method used in this paper, we compare and verify the unsteady aerodynamic data of NACA0012 airfoil under forced pitch vibration.

For a NACA0012 airfoil, we force it to oscillate pitching around the center of gravity in the calculation, and the variation rule of angle of attack is:

$$\alpha = \alpha_0 + \alpha_m \sin(\omega t) \quad (2)$$

Wherein, α_0 is the initial angle of attack, α_m is the amplitude, and $k = \omega c / 2V$ is the reduced frequency. The calculation condition is: $Ma = 0.6, \alpha_0 = 0^\circ, \alpha_m = 1^\circ, k = 0.808$. Compare the calculated data with the experimental results in the literature [9], and the results are shown in Figure 3. At some angles of attack, there are some errors between the numerical results and the experimental ones, but they are in agreement with the experimental ones on the whole, which can accurately reflect the flow field variation rule of the airfoil in unsteady motion.

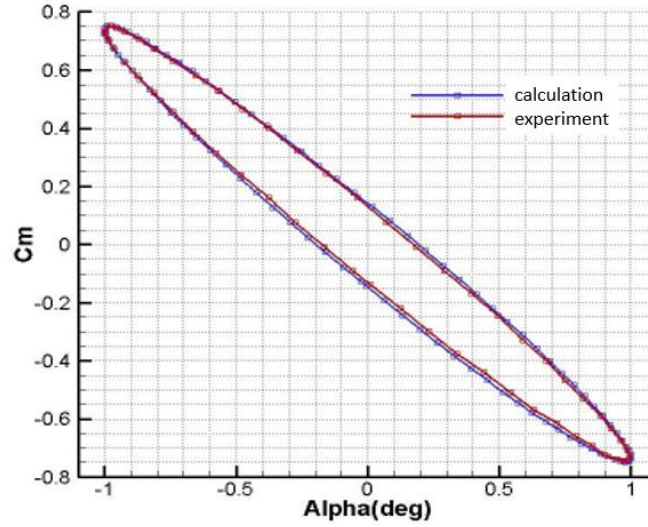


Figure 3 - Comparison of present numerical results with experimental results from Koopaee [9].

3. State Space Model Based on Nonlinear Regression

3.1 Principle and Improvement of State Space Model

The basic assumption of the state space model is that the hysteresis effect of aerodynamic forces at high angles of attack is mainly caused by the flow separation and vortex breakdown. Hence, the unsteady aerodynamic force can be represented by internal state variables related to the position of flow separation and vortex breakdown.

Goman [10] introduced the position of the flow separation point as an internal state variable into the unsteady aerodynamic modeling to fully demonstrate the unsteady flow behavior. We use a dimensionless quantity $\bar{x} = x/c \in [0,1]$ to describe the internal state variable, where x represents the distance from the separation point position to the leading edge of the wing, and c represents the chord length of the airfoil. When $\bar{x} = 1$, it means that the flow is attached at this time. On the contrary, when $\bar{x} = 0$, it means that the separation point is located at the leading edge of the airfoil, that is, the flow is completely separated. The analysis of a large number of experimental data shows that the motion of the flow field around the two-dimensional airfoil can be expressed by a first-order differential equation:

$$\tau_1 \frac{dx}{dt} + x = x_0 (\alpha - \tau_2 \dot{\alpha}) \quad (3)$$

Where τ_1 is the time constant in the process of changes of separation point position, and τ_2 represents the time constant of separation lag when the rate of change of angle of attack is not zero. In steady state, the position of airflow separation point is mainly determined by the angle of attack, and its relation is:

$$x_0(\alpha) = \frac{1}{1 + e^{\sigma(\alpha - \alpha^*)}} \quad (4)$$

Where α^* represents the angle of attack when the separation point reaches the midpoint of the airfoil string in the steady flow state, and σ is the slope factor.

When an aircraft flies at a high angle of attack, there are many parameters related to flight state, such as angle of attack α , pitch angle rate q , and so on. The aerodynamic forces and moments are not only related to the instantaneous value of these parameters, but also related to their unsteady change process. Therefore, when establishing an unsteady aerodynamic model with large angle of attack,

the condition $\dot{\alpha} = q$ should be taken into account. The expressions of aerodynamic forces and moments can be written as a function of flight state parameters and internal state variables:

$$\begin{aligned} C_y &= C_y(\bar{x}, \alpha, \dot{\alpha}); \\ C_x &= C_x(\bar{x}, \alpha, \dot{\alpha}); \\ C_m &= C_m(\bar{x}, \alpha, \dot{\alpha}); \end{aligned} \quad (5)$$

Taking the lift coefficient C_y as an example, in order to write the aerodynamic forces in the form of polynomials, we carry out the Taylor expansion of C_y and take it to the second order terms without dimensionalization:

$$C_y(\bar{x}, \alpha, \hat{\alpha}) = C_{y0} + C_{y\alpha}\alpha + C_{y\hat{\alpha}}\hat{\alpha} + \frac{1}{2} \left[C_{y\alpha^2}\alpha^2 + 2C_{y\hat{\alpha}\alpha}\hat{\alpha}\alpha + C_{y\hat{\alpha}^2}\hat{\alpha}^2 \right] \quad (6)$$

where $\hat{\alpha} = \dot{\alpha}t^*$ is a dimensionless form of pitch Angle rate, and t^* represents the characteristic time of flow, which is accurately defined as $t^* = c/2V$, and V represents the velocity of uniform flow in the test.

In the above formulas, although the specific form of each partial derivative is unknown, they are all functions of the internal state variable \bar{x} , so we can use quadratic polynomial to approximate:

$$\begin{aligned} C_{y\alpha}(\bar{x}) &= a_1 + b_1\bar{x} + c_1\bar{x}^2 \\ C_{y\hat{\alpha}}(\bar{x}) &= a_2 + b_2\bar{x} + c_2\bar{x}^2 \\ C_{y\alpha^2}(\bar{x}) &= 2(a_3 + b_3\bar{x} + c_3\bar{x}^2) \\ C_{y\hat{\alpha}\alpha}(\bar{x}) &= a_4 + b_4\bar{x} + c_4\bar{x}^2 \\ C_{y\hat{\alpha}^2}(\bar{x}) &= 2(a_5 + b_5\bar{x} + c_5\bar{x}^2) \end{aligned} \quad (7)$$

where $a_i, b_i, c_i, i = 1, 2, \dots, 5$ are all constants.

Similarly, the state space model of drag coefficient C_d and pitching moment coefficient C_m can also be obtained by this method.

So far, we have derived the relationship between aerodynamic forces, internal state variables and flight parameters, that is, the state space mathematical form of the unsteady aerodynamic coefficients for the aircraft at large angles of attack:

$$\begin{aligned} \tau_1 \frac{dx}{dt} + x &= x_0(\alpha - \tau_2 \dot{\alpha}) \\ C_y(\bar{x}, \alpha, \hat{\alpha}) &= C_{y0} + C_{y\alpha}\alpha + C_{y\hat{\alpha}}\hat{\alpha} + \frac{1}{2} \left[C_{y\alpha^2}\alpha^2 + 2C_{y\hat{\alpha}\alpha}\hat{\alpha}\alpha + C_{y\hat{\alpha}^2}\hat{\alpha}^2 \right] \end{aligned}$$

Where

$$\begin{aligned} C_{y\alpha}(\bar{x}) &= a_1 + b_1\bar{x} + c_1\bar{x}^2 \\ C_{y\hat{\alpha}}(\bar{x}) &= a_2 + b_2\bar{x} + c_2\bar{x}^2 \\ C_{y\alpha^2}(\bar{x}) &= 2(a_3 + b_3\bar{x} + c_3\bar{x}^2) \\ C_{y\hat{\alpha}\alpha}(\bar{x}) &= a_4 + b_4\bar{x} + c_4\bar{x}^2 \\ C_{y\hat{\alpha}^2}(\bar{x}) &= 2(a_5 + b_5\bar{x} + c_5\bar{x}^2) \end{aligned}$$

In the Goman's model, only the factor of angle of attack is taken into account as the airflow separation factor. In the rapid pitching motion, the pitching angle velocity has a certain damping effect on the changes of the angle of attack. Meanwhile, in order to reduce the error caused by the differential operation in the above formula, the revised formula of airflow separation point is introduced in this paper:

$$x(\alpha, \dot{\alpha}) = \frac{1}{1 + e^{\sigma(\alpha - \alpha^* - \tau_1 \dot{\alpha} - \tau_2 q)}} \quad (8)$$

The new separation point model is reconstructed by adding new parameters in combination with the characteristics of unsteady flow field based on the static separation point model. This model has clear physical meaning and solves the problem of complex differential equation calculation and large errors in the Goman's model.

3.2 Method and Process of Model Identification

After the mathematical form of the unsteady aerodynamic state space model is determined, the undetermined parameters in the model are obtained according to the identification criteria and numerical simulation data, namely parameter identification, which is an important step in model establishment [11]. Parameter identification includes identification criterion and optimization algorithm. Under unsteady conditions, the input is $\{\alpha(t), q(t), 0 < t < T\}$, and the aerodynamic coefficient obtained by the numerical simulation method at each time is $\{C_y(t_i), i = 0, 1, 2, \dots, n\}$. And the aerodynamic force coefficient under the corresponding condition is $\{\hat{C}_y(t_i), i = 0, 1, 2, \dots, n\}$, where $t_0 = 0 < t_1 < \dots < t_i < t_{i+1} < \dots < t_n = T$. Then get the quadratic difference of the model:

$$Q = \frac{1}{n+1} \sum_{i=0}^n [C_y(t_i) - \hat{C}_y(t_i)]^2 \quad (9)$$

We select a set of parameters to minimize the error Q as the final parameters of the mathematical model.

Considering the highly nonlinear characteristics of aerodynamic forces in unsteady state, we adopt the nonlinear regression analysis method for parameter identification to establish a more accurate model. The so-called nonlinear regression analysis is to establish the function expression of the regression relationship between the dependent variables and the independent variables by using mathematical statistics method on the basis of a large number of observed data. When this relationship is nonlinear, it is called nonlinear regression. For the general nonlinear regression problem, the nonlinear regression is transformed into linear regression by variable transformation, and then the linear regression method is adopted to solve it. However, for the nonlinear regression problem, which is often encountered in practical scientific research and cannot be dealt with linearly, a new solution is needed. In the modeling process, we take advantage of genetic algorithm, which has good global search ability and is not easy to fall into local optimal. We apply the genetic algorithm to the nonlinear regression analysis, aiming to identify the unknown parameters in the model and improve the accuracy of the model.

3.3 Comparison of State Space Model and Simulation Data

Based on the unsteady aerodynamic data calculated by the numerical method, we use nonlinear regression analysis approach to obtain the predicted values of the unsteady aerodynamic state space model of the aircraft at large angles of attack and compare them. The comparison between the predicted values of the state space model and the calculated value of CFD method is illustrated in Figure 4. The calculation conditions are: velocity $V = 68m/s$, law of model motion $\alpha = \alpha_0 + \alpha_m \sin(\omega t)$, where $\alpha_0 = 9^\circ, \omega = 10.88928$, $\alpha_m = 5^\circ, 10^\circ$. The corresponding reduced

frequency $k = \omega c/2V = 0.08$. C_l, C_d, C_m represents lift coefficient, drag coefficient and pitching moment coefficient, respectively.

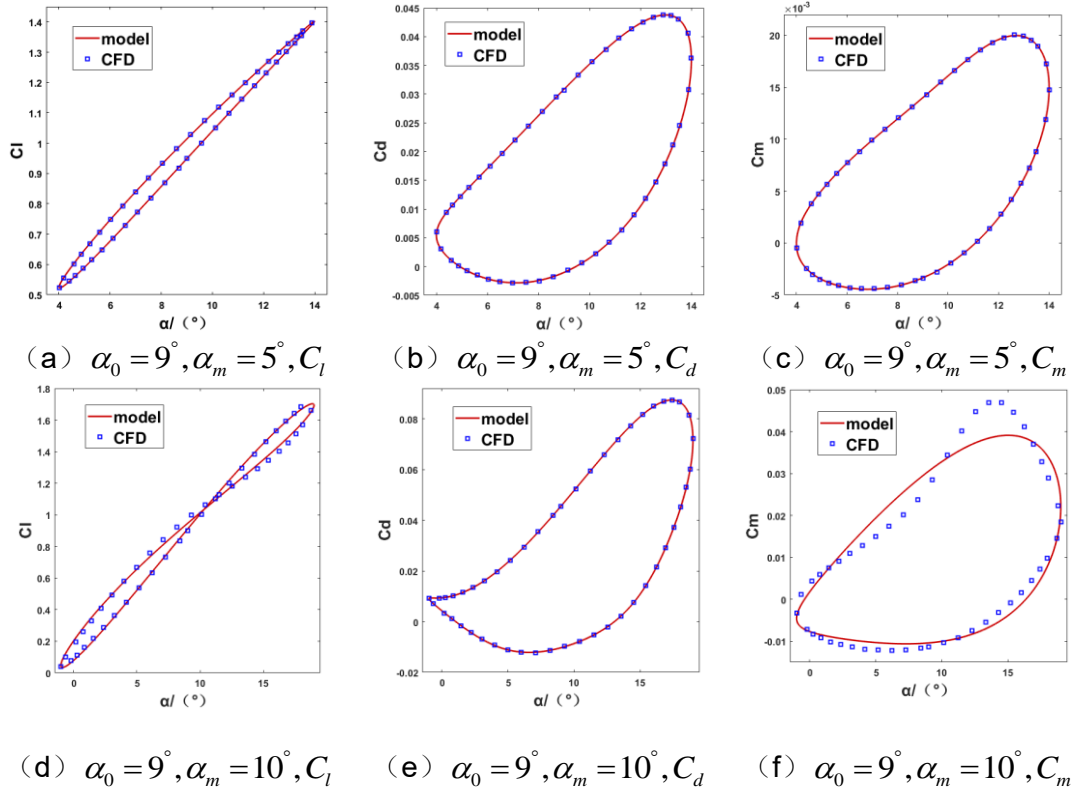


Figure 4 - Comparison between the predicted values of the state space model and the calculated value of CFD method

It can be seen from the figure that the improved state space model established by the nonlinear regression method has high prediction accuracy. Especially when the amplitude and angle of attack are small, the output error of the model can reach $10^{-4} \sim 10^{-6}$. When the amplitude and angle of attack increase, the model can also predict the corresponding aerodynamic forces well, and accurately reflect the variation trend of aerodynamic coefficients. However, it is noted that when the angle of attack increases further, there is a certain deviation between the output values of the model and the calculated values of CFD method, especially the prediction error of the pitching moment coefficient is relatively large. We analyze the reasons as follows: (1) The accuracy of the parameter identification method used in model establishment is limited, and the absolute value of the pitching moment coefficient is small compared with the lift coefficient and drag coefficient, resulting in the inability to obtain a completely accurate pitching moment coefficient model; (2) When the aircraft is pitching at a high angle of attack, the vortex structure on the upper surface of the airframe is asymmetrical from left to right, and the asymmetry of flow will produce non-negligible lateral moments. These all lead to a high degree of non-linearity of the aerodynamic force, so large errors will occur when establishing a aerodynamic model at large angles of attack. These possible reasons require further improvement of the expression form of unsteady aerodynamic state space model at large angles of attack and parameter identification method in the future research.

4. Intelligent Model Based on Nonlinear Regression

4.1 Principles of Neural Network Modeling

The basic idea of neural network is to simulate the nervous system of human brain from the perspective of bionics, so that machines can perceive things, reason logically and learn

autonomously just like human brains. In the field of function approximation, Back Propagation (BP) neural network, which is a multi-layer feed forward neural network, usually consists of input layer, hidden layer and output layer, is widely used [12]. It is named because the network training adopts error Back Propagation learning algorithm. By calculating the error between the actual output and the expected output of the network, the learning matrix is trained by using the learning algorithm with decreasing error gradient. When the output errors of all the samples are less than the set convergence error, the training ends, and then the actual prediction is performed according to the adjusted network model [13]. Figure 5 demonstrates a typical three-layer BP neural network.

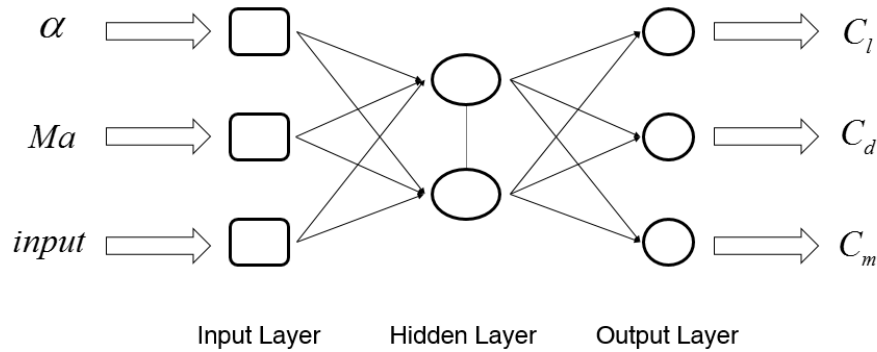


Figure 5 - Three-layer BP network structure diagram

The parameters of neural network mainly include the number of hidden layers, the number of hidden layer nodes, the learning rate and the number of training times.

According to the mathematical knowledge, when there are enough hidden layer nodes, the continuous function in any closed interval can be approximated by BP neural network with single hidden layer. When the number of hidden layers increases, the error of the model will decrease, but the complexity of the network will also increase.

The selection of hidden layer nodes is very important to the success or failure of BP network training. At present, there is no mature theory for people to fully refer to. In most cases, the selection of the number of hidden layer nodes is based on experience.

The learning rate determines the weight variation in each network training. With the increase of learning rate, the number of training times decreases, but the absolute convergence of the results cannot be guaranteed. When the learning rate decreases, the number of training times increases, the convergence rate slows down, but the stability of the network increases. The learning rate generally ranges from 0.01 to 0.8, which is usually selected according to the convergence speed and stability of the network.

The number of training times is also an essential parameter in neural networks. Too many training times will cause over-fitting of the network, leading to deviation of the results; Too few training times will make it difficult for the network to converge and fail to meet the expected requirements. At present, the improvement of computer performance makes the number of training times unrestricted, and the prediction accuracy of network is mainly considered when choosing training times.

The process of the aerodynamic neural network modeling algorithm is shown in Figure 6:

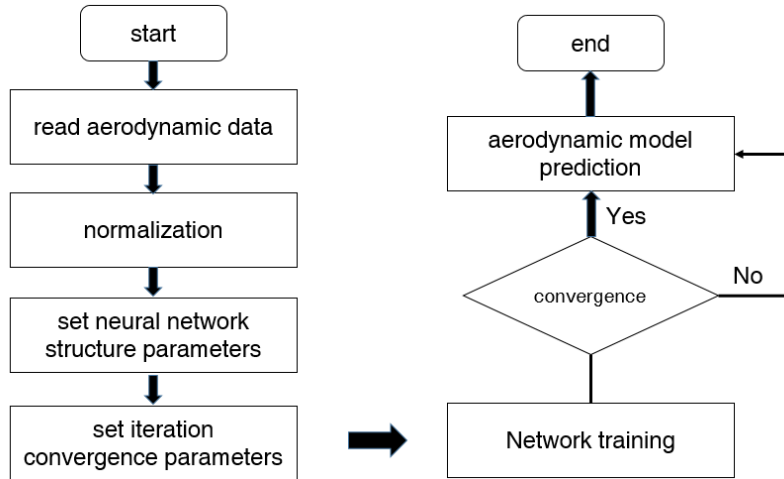
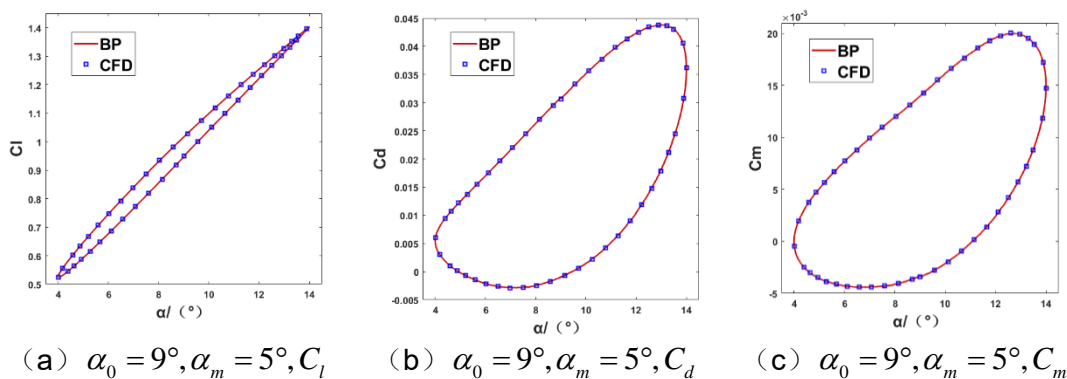


Figure 6 - Neural network modeling process

4.2 Comparison of Neural Network Model and Simulation Data

Based on the unsteady aerodynamic data calculated by the CFD method, we acquire the predicted values of the unsteady aerodynamic neural network model of the aircraft at large angles of attack by the nonlinear regression analysis method and compare them. The comparison between the predicted values of the neural network model and the calculated value of numerical simulation is shown in Figure 7. The calculation conditions are: velocity $V = 68m/s$, Law of model motion $\alpha = \alpha_0 + \alpha_m \sin(\omega t)$, where $\alpha_0 = 9^\circ$, $\omega = 10.88928$, $\alpha_m = 5^\circ, 10^\circ$. The corresponding reduced frequency $k = \omega c / 2V = 0.08$. The neural network adopts a three-layer structure. The input parameters of the BP network include time, angle of attack, and the pitching angle velocity. And the output parameters of the network are C_l, C_d, C_m that represent lift coefficient, drag coefficient and pitching moment coefficient, respectively. For the parameters of the neural network, through several attempts, the error is minimum when the number of hidden layer nodes is set as 9, so the number of hidden layer nodes is set as 9, the learning rate is set as 0.01, and the number of training times is set as 5000. The training dataset is the whole CFD result of corresponding case when we build the models. And the test dataset is the same as the training dataset. In the figure, BP represents the neural network model.



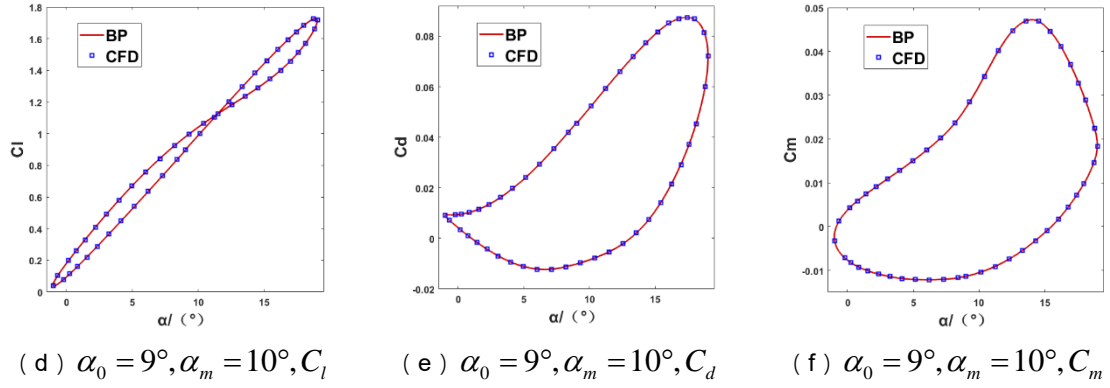


Figure 7 - Comparison of predicted values of neural network model and calculated values of CFD method

As can be seen from the figure, no matter when the amplitude and angle of attack are relatively small or large, the unsteady aerodynamic neural network model can predict the corresponding results well, and the accuracy is even higher. As for the highly nonlinear characteristics of aerodynamic force at high angles of attack, the neural network model can also handle it well. In order to better reflect the applicability of the neural network model, we calculate the aerodynamic forces of NACA0015 airfoil under forced pitch oscillation, and use the BP neural network to train and predict. The calculation conditions are: $V = 68m/s, \alpha = 9^\circ + 5^\circ \sin(9.52 * t)$, and the corresponding reduced frequency $k = 0.07$. The comparison between output values of the model and numerical simulation values is depicted in Figure 8:

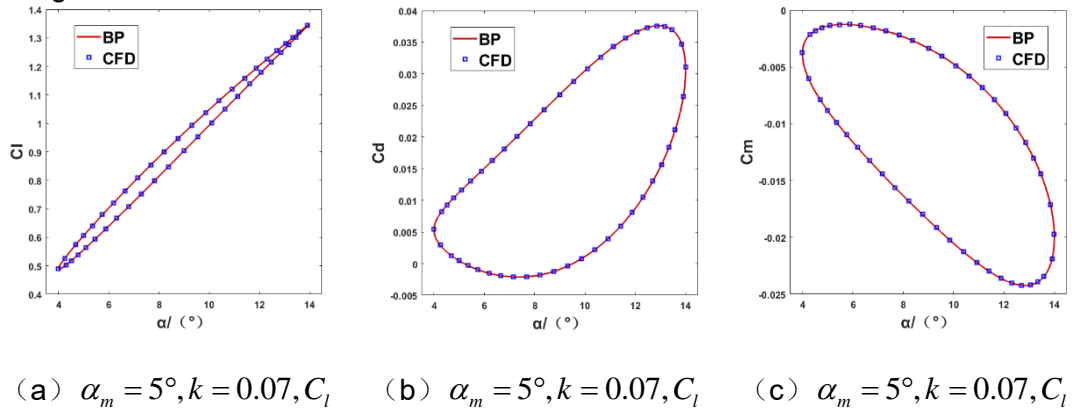


Figure 8 - Comparison of predicted values of neural network model and calculated values of CFD method for a NACA0015 airfoil ($k=0.07$)

It can be seen that the unsteady aerodynamic neural network model does not need to know the specific mathematical expression form of the model, but only needs a certain amount of aerodynamic data. Moreover, the neural network model has high accuracy and good applicability, without considering the intermediate complex process, so it has a good application prospect in the future.

5. Conclusion

In this paper, we use the nonlinear regression analysis method to establish the unsteady aerodynamic state space model and neural network model for the aircraft at large angles of attack. The following conclusions are obtained:

1) The traditional state space model has clear physical meaning. The precision of the model can be improved and the error can be reduced by adding parameters and changing the form of differential equation;

- 2) Applying the nonlinear regression analysis with genetic algorithm to the parameter identification of state space model can greatly improve the speed and precision of identification, and better deal with the highly nonlinear characteristics of aerodynamic force;
- 3) The neural network model based on nonlinear regression has high accuracy and good applicability at the same time, and it can also cope well with more complex situations. The neural network model will play an important role in future research;
- 4) It is feasible to apply mathematical nonlinear regression analysis to unsteady aerodynamic modeling at large angles of attack. It provides a new multi-disciplinary fusion idea for aircraft aerodynamic modeling and has reference significance for the design of new generation aircraft.

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