

MULTI-UAV TRAJECTORY PLANNING BASED ON COORDINATION VARIABLES

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Abstract

With the increase of complexity of unmanned aerial vehicles (UAV) tasks, trajectory planning algorithms, trajectory tracking algorithms, and coordination methods are becoming more and more important in multi-UAV collaborative task research. Based on the centralized coordination architecture, this paper studies the cooperative time constraint problem of multiple UAVs and establishes a minimum snap trajectory planning model for a single UAV which converts the trajectory planning problem into a quadratic programming problem. A trajectory tracking controller is used to track the planned flight trajectory. In multi-UAV coordination with cooperative time constraints, coordination variables and coordination functions are selected as the multi-UAV cooperative strategy, and the time generated by the trajectory planning algorithm is used as the coordination variable. After the coordination function has coordinated the coordination variables, a unified time will be generated and returned to each UAV for replanning and tracking. The simulation result shows that multiple UAVs can track the planned trajectory with a small tracking error and complete the expected mission of reaching the same target point simultaneously.

Keywords: Minimum Snap, trajectory tracking, multi-UAV coordination

1. General Introduction

An unmanned Aerial Vehicle (UAV) is a kind of unmanned, recyclable, and reusable aircraft [1]. Due to its lower cost, higher maneuverability, strong adaptability, and excellent stealth performance, UAV is more suitable for performing missions in high-risk environments such as intelligence surveillance, target tracking, communication relay, and ground target attack. However, in the face of more complex and diversified mission requirements, due to the restriction of load, volume, energy, and other conditions, the functions of a single UAV are limited, so it is challenging to complete the mission alone. As a result, multi-UAV collaboration has become a research hotspot in the theory and application of UAVs.

Multi-UAVs can enable the team to obtain broader coverage, detailed and accurate external information through communication. Furthermore, when multiple UAVs perform missions, the vacancies in missions for a single UAV caused by various failures can be filled by other UAVs. The multi-UAV framework ensures the completion of the mission, which shows that cooperation between UAVs can increase the probability of task completion and improve the quality of task completion. Through task allocation and optimization, each UAV is assigned to different tasks to execute simultaneously, which will significantly shorten the task completion time.

1.1 Trajectory Planning Algorithm

In multi-UAV cooperative missions, coordinated trajectory planning is essential for guiding the aircraft to the target point after completing the task assignment. It means that based on satisfying various constraints, the planning system plans a flyable trajectory for each UAV through the necessary path points according to the specific task. And the flyable trajectory is optimal or suboptimal [2]. Multi-UAV coordinated trajectory planning is essentially an optimization problem for multiple goals. It is

necessary to generate a flyable trajectory for each UAV and meet the coordination between multiple UAV trajectories.

There are several typical path planning algorithms, such as visibility graphs [3], random sampling search algorithms like rapid exploration random tree (RRT) [4] and the probabilistic road map (PRM) [5], optimal search algorithms like Dijkstra algorithm [6], A* [7] and D* [8]. There are also biological heuristic planning algorithms such as evolutionary algorithm (EA) [9] and neural network (NN) algorithm [10]. A typical algorithm based on mathematical models is the optimal control method [11]. This type of method considers kinematics and dynamics constraints and then combines the cost function with all inequalities or equalities to obtain the optimal solution. In a complex three-dimensional environment, the goal of path planning algorithms is not only to find obstacle avoidance paths but also to minimize the cost function.

1.2 Trajectory Tracking Algorithm

Trajectory tracking is another essential function of UAV. The trajectory tracking problem-solving strategy is divided into geometric method and control theory [12]. Geometric methods include the pure tracking method, line-of-sight method, et al. The trajectory tracking algorithm based on pure tracking and line-of-sight guidance law uses a virtual target point (VTP) on the path. The guidance law guides the aircraft to chase the virtual target point and finally guides the aircraft to the trajectory that needs to be tracking. In addition to using the tracking method or line-of-sight guidance law, literature [13] developed a nonlinear guidance law (NLGL) that uses virtual target points. Another option for the guidance law is a vector field-based method [14]. Since all of the above techniques use geometric methods, calculating the expected heading angle is fast and easy to implement.

Control technology, especially nonlinear control technology, is prevalent in trajectory tracking applications. They are robust to wind disturbances. A common method in path tracking is based on proportional-integral-derivative (PID) control, but PID control performance is not as good as nonlinear guidance law. A technique that uses a PID controller with feedforward capability performance is better than nonlinear guidance law [15]. Some well-known control techniques include linear quadratic regulator, sliding mode control, model predictive control, backstepping control, gain scheduling theory, adaptive control, and dynamic programming. Some other control methods, such as segmented affine control, are also used to track a pre-defined path. In trajectory tracking, the path is time parameterized, which is not considered in path following. The trajectory tracking controller can be re-parameterized to produce a path-following controller.

1.3 Cooperative Trajectory Planning

One of the research focuses of cooperative trajectory planning is time coordination. Time coordination generally requires multiple UAVs to arrive at the mission area at the same time or sequentially to perform tasks. Multiple aircraft attacking targets from different directions at the same time is a crucial combat strategy in military applications. There are two ways to achieve simultaneous attacks. One is that all aircraft reach the target independently at a predetermined time [16]. However, it should be noted that due to the aircrafts' initial range and speed limitations, there may not be a guidance law that meets the time limit. Another method is a coordinated attack. That is, the aircraft can attack the target at the same time through coordination. This method requires the aircrafts to exchange information through a communication network and synchronize their influence time. Specifically, the aircrafts share their estimated remaining flight time information. Then the aircrafts with longer flight time accelerate themselves or shorten the path, while other aircrafts with shorter flight time decelerate or detour. Coordinated simultaneous attacking is a consensus problem, and the flight time is a consensus variable. The main difficulty of this problem is processing time. Flight time is a variable related to future flight conditions. Therefore, the current flight time cannot be accurately derived and can only be predicted by assuming that the aircraft will fly according to specific rules, such as flying at a constant speed.

1.4 Paper Structure

In the remainder of this article, we model and solve the UAV's trajectory planning problem, and then based on the generated flyable trajectory, apply the tracking controller for trajectory tracking (Section

2). After that, the trajectory planning algorithm is combined with the coordinated variable method to quickly coordinate the flight time of multiple UAVs to achieve the goal of reaching the target point at the same time (Section 3). Then the simulation of the entire task is implemented and verified.

2. Trajectory Planning and Tracking Control

In 2011, Daniel Mellinger and Vijay Kumar et al. proposed the minimum snap algorithm. This algorithm generated the optimal trajectory in real-time through a series of three-dimensional points and yaw angles while ensuring safe passage through a designated corridor and meeting speed acceleration and input constraints [17]. This section briefly reviews the minimum snap trajectory planning algorithm, including the modeling of cost functions, mandatory equality constraints, and non-mandatory inequality constraints.

2.1 Minimum Snap Algorithm

The minimum Snap algorithm is founded on the basis that the quadrotor dynamic with the four inputs is differentially flat. It means that the states and the inputs can be written as algebraic functions of four carefully selected flat outputs and their derivatives. The property of differential flatness facilitates the automated generation of trajectories since any smooth trajectory (with reasonably bounded derivatives) in the space of flat outputs can be followed by the underactuated quadrotor [17]. For quadrotor, the choice of flat outputs is given by

$$p = [x, y, z, \psi]^T \quad (1)$$

where $r = [x, y, z]^T$ is the coordinate of the center of mass in the world coordinate system, and ψ is the yaw angle of the quadrotor. Then we have to define a trajectory function, $p(t)$, in the space of flat outputs.

In order to avoid the Runge phenomenon that occurs with high-degree polynomials, it's naturally represent the trajectory as piecewise polynomial functions of order n over m time intervals as:

$$p(t) = \begin{cases} [1, t, t^2, \dots, t^n] \cdot p_1, t_0 \leq t \leq t_1 \\ [1, t, t^2, \dots, t^n] \cdot p_2, t_1 \leq t \leq t_2 \\ \dots \\ [1, t, t^2, \dots, t^n] \cdot p_m, t_{m-1} \leq t \leq t_m \end{cases} \quad (2)$$

among them, $p_1, \dots, p_m \in R^{1 \times (n+1)}$ are unknown trajectory parameters vector. Due to the convenience provided by differential flatness, the kinematic parameters of each order of the derivative curve for any time t during the movement can be calculated as

$$\begin{cases} v(t) = p'(t) \\ a(t) = p''(t) \\ jerk(t) = p^{(3)}(t) \\ snap(t) = p^{(4)}(t) \end{cases} \quad (3)$$

where $jerk(t)$ is the derivative of the acceleration vector with respect to time, and $snap(t)$ is the second order derivative of the acceleration vector, equivalently, is the fourth derivative of the position vector with respect to time.

The trajectory needs to meet a series of constraints in actual problems, such as the position, velocity, and acceleration value of the starting point and ending point. Two adjacent segmented trajectories require smooth connection, which means continuous position and velocity. The dynamic of the aircraft also requires the trajectory to meet the constraints of maximum velocity and maximum acceleration. In some special cases, the trajectory also needs to pass through several specific path points, and it is even desired that the trajectory should be in a certain constraint space (such as a corridor). Therefore, it is necessary to construct an optimal function to find the optimal trajectory that satisfies the constraints in the solution space of the trajectory parameters. Therefore, the problem can be formulated as a constrained optimization problem, such as:

$$\begin{aligned} & \min f(p) \\ & \text{s.t. } A_{eq}p = b_{eq} \\ & \quad A_{ieq}p \leq b_{ieq} \end{aligned} \quad (4)$$

The objective function to minimize in the minimum snap algorithm is the quadratic form of $snap(t)$, which is:

$$\min f(p) = \min \left(p^{(4)}(t) \right)^2 \quad (5)$$

Combining the equation (5) with equation (2), the optimization problem expressed in equation (5) can be transformed into a quadratic programming problem:

$$\min f(p) = \min p^T \cdot Q \cdot p \quad (6)$$

where

$$Q = \begin{bmatrix} Q_1 & & & \\ & Q_2 & & \\ & & \dots & \\ & & & Q_m \end{bmatrix} \quad (7)$$

Q_i represents the known parameters of each segment trajectory. Q_i is calculated as

$$\begin{aligned} Q_i &= \int_{t_{i-1}}^{t_i} \left[0, 0, 0, 0, 24, \dots, \frac{n!}{(n-4)!} t^{n-4} \right]^T \left[0, 0, 0, 0, 24, \dots, \frac{n!}{(n-4)!} t^{n-4} \right] dt \\ &= \begin{bmatrix} 0_{4 \times 4} & 0_{4 \times (n-3)} \\ 0_{(n-3) \times 4} & \frac{j!}{(j-4)!} \frac{k!}{(k-4)!} \frac{1}{(j-4)+(k-4)+1} \left(t_i^{j+k-7} - t_{i-1}^{j+k-7} \right) \end{bmatrix} \end{aligned} \quad (8)$$

where j, k is the row index and column index of the matrix respectively (the index starts from 0).

The beginning and ending points of the trajectory and the intermediate points that need to be precisely specified between segments can be written as equality constraints. The strong constraint of passing through specific waypoints may cause excessive curvature of the trajectory. Thus a feasible channel constraint is introduced to limit the shape of the trajectory. That is, the trajectory planned by the algorithm must be in the corridor. Therefore, if the feasible channel is modeled as an inequality constraint and added to the quadratic programming problem, the trajectory obtained would be naturally constrained in the corridor. At this time, the intermediate point does not need to be specified accurately, which relaxes the strong constraint of the trajectory.

Another key part of the minimum snap algorithm is time allocation. Initial time allocation is required at the beginning of trajectory planning. Calculate the straight-line distance between each path point, and then we can get the total time with the given initial average speed. There are two methods to allocate the whole time of the entire trajectory. One is uniforming time allocation. Assuming that speed in each segment of trajectory is constant, and the distance of each segment determines the time of each segment. Another is trapezoidal speed curve time allocation. Assuming that the speed curve in each segment is accelerated from 0 to the maximum velocity v_{max} with a constant acceleration a , and then decelerated to 0 with $-a$, where the acceleration a and the maximum velocity v_{max} are preset. With these two methods, we can obtain the coordination time for every trajectory.

2.2 Trajectory Tracking Controller

PID controller is the most commonly used trajectory tracking controller. Combined with the characteristics of the minimum snap trajectory planning algorithm that can directly give the each order derivatives of the position of the trajectory, the PD controller with the acceleration feedforward term is used in the tracking control to perform multi-UAV coordinated trajectory tracking tasks. The acceleration command is:

$$a = a_e + k_p (p_{des} - p_{current}) + k_d (v_{des} - v_{current}) \quad (9)$$

The acceleration calculated by the minimum snap algorithm is used for feedforward control, which is added with the position error and the velocity error to form the control command of the tracking controller.

3. Multi-UAV Cooperative Strategy

With trajectory planning and tracking for a single UAV, applying a coordinate strategy to multiple UAVs can perform multi-aircraft coordination tasks. In 2005, Timothy W. McLain and Randal W. Beard introduced a solution to achieve time coordination between aircraft [18], which is based on the concept of coordination variables and coordination functions and improves coordination efficiency by effectively using communication and computing resources. It has been applied to the time-constrained trajectory planning problem for a UAV group.

This section first introduces the conception of coordination variables proposed by McLain et al. and mathematically models coordination variables and coordination functions. Afterward, coordinated variables are used through the communication between multiple UAVs to realize multi-UAV cooperative trajectory planning.

3.1 Coordination strategy: Coordination variables and coordination functions

The decisive information that each UAV must share in the collaborative process is called coordination variables. Coordination variables represent the minimum amount of information that needed to be exchanged to achieve specific coordination tasks. The coordination function parameterizes the impact of coordination variables on each aircraft and then uses this information to provide a feasible coordination variable value by optimizing. In this paper, the coordination variable is defined as the time information required by the cooperative task, such as the estimated time of arrival (ETA) to the specified destination. The coordination function describes the cost of the UAV in different coordinated variable values.

To represent the variables in the coordination process, χ_i is defined as the state space of the i -th aircraft, and $x_i \in \chi_i$ is taken as the specific state of the i -th aircraft. Let $U_i(x_i)$ be the set of feasible decision values for the state x_i , and let $u_i \in U_i(x_i)$ be the specific decision variable of the i -th aircraft. Generally, the set of feasible decision values for the aircraft is the flyable path generated by the waypoint and trajectory planning algorithm. The minimum amount of information required by the UAV team to achieve coordination is called coordination variable, which is represented by the symbol θ . If multiple UAVs arrive at a certain location at the same time, the coordination variable is the flight time of the aircraft to the destination. If $f_i : \chi_i \times U_i \rightarrow IR^c$ is a function that maps the state variables and decision vector to the coordination space, then the set of feasible coordination variables for the i -th aircraft in the state variable x_i is

$$\Theta_i(x_i) = \bigcup_{u_i \in U_i(x_i)} f_i(x_i, u_i) \quad (10)$$

For a specific trajectory and speed selection given by u_i , the coordination variable has a unique value $\theta_i = f_i(x_i, u_i)$.

Assuming that f_i is pseudo-invertible, there is a pseudo-inverse function f_i^\dagger of $f_i : \chi_i \times U_i \rightarrow \Theta_i$, so that for every $\theta_i \in \Theta_i(x_i)$ there is $f_i(x_i, f_i^\dagger(x_i, \theta_i)) = \theta_i$. In other words, if the state variables and coordination variables are known, then the decision variables are also unique. For the time constraint problem considered in this paper, this means that if the expected arrival time can be given, a specific path and speed that meet the arrival time can be determined.

In practice, there may be multiple decision variable values corresponding to a single coordination variable value, that is, different route choices may correspond to the same arrival time. Therefore, the reversibility of f_i is determined by the optimal decision variable value for the same coordinated variable value. Like coordination variables, the target cost of a single UAV depends on the state variables and decision vector. For the i -th UAV, the target cost can be represented by the function $J_i : \chi_i \times U_i \rightarrow IR$. For a given value of θ_i , find the decision variable u_i by choosing the lowest cost

$$u_i = f_i^\dagger(x_i, \theta) = \arg \min_{u_i \in U_i(x_i)} J_i(x_i, u_i) \quad (11)$$

By applying the relationship $u_i = f_i^\dagger(x_i, \theta_i)$ to each $\theta_i \in \Theta_i(x_i)$, the cost can be parameterized as a function of coordination variables:

$$\phi_i(x_i, \theta_i) = J_i(x_i, f_i^\dagger(x_i, \theta_i)) \quad (12)$$

The function $\phi_i : \chi_i \times \theta_i \rightarrow IR$ given by equation above is called the coordination function of the i -th aircraft.

For multi-UAVs, using coordination variables and coordination functions we can obtain:

$$\theta^* = \arg \min_{U_1 \times \dots \times U_N} J_T (J_1(x_1, u_1), \dots, J_N(x_N, u_N)) \quad (13)$$

When the constraints of the equation above are satisfied, the goal of coordination can be achieved. In the decomposition of the equation above, the cooperative time constraint is implicitly satisfied by choosing a unified coordination variable value θ^* . In the cooperative time constraint problem, the optimal solution could meet the time limit while minimizing the flight cost of the UAV team. When the optimal value of UAV team coordination variables θ^* is solved, the single UAV decision variables can be obtained by solving the decision variables in the equation below:

$$u_i = f_i^\dagger(x_i, \theta^*) \quad (14)$$

3.2 Cooperative Control Strategy of UAV Team

UAV swarm coordination strategies are generally divided into centralized coordination strategies and decentralized coordination strategies. If a centralized coordination unit in the group can obtain global information, then a globally optimal solution can be obtained. Suppose there is no centralized coordination unit, and global information cannot be obtained, it means that a single UAV can only receive the information of its neighboring UAVs. In that case, a decentralized coordination strategy needs to be designed.

In a centralized coordination strategy, there is a centralized coordination unit that can interact with all aircraft. All aircrafts transfer the state information required for cooperative control to the centralized coordination unit, which calculates a unified coordination variable value θ^* , and then returns it to all aircrafts. Each aircraft makes corresponding adjustments according to the calculated and coordinated variable, and then the cooperative control of the entire group is realized. Although the centralized coordination strategy is easy to implement, it is not easy to achieve the requirement that the centralized coordination unit obtains the global information of the huge group in practical applications. In addition, the centralized coordination unit is the core of the group, receiving data from all aircraft in the team and returning the processed information. Therefore, the failure of the centralized coordination unit will lead to paralysis of the entire system. Thus, the reliability and robustness of the centralized coordination strategy are poor.

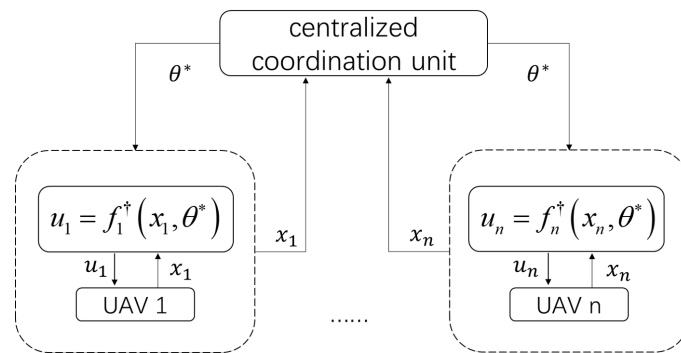


Figure 1 – Centralized coordination strategy structure.

In the decentralized coordination strategy, there are decentralized coordination units in each aircraft. When applying coordination variables and coordination function methods, each aircraft in the group first generates a local coordination variable θ_i according to its state variables and decision variables. Then each aircraft continuously adjusts its coordination variable value θ_i according to the θ_j of neighboring aircraft. Finally, the coordination variables of each aircraft should converge to a uniform θ^* , which is the so-called coordinated consensus algorithm.

In the centralized coordination strategy, the unified coordination variable value θ^* can be directly calculated, while in the decentralized coordination strategy, θ^* gradually converges to a unified value.

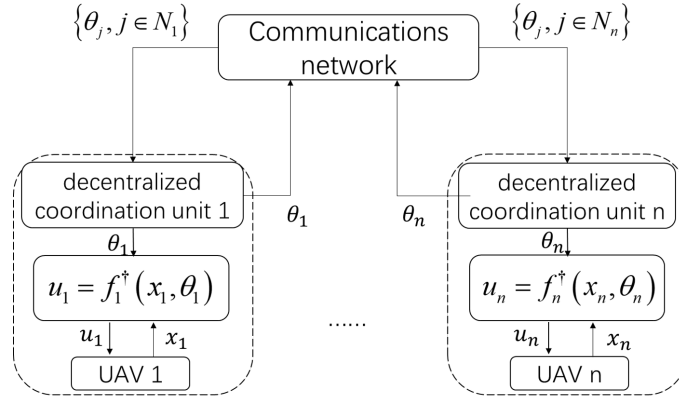


Figure 2 – Decentralized coordination strategy structure.

Therefore, the centralized coordination strategy can obtain consistent coordination variable values faster. In contrast, the decentralized coordination strategy does not require the global information of all aircraft, which is easier to implement in practice.

3.3 Implementation of multi-UAV cooperation

Figure 3 illustrates the trajectory tracking architecture of a single UAV, which consists of three main elements, collaborative management (CM), trajectory planner (TP), and trajectory tracker (TT). TP uses the minimum snap algorithm for trajectory planning. CM transmits local coordination variables to the centralized coordination unit. After processing by the centralized coordination unit, CM receives the values of the unified coordination variables. After receiving the unified, coordinated variable and replanning the trajectory, TT uses the PD controller with an acceleration feedforward to control the UAV to complete a single UAV trajectory tracking.

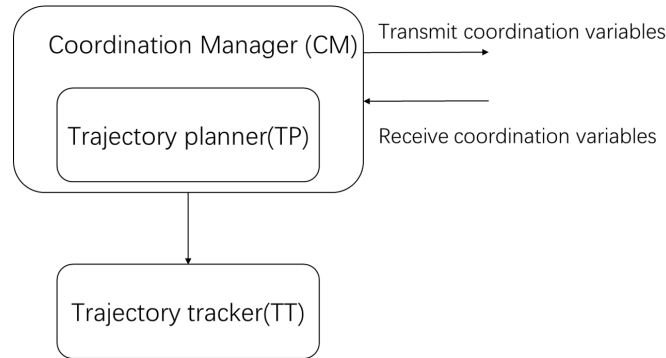


Figure 3 – Single UAV trajectory tracking architecture.

Consider the actual problem of multiple UAVs arriving at the same place at the same time, this paper chooses a centralized coordination strategy and uses a centralized coordination unit to process the coordination variables $\theta_i, i \in 1, \dots, n$ transmitted by multiple UAV.

The path point of each UAV has been preset before the trajectory planning. In the problem of this paper, the ending point of the two UAVs must be the same. In the coordination time problem described in this paper, the key of coordination depends on the time to reach the same target point. Therefore, the coordination variable θ_i is the initial time allocation of the UAV at the initial average speed v_{pre} . For the straight path w , which is formed by the given path points, assuming the path length is $L(w)$, then the coordination variable is given by equation (15)

$$\theta_i = \frac{L(w)}{v_{pre}} \quad (15)$$

Since the minimum snap algorithm takes the minimum value of the fourth-order derivative $snap(t)$ of the position as the optimization objective function, the trajectory planned by the algorithm is already

the optimal solution. Thus, this paper does not design the path cost function J_i . After each UAV transmits the local coordination variable θ_i , the coordination function in the centralized coordination unit compares the initial time of each UAV and determines the longer initial flight time as a unified coordination variable θ^* , and return its value to each UAV.

$$\theta^* = \max\{\theta_i\} \quad i = 1, 2, 3, \dots, n \quad (16)$$

The coordinated management (CM) of each UAV receives the returned unified coordinated variable value θ^* , which is

$$\theta_1 = \theta_2 = \theta^* \quad (17)$$

Adjust the initial average speed v_{pre} to v_{pre}' according to θ^* , and reallocate time from the new initial average speed v_{pre}' , then the trajectory needs to be replanning. After that, the coordinated task of two UAVs arriving at the same place at the same time is completed.

4. Numrical Simulations

This simulation is aimed at the cooperative task of two UAVs in three-dimensional space. The model used in the simulation is the quadrotor robot model developed by the *ROBOTICS* course of the University of Pennsylvania. The specific simulation parameters are as follows:

Table 1 – Simulation parameters for UAV 1 and UAV 2.

Parameter	UAV 1 and UAV 2
Path point for UAV 1	$waypoints1 = [0, 1, 0; 1, 1, 1; 0.5, 1, 2; 0.5, 1, 3; 0, 2, 3]'$
Path point for UAV 2	$waypoints2 = [-1, 1, 0; -1, 1.5, 1; -0.5, 1.5, 2; 0, 1.5, 2; 0, 2, 3]'$
Starting point velocity	$v_0 = [0, 0, 0]$
Starting point acceleration	$a_0 = [0, 0, 0]$
End point velocity	$v_1 = [0, 0, 0]$
End point acceleration	$a_1 = [0, 0, 0]$
Polynomial order	$n_{order} = 7$
Initial average velocity	$vel_{pre} = 0.5m/s$

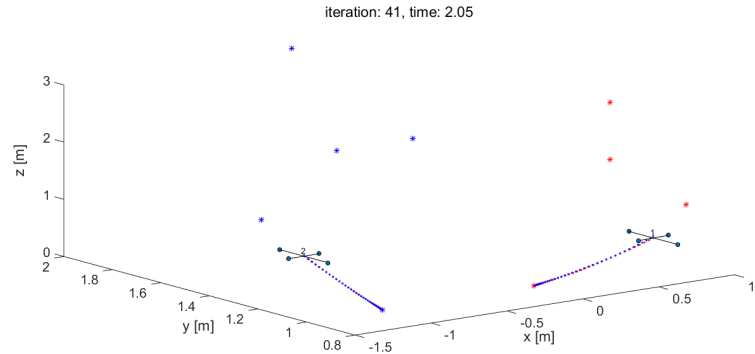
The simulation results of multi-UAV cooperation in three-dimensional space are shown in Figure 4 to Figure 6.

From Figure 4 to Figure 6, it can be seen that even if the trajectory is tortuous, the two UAVs can still track the trajectory well under the control of the PD controller with feedforward. From the development process of Figure 4, we can find that it has a good performance for the two UAVs to reach the same place at the same time with cooperative time constraints. The PD controller with feedforward can ensure the UAV quickly tracking the trajectory when the desired position is continuously changing, which can be seen from Figure 5 to Figure 6.

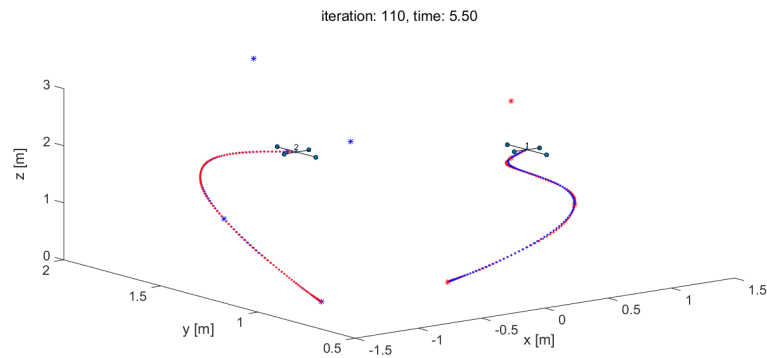
5. Conclusion

Based on the three aspects of UAV trajectory planning, trajectory tracking, and multi-UAV cooperative methods, this paper completes the multi-UAV cooperative task with cooperative time constraints. The modeling and simulation of the minimum snap trajectory planning algorithm are implemented on the quadrotor. Then the PD controller with acceleration feedforward is used for trajectory tracking. Finally, a centralized collaboration strategy is adopted to complete the cooperative time constraint task by coordinating cooperative variables. However, due to the complexity of the actual task, this paper is still insufficient in the depth of research. In the future, we will continue to research the following several aspects. First, in the multi-UAV cooperative time constraint task, we would consider the overall trajectory cost of the two UAVs to optimize the flight time. Second, in the trajectory tracking research, model predictive control would be used in subsequent research to improve the trajectory tracking accuracy.

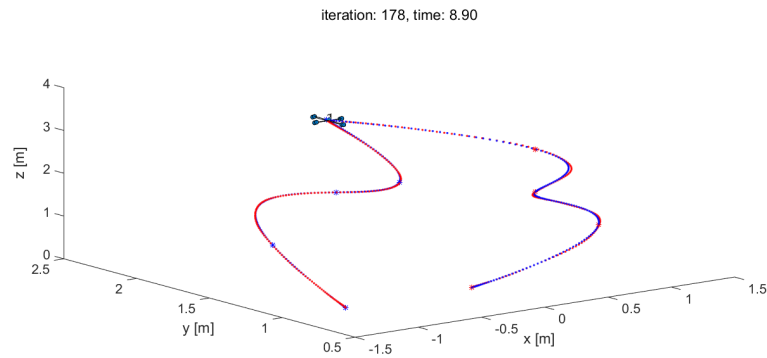
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(a) Early in the flight

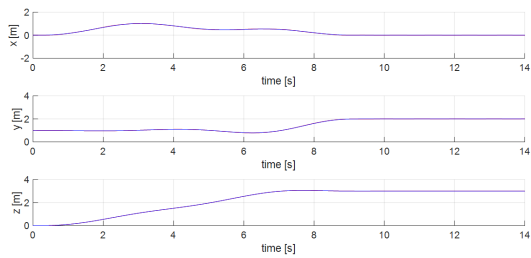


(b) Middle in the flight

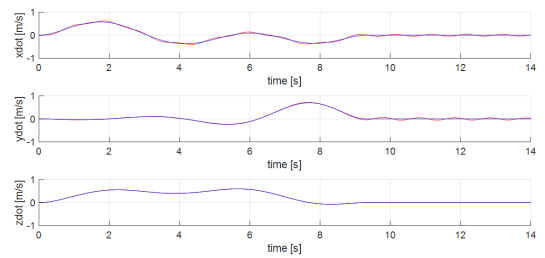


(c) Late in the flight

Figure 4 – Dynamic simulation diagram of multi-UAV cooperation



(a) Position curve



(b) Velocity curve

Figure 5 – Position and velocity in three-dimensional of UAV 1

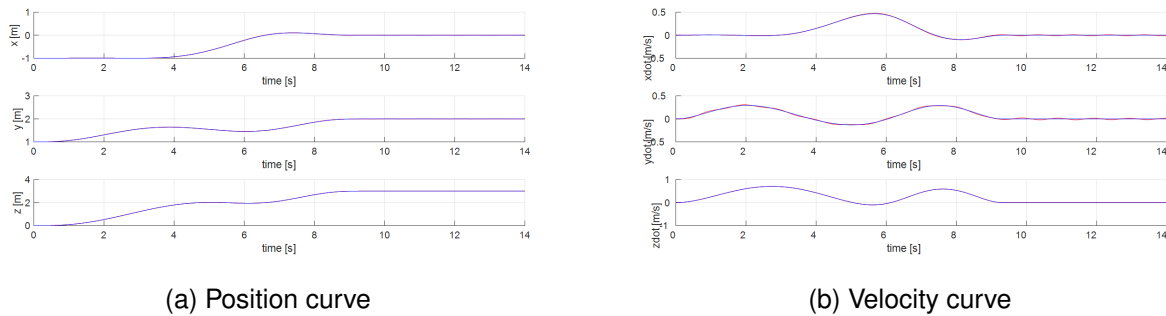


Figure 6 – Position and velocity in three-dimensional of UAV 2

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References

- [1] Valavanis KP, Vachtsevanos GJ, editors. *Handbook of unmanned aerial vehicles*. Dordrecht: Springer Netherlands; 2015 Jan.
- [2] Li W, Cassandras CG. Centralized and distributed cooperative receding horizon control of autonomous vehicle missions. *Mathematical and computer modelling*. 2006 May 1;43(9-10):1208-28.
- [3] Schøler F, la Cour-Harbo A, Bisgaard M. Generating configuration spaces and visibility graphs from a geometric workspace for uav path planning. *Autonomous Robots*, 2012.
- [4] Yang K, Sukkarieh S. Real-time continuous curvature path planning of UAVs in cluttered environments. *In 2008 5th International Symposium on Mechatronics and Its Applications*. 2008 May 27 (pp. 1-6). IEEE.
- [5] Yan F, Liu YS, Xiao JZ. Path planning in complex 3D environments using a probabilistic roadmap method. *International Journal of Automation and computing*. 2013 Dec 1;10(6):525-33.
- [6] Musliman IA, Rahman AA, Coors V. Implementing 3D network analysis in 3D-GIS. *International archives of ISPRS*. 2008 Jul;37(part B).
- [7] De Filippis L, Guglieri G, Quagliotti F. Path planning strategies for UAVS in 3D environments. *Journal of Intelligent & Robotic Systems*. 2012 Jan;65(1):247-64.
- [8] Carsten J, Ferguson D, Stentz A. 3d field d: Improved path planning and replanning in three dimensions. *In 2006 IEEE/RSJ international conference on intelligent robots and systems*. 2006 Oct 9 (pp. 3381-3386). IEEE.
- [9] Hasircioglu I, Topcuoglu HR, Ermis M. 3-D path planning for the navigation of unmanned aerial vehicles by using evolutionary algorithms. *In Proceedings of the 10th annual conference on Genetic and evolutionary computation*. 2008 Jul 12 (pp. 1499-1506).
- [10] Kroumov V, Yu J, Shibayama K. 3D path planning for mobile robots using simulated annealing neural network. *International Journal of Innovative Computing, Information and Control*. 2010 Jul 1;6(7):2885-99.
- [11] Miller B, Stepanyan K, Miller A, Andreev M. 3D path planning in a threat environment. *In 2011 50th IEEE Conference on Decision and Control and European Control Conference*. 2011 Dec 12 (pp. 6864-6869). IEEE.
- [12] Sujit PB, Saripalli S, Sousa JB. Unmanned aerial vehicle path following: A survey and analysis of algorithms for fixed-wing unmanned aerial vehicles. *IEEE Control Systems Magazine*. 2014 Jan 14;34(1):42-59.
- [13] Park S, Deyst J, How JP. Performance and lyapunov stability of a nonlinear path following guidance method. *Journal of guidance, control, and dynamics*. 2007 Nov;30(6):1718-28.
- [14] Nelson DR, Barber DB, McLain TW, Beard RW. Vector field path following for miniature air vehicles. *IEEE Transactions on Robotics*. 2007 Jun 25;23(3):519-29.
- [15] Rhee I, Park S, Ryoo CK. A tight path following algorithm of an UAS based on PID control. *In Proceedings of SICE Annual Conference 2010*. 2010 Aug 18 (pp. 1270-1273). IEEE.

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- [16] Jeon IS, Lee JI, Tahk MJ. Impact-time-control guidance law for anti-ship missiles. *IEEE Transactions on control systems technology*. 2006 Feb 21;14(2):260-6.
- [17] Mellinger D, Kumar V. Minimum snap trajectory generation and control for quadrotors. *In 2011 IEEE international conference on robotics and automation*. 2011 May 9 (pp. 2520-2525). IEEE.
- [18] McLain TW, Beard RW. Coordination variables, coordination functions, and cooperative timing missions. *Journal of Guidance, Control, and Dynamics*. 2005 Jan;28(1):150-61.