ICAS 2020 SHANGHAI
International Council of the Aeronautical Sciences
32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

FLIGHT PERFORMANCE AND TRAJECTORY PREDICTION OF THE COAXIAL HELICOPTER IN AUTOROTATION

S-Z. Zhao*, X. Zhao*, R. Yao* & Y-F. Lin**

*Northwestern Polytechnical University, Xi' an, Shaanxi, 710072, China
**China Helicopter Research and Development Institute, Jingdezhen, Jiangxi, 333001,China

Abstract

In this paper, the flight dynamics and kinematics models were established for the coaxial rigid helicopter in autorotation process, based on the principle of point-mass model. The XH-59A helicopter was chosen to be an example. The aerodynamic forces of rotor and fuselage were superimposed. The rotor performance prediction was based on engineering empirical formula. The dangerous region of the helicopter is determined, and the parameters varying with descent angle in stable autorotation state were calculated for flight speed 10m/s and 30m/s respectively. Result shows that with the increase of descent angle, the rotor thrust, the collective pitch and the helicopter horizontal velocity are decreased, while the rotating speed of the rotor and descent rate are increased. The higher the flight speed is, the higher the thrust and descent rate are, and the smaller the pitch angle is.

Keywords: autorotation, dangerous region, aerodynamic performance, stable autorotation

Nomenclature

e =Oswald efficiency factor

f =equivalent flat-plate drag area,ft^2

$h.p.$ =required horsepower, h.p.

A =rotor-disk area, ft^2

A_b =total rotor-blade area, ft^2

C_T, C_w =tensile coefficient, tensile coefficient calculated by total weight

$G.W.$ =total weight of the helicopter

I_R =total rotational inertia of the rotor system

J =rotor rotating inertia, slug-ft^2

P_{mr} =required power of rotor

P_s =available power of the helicopter

$V_{L.G.}$ =landing-gear design vertical impact speed, ft/sec

Ω =rotor speed, radians/second^2

Ω_d =design rotor speed, radians/second^2

ρ =ambient air density at any altitude, slug/ft^3

ρ_0 =ambient air density at sea level, slug/ft^3

Λ =ground effect parameter

1. Introduction

Engine failure is one of the most important reasons for helicopter flight accidents. There is one engine failure in every 10,000h of operation according to statistics provided from authorities. The rotor speed decays rapidly, the helicopter deviates from the equilibrium state, and the pilot feels obvious yaw and roll angular velocity and lateral acceleration due to power loss after engine failure. Besides, autorotation is the maneuverable and fastest method of descent. Therefore, autorotation is an important issue of the flight safety.

Rotor utilizes the gravitational potential energy of the helicopter released by the decline to drive itself to rotate. Therefore, the rotor thrust is controllable and the flight control can be maintained when the engine is in failure. The autorotation process can be mainly divided into five stages: (1) engine failure, (2) autorotation entry, (3) stable autorotation, (4) deceleration and leveling, (5) collective pitch increasing. Generally, (4) and (5) in the **Figure 1** are collectively called landing.

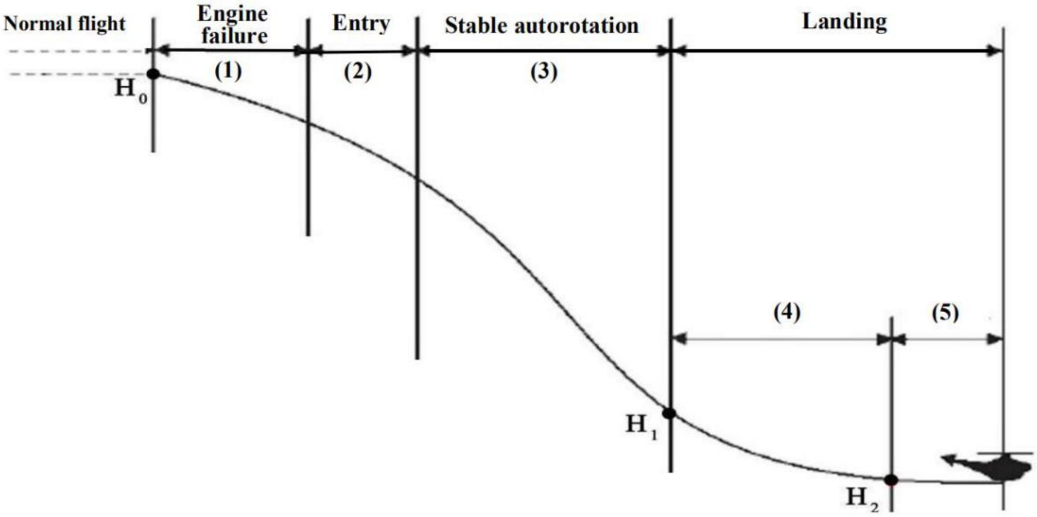

Figure 1-Autorotation process of the helicopter.

For helicopter, the movement of the main rotor needs to be controlled instanously to provide a desired control force and direction. The rotation speed, shaft angle, collective pitch and cyclic pitch change of the rotor bring a complicated flow field change, resulting in dynamic change of the helicopter. Unsteady and non-uniformity flow field of helicopter and strong flow interference between the rotor, fuselage and tail even bring the flow phenomenon more complicated. In autorotation, the collective pitch and cyclic pitch of the rotor need to be controlled by the pilot, which changes in accordance with the law at different times, resulting in the changes of rotor speed, helicopter attitude and speed. It can be seen that helicopter flight dynamics and kinematics are coupled together and need to be solved iteratively. Therefore, the accuracy and rapidity of the prediction of rotor aerodynamic force and torque are very important to the movement and flight characteristics of the helicopter.

The research on the autorotation of helicopters can be roughly divided into several categories. (1) Flight test can provide accurate result but limited. Niu and Yang [1] studied the autorotation characteristics and autorotation landing of the Z11 helicopter through the flight test, and proposed that during the approach and landing phase of autorotation, the economic speed should be maintained and the rotor speed should be kept within a certain range, so that the rotor can provide the maximum thrust and reduce the descent rate. (2) Mathematical models were applied for versatile application. Wang and Deng [2] numerically calculated the autorotation landing trajectory of a light coaxial helicopter that can provide a reference for pilots by using the control model. (3) The semi-empirical method is also applied to predict based on the method of minimizing the dangerous region. Okuno [3, 4] established a longitudinal three-degree-of-freedom rigid body model after helicopter engine failure. Predicted the positions of high hover points, inflection points and low hover points, and calculated the low-speed height-velocity curve by using the total rotor pitch and the longitudinal periodic pitch as the control variables based on the method of minimizing the dangerous region.

Coaxial rigid rotor helicopter has its particularity compared with single rotor helicopter. (1) There is no tail rotor to balance torque generated by main rotors. (2) The flow interference is serious due to the short distance between upper and lower rotors. (3) The fuselage is short, resulting in small value of moment of inertia, causing a rapid change in pitch attitude. Unfortunately, the research literature on the autorotation of a coaxial rigid helicopter is limited.

This paper takes the XH-59A helicopter as the research object, the aerodynamic model includes three parts: rotor, fuselage and propeller. The propeller radius of the helicopter is 5.486 m, the weight is 5900 kg, and the speed is 60 m/s in level flight. A contraprop was used to replace the auxiliary thrust system, and the contraprop was adjusted to windmilling after the engine failure for coaxial rotor helicopters in this paper.

2. Prediction of the dangerous region

There are many factors that affect safety when autorotation happens: such as flight height, speed, and rotor speed. Each helicopter has its own dangerous region. When flying within the height and speed range of the dangerous region, not only does the pilot have no time to steer the helicopter into a stable autorotation after the engine failure, but also the force applied on the helicopter when it touches the ground will exceed the maximum impact force that the body structure can withstand. **Figure 2** shows the general dangerous region for helicopters.

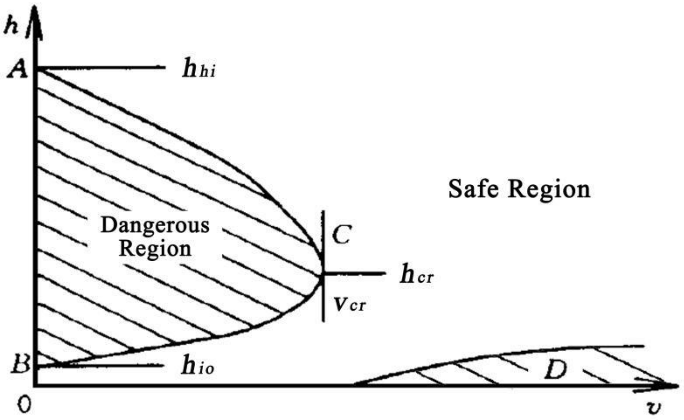

Figure 2 -Typical height-velocity diagram.

In this paper, the dangerous region of the selected helicopter was predicted firstly in order to ensure the rationality and effectiveness of the selected operating conditions. Method of the dangerous region prediction are as follows. Taking a single-engine helicopter as an example, three special heights and one speed need to be determined, namely the low hover height h_{io}, the high hover height h_{hi}, a speed above which a power-off landing can be made at any height V_{cr}, and the geometric height at V_{cr} as h_{cr}.

h_{io} can be obtained by the **Formula (1)**, assuming that the vertical descent rate equals to the decline rate, and C_T/σ does not exceed 0.2. More detail can be referred from a dimensionless height-velocity diagram established using data from the FAA (Federal Aviation Administration) flight test project.

Experiments have shown that the speed V_{cr} is a function of the minimum power speed V_{min} and the total weight. V_{min} can be obtained by the minimum power condition which is shown in **Formula (2)**. V_{cr} can be obtained from the literature [5] [6] after the V_{min} is determined, and the critical height h_{cr} is 120 feet (36.574 m) from the military hysteresis specification. Then h_{hi} can be obtained. Table shows the detailed results of the calculation for XH-59A helicopter.

$$h_{io} = \frac{V_{L.G.} J\Omega_d^2 [1 - \sqrt{\frac{C_w/\sigma}{0.2}}]}{1100 h.p.\Lambda} \qquad （1）$$

$$V_{\min}^4 + \frac{1}{2}\frac{A_b}{f}\Omega R C_d V_{\min}^3 = \frac{(\frac{G.W.}{\rho_0})^2}{3(\rho/\rho_0)^2 efA} \qquad （2）$$

Table 1-Parameters for the dangerous region calculation of XH-59A helicopter

I_R/(kg·m^2)	V_{vd}/(m/s)	Ω_d/(rad/s)	H_{preq}	C_T	σ	Λ
615.508	2	36.1	1500	0.01	0.1267	0.75
R/(m)	A_b/(m^2)	C_d	f/(m^2)	W/(kg)	$\rho/\rho0$	ρ/(kg/m^3)
5.486	11.979	0.013	3.963	5900	0.956	1.171

Through the parameters in **Table 1** and **Formulas (1)** and **(2)**, h_{io} was calculated as 2.1332 m, the V_{min} was 42.662 m/s, V_{cr} was 49.410 m/s, and h_{hi} was 356.402 m.

Figure 3 shows the calculated diagram of the dangerous region for the XH-59A helicopter height-velocity diagram.

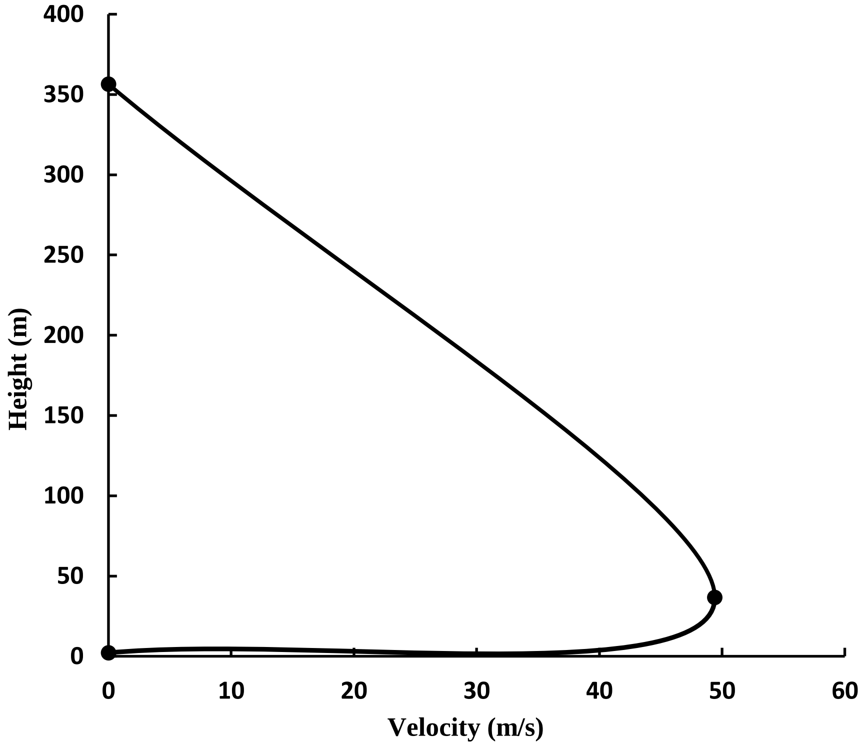

Figure 3- XH-59A helicopter height-velocity diagram.

When the helicopter descends at an economic speed, the required power is the least. It is easy to enter the autorotation. Therefore, 60m/s and 1000m were selected as the economic speed and the initial height H_0 within the safe region for the XH-59A helicopter studied in this article.

3. Analysis of the autorotation process

3.1 Classification of the subregions in the process

When the collective pitch of the rotor is small, the total aerodynamic force of the blade tilts forward and accelerates the rotation. On the contrary, a large collective pitch helps in the aerodynamic force of the blade to tilt it backward and decelerate the rotation. Therefore, it is necessary to select an appropriate pitch to maintain a steady rotating. Besides, the autorotation with forward speed will cause the aerodynamic balance position of the blade to shift from the forward side to the backward side.

The changes in the handling and attitude of the helicopter during the autorotation process was discussed next.

(1) Engine failure.

The pilot needs to react within the allowable lag time (collective pitch is 2s, and other operations are 1s) at this stage, which will not be discussed in details in this article.

(2) Autorotation entry.

At this stage, manipulated variables include collective pitch and longitudinal cyclic pitch. Forward speed was selected as 60m/s , the minimum collective pitch was 5.5° and the pitch angle required to enter stable autorotation was 5° , without considering the waving movements and decoupled of lateral and longitudinal motion. The helicopter's parameters and trajectory changes at this stage were calculated by using the momentum blade element theory and the dynamic equation, based on the established airfoil database.

(3) Stable autorotation.

This stage is the stable state at the end of the previous stage. The rotor speed, descent rate and forward speed were calculated.

(4) Deceleration and leveling.

In this phase, the altitude of the helicopter is H_1 and manipulated variables is longitudinal cyclic pitch. The pilot can pull the steering stick to manipulate the helicopter, to increase the pitch angle and reduce the forward speed and descent rate.

(5) Collective pitch increasing.

In the final phase, the altitude of the helicopter is H_2, and manipulated variables include collective pitch and longitudinal cyclic pitch, constrained by the allowable overload and grounding speed. The pilot can adjust the landing attitude by pushing the stick to avoid its tail hitting the ground, and quickly increase the collective pitch to the maximum at the same time.

Different H_1-H_2 combinations would make the helicopter reach different descent rates and forward speeds in landing. Therefore, this research would be helpful for the pilot to judge how to make the helicopter get the best descent rate within the safe region.

3.2 Force analysis on the helicopter

The trajectory of the helicopter bends downward during the autorotation process, and the force analysis of the helicopter is shown in **Figure 4** with flight speed V. Only gravity G, fuselage drag D_F and rotor thrust T were considered. The fuselage coordinate system is adopted, and the origin of the coordinate is the center of mass of the helicopter, θ in the **Figure 4** indicates the inclination angle of the track, ϑ represents the helicopter pitch angle, α is the fuselage angle of attack, α_s is the rotor angle of attack, δ is the forward tilting angle of rotor shaft (because of the rigid rotor, δ is assumed to be 0), β_s is the angle of fuselage sideslip (0 for longitudinal maneuver). Assume the pitch angle of the helicopter is zero, that is $\vartheta = 0$, then $\alpha = \theta$.

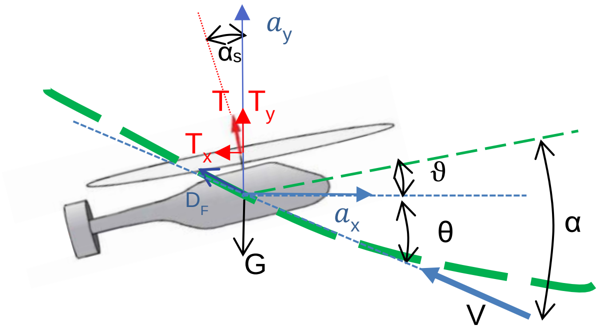

Figure 4- Force analysis of the helicopter in autorotation.

Horizontal and vertical acceleration a_x and a_y produces a non inertial force (opposite to the direction of acceleration) as shown in **Formula (3)** and **(4)**. The rotor thrust is decomposed into two components along the tangential and normal phases of the rotor tip path plane (TPP), which are T_x and T_y, respectively. The drag of fuselage are assumed to follow the **Formula (5)** regarding as an equivalent plate, where the equivalent area f_e is 3.83 m^2. The gravity is known, the following equation can be obtained and the rotor thrust component can be determined. The magnitude and direction of the rotor thrust force are determined according to **Formula (6)** and **(7)** (the pitch angle of the helicopter can be determined according to the rotor thrust direction α_s). The relationship of the fuselage angle of attack is given by **Formula (8)**.

$$T_x - D_F \cos\theta = ma_x \quad \textbf{(3)}$$

$$T_y + D_F \sin\theta - G = ma_y \quad \textbf{(4)}$$

$$D_F = \frac{1}{2}\rho f_e V^2 \quad \textbf{(5)}$$

$$T = \sqrt{T_x^2 + T_y^2} \quad \textbf{(6)}$$

$$\alpha_s = -\arctan\left(T_x/T_y\right) \quad \textbf{(7)}$$

$$\alpha = \theta \quad \textbf{(8)}$$

In the stable autorotation state, the acceleration in both directions is 0, the following formula can be obtained.

$$T_x = D_F \cos\theta \quad \textbf{(9)}$$

$$T_y = G - D_F \sin\theta \quad \textbf{(10)}$$

Then the thrust *T* will be calculated.

Formula (11) shows the relationship between rotor speed, available power of the helicopter and required power of the rotor. Additional constraint is added for the stable autorotation state, i.e. **Formula (12).**

$$\dot{\Omega} = \frac{1}{\bar{I}_R\Omega}\left[P_s - \frac{1}{\eta}P_{mr}\right] \approx -\frac{1}{\bar{I}_R\Omega}P_{mr} \quad \textbf{(11)}$$

$$P_{mr} = TV\cos\alpha\sin\alpha_s - TV\sin\alpha\cos\alpha_s - Tv_i + \frac{1}{8}\sigma c_d\left[\rho\pi R^2(\Omega R)^3\right] = 0 \quad \textbf{(12)}$$

In **Formula (11),** c_d is the drag coefficient of the rotor blade, given by 0.15 (airfoil NACA 23012 at 10° angle of attack). σ is the rotor solidity ratio, given by 0.1267. The induced velocity v_i follows **Formula (13)**, where C_T is thrust coefficient, the K_{ind} is the ratio of nonuniform inflow to uniform inflow induced power requirements, given by 1.13. The induced velocity parameter f_I is given by the **Formula (15)**, where the parameters $\overline{x_1}$ and $\overline{x_2}$ are defined by **Formula (16)**.

$$v_i = K_{ind} f_I \Omega R\sqrt{C_T/2} \quad \textbf{(13)}$$

$$C_T = \frac{T}{\rho \pi R^2 (\Omega R)^2} \tag{14}$$

$$f_I = \begin{cases} 1/\sqrt{[\bar{x}_2^2 + (\bar{x}_1 + f_I)^2]} & (2\bar{x}_1 + 3)^2 + \bar{x}_2^2 \geq 1 \\ \bar{x}_1 (0.373 \bar{x}_1^2 + 0.598 \bar{x}_2^2 - 1.991) & (2\bar{x}_1 + 3)^2 + \bar{x}_2^2 < 1 \end{cases} \tag{15}$$

$$\begin{cases} \bar{x}_1 = (V \cos\alpha \sin\alpha_s - V \sin\alpha \cos\alpha_s)/(\Omega R \sqrt{C_T/2}) \\ \bar{x}_2 = (V \cos\alpha \cos\alpha_s - V \sin\alpha \sin\alpha_s)/(\Omega R \sqrt{C_T/2}) \end{cases} \tag{16}$$

Otherwise, the collective pitch at 75% span, $\theta_{0.75}$, is defined by **Formula (17)** from the literature [7]**.** a is the rotor blade two-dimensional lift curve slope (airfoil NACA23012), given by 6.1.

$$\theta_{0.75} = \frac{6C_T}{a\sigma} + \frac{3}{2}\left(\frac{v_i - V}{\Omega R}\right) \tag{17}$$

Therefore, for the stable autorotation state, according to the formulas (9-10), T_x and T_y can be determined first if the helicopter flight speed V and the inclination angle are known. Then T and α_s can be obtained. Use the **Formula (12)**, with the iteration of induced velocity, the rotating speed Ω can be determined. Finally the collective pitch can be determined.

3.3 Aerodynamic performance and trajectory response with manipulation variables

To start with, a reasonable stable slip condition was calculated as the initial condition. Then the collective and cyclic pitches were taken as the input conditions according to the program. Meanwhile, zero balance of the force on helicopter and tangential resultant moment of the entire rotor were taken as the constraint condition. Through the flow-process diagram in **Figure 6**, the calculation program of the autorotation glide trajectory and aerodynamic characteristics was compiled. According to the iterative solution, the relationship between helicopter attitude and trajectory with time can be obtained. The helicopter aerodynamic characteristics and some flight performance were given, and the specific work will be carried out.

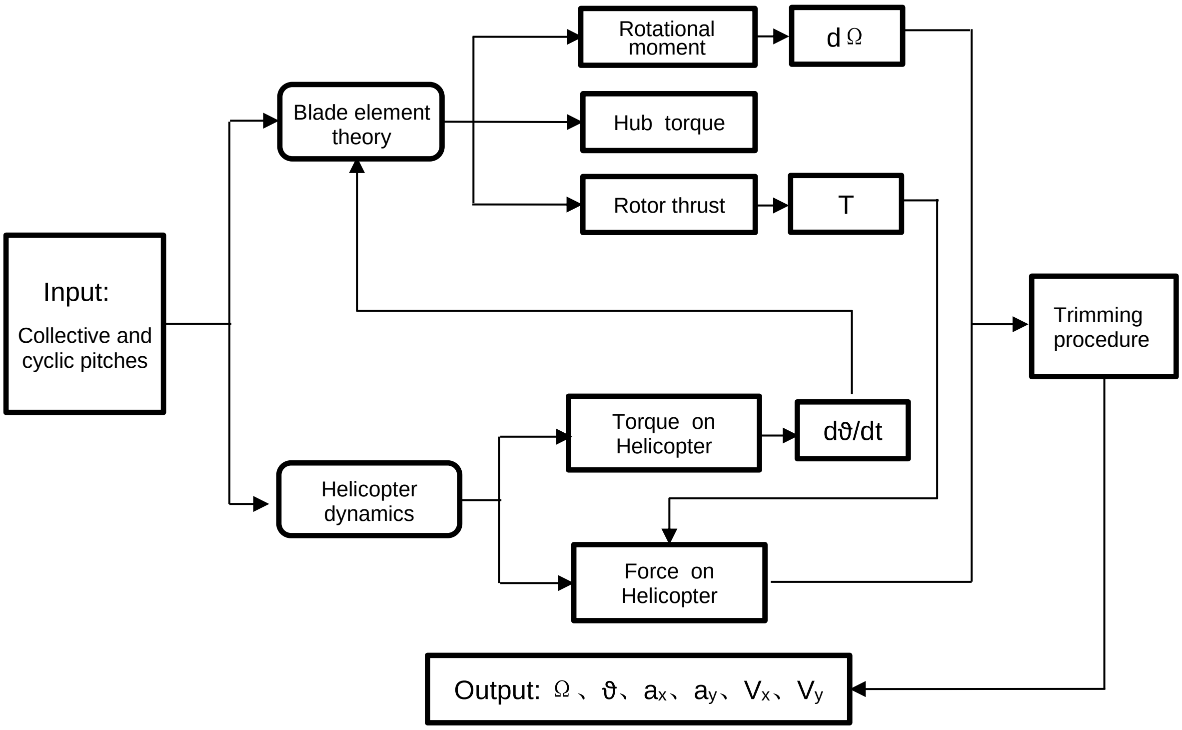

Figure 6- Program chart of autorotation calculation.

3.4 Results and discussions

3.4.1 Stable autorotation state

When the helicopter enters a stable autorotation state, the rate or angle of descent was calculated by adjusting different collective pitch input after the engine failure. In this paper, the flight speed of the helicopter was assumed to be 10 m/s and 30m/s separately, the original rotor rotating speed is 35 rad/s. Based on point-mass model, according to the **Formula (9), (10)** and **(12)**, the variation of rotating speed, main rotor thrust, collective pitch, descent rate and horizontal velocity with descent angle can be determined. **Figure 7** shows the parameters change for 10m/s flight speed. **Figure 8** shows the parameters change for 30m/s working condition.

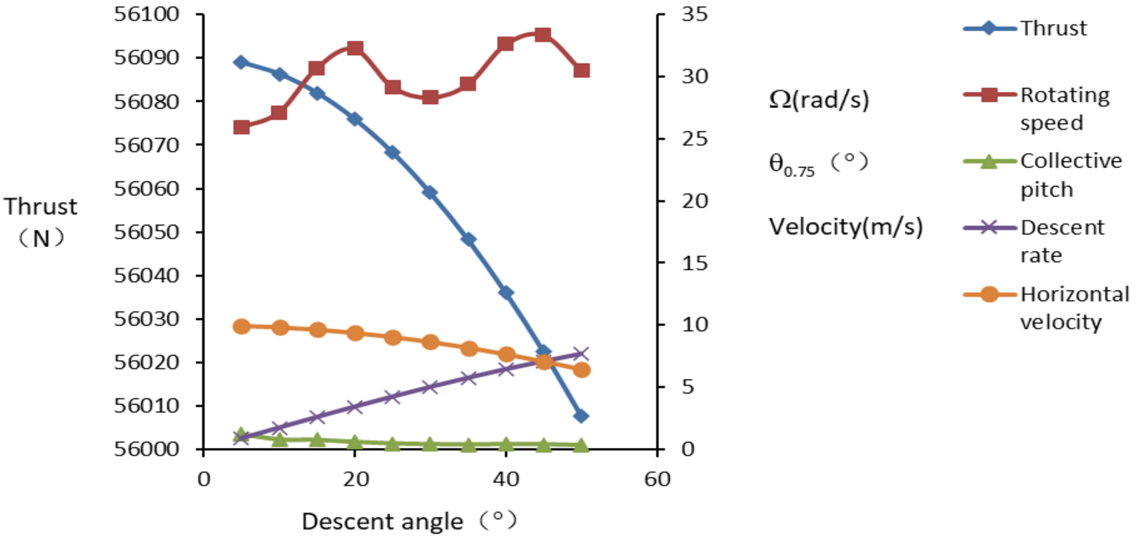

Figure 7-Parameters change with descent angle for flight speed 10 m/s.

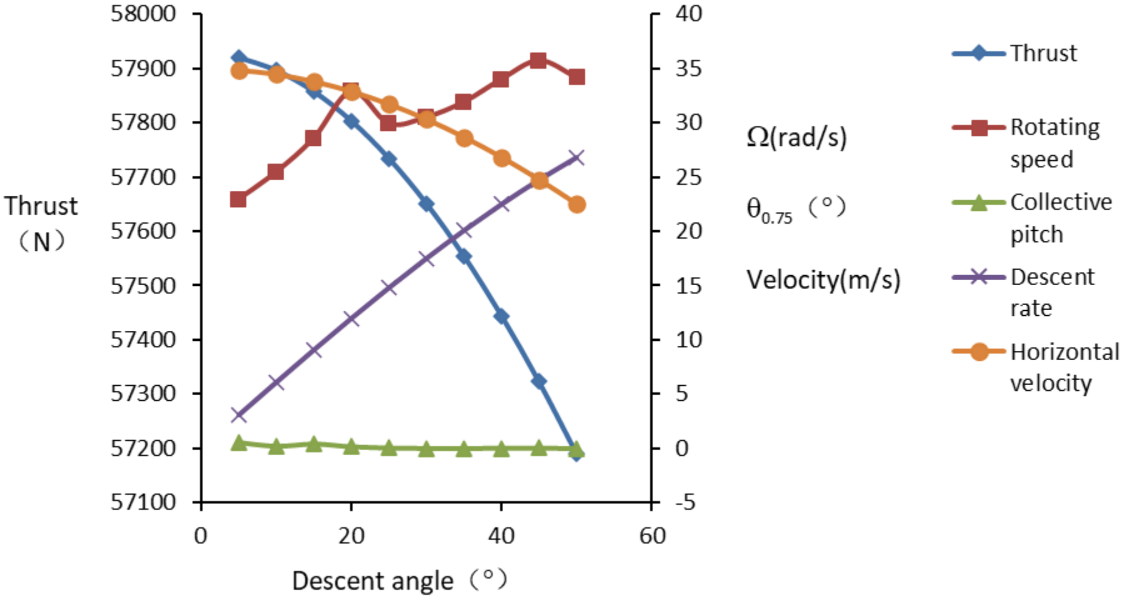

Figure 8- Parameters change with descent angle for flight speed 30 m/s.

As can be seen from **Figure 7**, with the increase of descent angle, the thrust is decreased in range 56000~56089 N, rotating speed is increased in range 22~35 rad/s as a whole, the collective pitch is decreased in range 0.32~1.26, descent rate is increased in range 0.8~7.7 m/s, the horizontal velocity is decreased in range 6.4~10m/s. Those variation of parameters are reasonable, because when the descent angle is increased, the helicopter drag component of in the vertical direction is increased. Therefore, the thrust component on vertical direction is decreased, together with the decrease of collective pitch. Meanwhile, the increase of descent rate results in the energy created from the wind to the main rotor increased, thus the rotating speed is increased.

Figure 8 show the similar variation trend with **Figure 7**. There are small differences between parameter values range. With the increase of descent angle, the thrust is decreased in range 57200~57957 N, rotating speed is increased in range 22~35 rad/s (same with 10m/s working condition) as a whole, the collective pitch is decreased in range -0.03~0.56, decent rate is increased in range 3~27 m/s, the horizontal velocity is decreased in range 22~35 m/s.

The comparison of **Figure 7** and **8** shows that when flight speed is increased at same descent angle, the thrust is increased, collective pitch angle is decreased and descent rate is increased. The autorotation state results also show that, a smaller collective pitch corresponds to a larger descent rate and larger track tilt angle.

3.4.2 The manipulation variables change in rotation process

If in **Formula (3)** and **(4)**, the acceleration variable are applied. According to the primary configuration in **Figure 6**, the variable change with time can be obtained which includes helicopter attitude, descent rate, trajectory, speed and rotating speed, varying with time were predicted under the condition that the initial state and control parameters of the helicopter were known. Further work will be carried on in the research group in the future.

4. Conclusion

The dangerous region of the XH-59A helicopter was calculated according to general empirical formula, which provided the safety region for the trajectory prediction.

The point-mass model of the helicopter had been created with force and moment analysis on it to predict trajectory and dynamic response. Aerodynamic force distribution on blade element was linked with helicopter's working conditions. The manipulation variables were selected as collective and cyclic pitches. The relationship between manipulation variables and helicopter performance were created using MATLAB program.

Parameters varying with descent angle in stable autorotation state were calculated for flight speed 10m/s and 30m/s respectively. Result shows that with the increase of descent angle, the rotor thrust, the collective pitch and the helicopter horizontal velocity are decreased, while the rotating speed of the rotor and descent rate are increased. The higher the flight speed is, the higher the thrust and descent rate are, and the smaller the pitch angle is.

5. Contact Author Email Address

Postgraduate, School of Aeronautics: Zsz-1997@mail.nwpu.edu.cn
Associate Professor, School of Aeronautics: xuzhao@nwpu.edu.cn
Postgraduate, School of Aeronautics: 15110377972@163.com
Senior engineer: lenovo37213618@sina.com

6. Copyright Statement

7. Acknowledgments

This work was supported by the Aeronautical Science Foundation of China 20175753030 .

References

[1] Y-H Niu, B-X Yang, Flight test of Z11 helicopter autorotation landing[J]. *Flight Dynamics*, 2001(03):74-76+83.

[2] X-Y Wang, Y-M Deng, Autorotation Flight Path Calculation of a Coaxial Helicopter[J]. *Flight Dynamics*, 2002(04):10-13.

[3] Okuno Y, Kawachi K, Azuma A, et al. Analytical Prediction of Height-Velocity Diagram of a Helicopter Using Optimal Control Theory[J]. *Journal of Guidance, Control, and Dynamics*, 1991, 14(2): 453-459.

[4] Okuno Y, Kawachi K. Optimal Control of Helicopters Following Power Failure[J]. *Journal of Guidance, Control, and Dynamics*, 1994, 17(1): 181-186.

[5] Pegg, An Investigation of the Height - Velocity Diagram Showing Effects of Density Altitude and Gross Weight, NASA TND-4536, 1968.

[6] Condon, Bailes, & Connor, Height-Velocity Test, AH-1G Helicopter[R], USAASTA Project 69-13, 1971.

[7] Lee, Allan Y. Optimal autorotational descent of a helicopter with control and state inequality constraints[J]. *Journal of Guidance Control and Dynamics*, 2015, 13(5):922-924.

[8] Lee, A. Y., Bryson, A. E., Jr., and Hindson, W. S., "Optimal Landing of a Helicopter in Autorotation," *Journal of Guidance, Control, and Dynamics,* Vol.11, No.1, 1988, pp. 7-12.

[9] Y-H Zhang. An optimal control of the landing of a helicopter in autorotation [D]. Nanjing University of Aeronautics and Astronautics, 2004.

[10]Bibik P, Narkiewicz J. Helicopter Optimal Control after Power Failure Using Comprehensive Dynamic Model[J]. *Journal of Guidance, Control, and Dynamics*,2012, 35(4): 1354-1362.

[11]Stack J, Caradonna F, Savas O. Flow visualizations and extended thrust time histories of rotor vortex wakes in descent. 4th American Helicopter Society Decennial Specialists' Conference on Aeromechanics[J]. *Journal of the American Helicopter Society,*2005, 50(3): 279–288.

[12]Green R B, Gillies E A, Brown R E. The flow field around a rotor in axial descent[J]. *Journal of Fluid Mechanics*, 2005, 534: 237–261.

[13]W-L Meng, R-L Chen. Study of helicopter autorotation landing following engine failure based on a six-degree-of-freedom rigid-body dynamic model[J]. *Chinese Journal of Aeronautics*, 2013, 26(6): 1380-1388.

[14]Felker F F. Performance and loads data from a wind tunnel test of a full-scale, coaxial, *hingeless rotor helicopter*[R]. NASA technical Memorandum 81329, 1981.