ICAS 2021 SHANGHAI
International Council of the Aeronautical Sciences
32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

RATIONAL DETERMINATION OF WING JIG SHAPE OF A LONG-RANGE AIRCRAFT WITH TRANSONIC CRUISE REGIME

Bezuevsky A.V.[1], Grigorev A.V.[1]

[1] Central Aerohydrodynamic Institute (TsAGI), Zhukovsky, Russia

Abstract

Iterative procedure for the long-range airplane wing jig shape calculation was developed and presented. Parametric study results of the effect of the aircraft wing jig shape on structural mass and fuel consumption were given. Recommendations for rational determination of the wing jig shape.

Key words: jig shape, static aeroelasticity, long-range airplane, iterative algorithm, structural mass

1. Introduction

According to the current tendencies it is logical to assume that the ecological requirements for the airplanes will only become stricter with time due to the need of further diminishing the environmental impact of aerial transport. So, increasing of lift-to-drag ratio is the one of the most important goals during the design phase of transport aircraft development. The majority of large transport airplanes operate in 18-19 range of maximum lift-to-drag ratio values if configuration with traditional wing and cylindrical fuselage is used (tube-and-wing). The lift-to-drag ratio is affected by multiple factors such as: aircraft layout, flight regimes, airfoils on lifting surfaces, surface smoothness, balancing conditions and so on. Since the beginning of air travel history a correct choice of these factors always received close attention. In recent years the implementation of light-weight materials such as composites made it possible to significantly diminish structural mass and balance drag which, in turn, decreased fuel consumption. Research and development of airplanes with high-aspect-ratio wings demands a lot of attention towards the studies of aeroelasticity. That is due to fact that the rise of wing aspect ratio in turn increases its elasticity

One of the actual tasks in designing a long range aircraft (LRA) wing is the correct choice of jig shape, taking into account the elastic deformations of the structure. Usually it is determined from the condition of the minimum difference between the flight twist of the wing and the theoretical twist, determined from the condition of maximum lift-to-drag ratio in the design cruise flight regime. The task has unique solution for one design weight and one flight regime.

In real flight, the aircraft mass and its distribution change, which leads to a change in the elastic deformations of the structure. Part of the flight may be performed in non-optimal flight regime on Mach number and altitude. The choice of wing jig shape affects the load distribution in the design load cases, and consequently the parameters of the primary structure and the stiffness of the wing. The interrelation of these factors complicates the problem of choosing the jig shape.

2. Overview of current studies in this area

In the research articles of different authors various aspects of jig shape determination were considered such as: minimization of a efficiency loss due to induced drag at off-design regimes by tailoring wing structures [1]; using additional control surfaces along the trailing edge [2, 3]; implementation of multi-disciplinary optimization of wing structure [4, 5].

In NASA Ames Research Center (Moffett Field, California) a team of researchers developed methods and tools for aeroelasticity studies of modern high-aspect-ratio wings (Transonic Truss-Braced Wing - TTBW). The paper [6] presents a jig twist optimization study of Mach 0.745 TTBW aircraft using an in-house developed aero-structural analysis software. A beam finite-element model was coupled with a vortex-lattice model for fast calculation of aeroelastic properties. For the result validation a

CFD computations were used as well as transonic wind-tunnel test data. Elastic properties were obtained using finite-element model analysis. Twist was represented by a sum of functions of shape (Chebyshev cubic polynomial fit), that allowed to model a smooth distribution of twist while using finite set of parameters. The result of application of optimization algorithm was the reduction of drag coefficient by 0.0008. That result was verified by CFD-simulation.

At the same NASA Ames Research Center, the research was performed under the title of Elastically Shaped Air Vehicle Concept [7]. Three main directions were considered: non-planar wing optimization; drag reduction through a new concept of elastic wing shaping control; implementing new aerodynamic control effector called a variable camber continuous trailing edge flap. For the optimization procedure the prototype was the research platform, that included both a wind tunnel model and remotely controlled unmanned flying vehicle. The effect of implementing all this means is pretty substantial, but requires use of additional control surfaces or significant changes in aircraft construction.

At University of Michigan, (Ann Arbor, Michigan) the studies were performed on elastic high-aspect-ratio wings [8]; coupled aero-propulsion effects from boundary layer ingestion; complex non-linear flutter behavior for truss-braced wings. Optimization was performed for a cruise regime and two off-design load cases to insure buffeting safety. The minimization of aerodynamic drag was performed at fixed lift. The aircraft was in a balanced state, as a variables were used such parameters as tail rotation angle, multiple wing shape parameters, twist angles, angle of attack values. There were constraints like lower limit to airfoil thickness and wing inner volume. The model with parameters is shown on Figure 1.

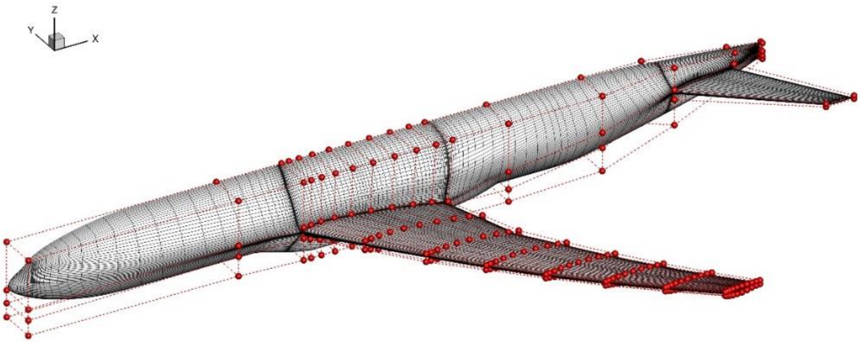

Figure 1 - Model and optimization parameters.

A sum of drag coefficients with weights applied for several load cases was a function that minimized. When the take-off weight was used as goal function, the result of the optimization procedure was a wing with aspect ratio of 9.08 and lift-to-drag ratio of 18.83 at cruise regime. When the fuel consumption rate was used as a goal function, the wing with aspect ratio of 12.59 was produced and lift-to-drag ratio of 23.6.

In the research work [9] authors' goal was reduction of fuel consumption by making use of the Individual Discipline Feasible architecture, which decouples the aerodynamic and structural disciplines from each other. Using this approach, the consistency of the system as a whole is maintained by the use of equality constraints on surrogate design variables. No coupled sensitivity information is required because of this decoupled system. This makes the system not only simpler, but also provides more freedom in software choice. Furthermore, the time to perform optimization is reduced with respect to the traditional Multidisciplinary Feasible architecture as the work of making the system consistent is removed from the computationally expensive individual disciplines and put it in the hands of the cheap optimization algorithm. As design variables the twist angles were taken, as well as planform parameters and wing-box panel thicknesses. As constraints were used the strength limitations as well as buckling conditions and fatigue. Aside from those there was maximum wing load and minimum aileron efficiency constraints. The analysis of aerodynamic properties was performed for cruise regime. Loads were calculated for pull up, push over and gust load cases. There was also load case for aileron roll efficiency calculation. As the reference aircraft the Airbus A320 was used. The Sparse Nonlinear Optimizer, based on the Sequential Quadratic Programming optimization algorithm, was used for optimization.

The analysis of aforementioned works led to the conclusion that multi-disciplinary optimization of lift-to drag ratio or fuel consumption ratio is mandatory. Software, used for calculations, should not require substantial computation power, since more precise algorithms may need hundreds and thousands of machine-hours of powerful modern computers for each run of optimization procedure. To ensure sufficient precision it is possible to verify results of computation, made by standard software tool, for several separate load cases by more resource-hungry programs or by using the experimental data.

3. The algorithms and software used

The calculations were performed with TsAGI in-house ARGON software package [10] which is based on polynomial Ritz method. For deformation representation this software uses beam and plate scheme of aircraft aggregate modelling. Aerodynamic properties could be determined by using in-built tools of this software. There is also a possibility to use the calculation results of external specialized software products.

3.1 Algorithm for rational wing jig shape determination

Algorithm has three levels. The first level is the iterative determination of preliminary wing-box structural parameters for conditions of chosen typical load cases.

Preliminary parameters and strength of wing-box structure are defined by the loads of rigid aircraft for load cases of quasi-static maneuvers. Critical load cases for maneuver loads are determined as a result of preliminary research. The load calculation is performed for chosen mathematical model of airplane for given flight parameters and control surfaces' deflection angles. It is done by using the module for quasi-static loads' calculation of ARGON software package. During the first step the rigid airplane loads are determined for created computational model. Model structural design is performed under conditions of strength based on integral loads' envelope. In this case the design procedure consists of required panel thickness calculation that meet strength requirements for integral loads in beam approximation. During the second step the wing-box designed for rigid airplane loads is redesigned for loads of flexible airplane. The procedure of redesigning for flexible airplane loads is iterative and 2-3 iterations are required.

After that the jig shape is calculated for the model optimized for loads of flexible airplane. Cruise twist is a jig shape of preliminary model and flexible twist angles are determined from calculations of loads and deformations at cruise flight regime. Jig shape is determined from those by using the following formula:

$$\boldsymbol{\alpha}_{\mathrm{jig}} = \boldsymbol{\alpha}_{\mathrm{cruise}} - \boldsymbol{\alpha}_{\mathrm{flex}}$$

Here: α_{jig} is jig shape angle at given wing section; α_{cruise} – the shape at cruise flight regime; α_{flex} – stream-wise twist angle at the same section.

After the first iteration instead of cruise angles the calculated jig angles are inserted and the calculation of twist angles is repeated for modified model. The process is repeated until the difference between jig shapes for two consecutive iterations lies inside of allowable error values (3-4 iterations are usually needed). For the preliminary model the cruise shape is used. For obtained flexible model with calculated jig shape further wing-box redesign is performed for flexible airplane's loads, which requires 2 to 3 iterations. The resulting model has the wing-box, optimized for design loads and which jig-shape is optimal for chosen cruise regime and aircraft flexibility taken into account. Resulting flow-chart shown on Figure 2 demonstrates the algorithm of jig shape calculation at which the maximum lift-to-drag ratio for given flight regimes is reached.

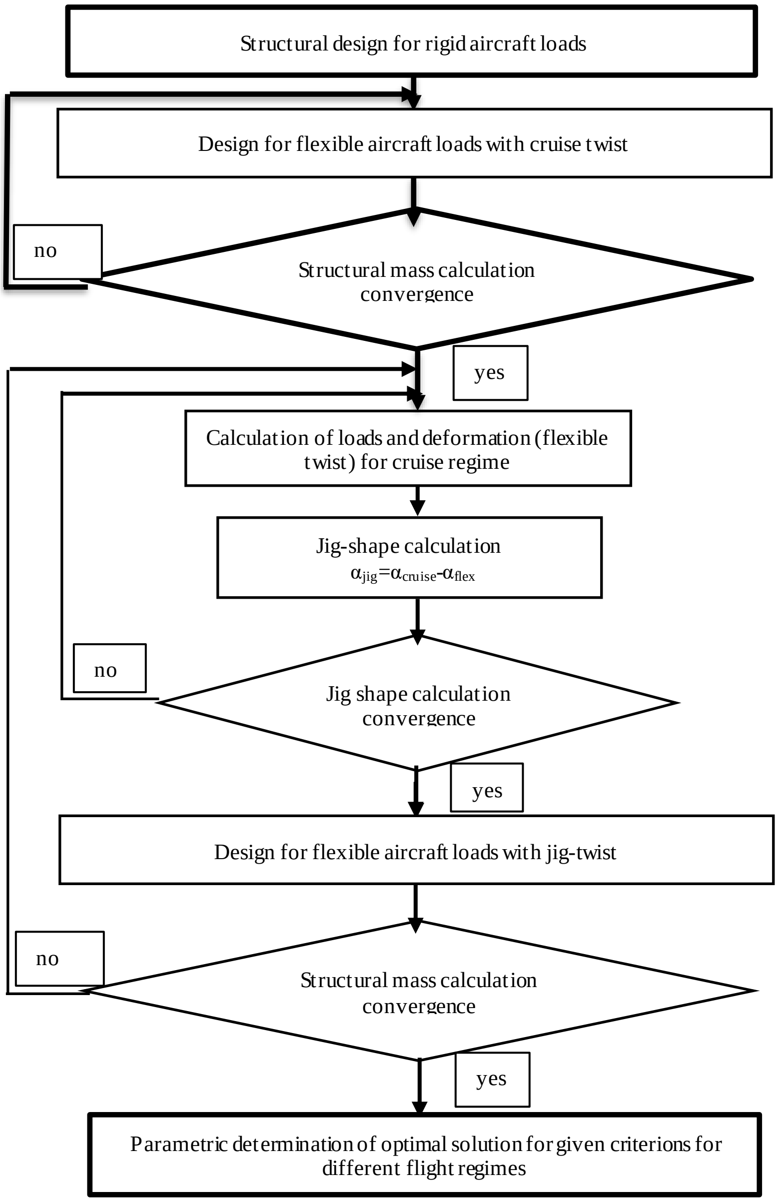

Figure 2 - Algorithm of structural and jig-shape optimization for given flight regime.

3.2 Aerodynamic calculations

In considered range of evaluative calculations the precision of panel aerodynamics is sufficient, so for it is used for the majority of calculations here.

To take peculiarities of trans-sonic aerodynamics into account BLWF software package is used. The calculation process in this software is performed by numerical integration of conservative form of finite-difference analogue of Euler's system of equations. Computational algorithm is based on iterative technique of viscous-inviscid interaction in shear layer theory. Laminar and turbulent spatial shear layer on wing surfaces is calculated using by finite-difference method based on predictor-corrector scheme [11]. The loads obtained as a result of aerodynamic calculations transferred to model of the structure for deformations' calculations.

3.3 Wing-box design

For design studies of high-aspect-ratio wing it is preferable to use the beam theory. Beam approximation in structural design assumes use of bending and torsion moments' distribution, which resulted from of active aerodynamic and inertial forces. For aerodynamic loads' distribution analysis the pressure distributions are used that are calculated beforehand for separate cases. After that the aerodynamic forces for each panel are determined (normal to the surface). For inertial forces calculation the distributed masses are translated into point (concentrated) masses. Forces could be integrated along given lines, presented as diagrams for distributed bending and twist moments as well as shear forces and then used for analysis and design of model in beam approximation. The design technique is then simply a computation of panel thickness that is sufficient to meet the strength requirements under integral loads

For composite materials the criterion of limits of allowed strain is used to design a wing-box in beam approximation. In this case during the loads' calculation it is determined which (of bending and torsion) are the defining deformations. If the defining deformations are bending, then the defining load is a bending moment M_{bend}. Then, form the cross-section balance condition the following equation is obtained:

$$\left(\sigma_{panel}F_{panel}+\sigma_{beam}F_{beam}\right)H=fM_{bend} \qquad (1),$$

In which $F_{panel}=tb$ – skin section area, b – chord, t – skin thickness, σ_{panel} – normal stress in panels, σ_{beam} – normal stress in beams, f – safety coefficient, $F_{beam}=k_bF_{panel}=k_btb$ – spar cap area.

From beam and panel strain compatibility condition: $\frac{\sigma_{panel}}{E_{panel}}=\frac{\sigma_{beam}}{E_{beam}}=\varepsilon_a$, where ε_a – allowed strain, then $\sigma_{beam}=\frac{E_{beam}}{E_{panel}}\sigma_{panel}$.

By putting this into (1), we obtain: $\sigma_{panel}tb\left(1+\frac{E_{beam}}{E_{panel}}k_b\right)H=fM_{bend}$

Then:

$$t=\frac{fM_{bend}}{\sigma_{panel}b\left(1+\frac{E_{beam}}{E_{panel}}k_b\right)H}$$

Or, if we assume that

$$\sigma_{panel}=E_{panel}\varepsilon_a$$

$$t=\frac{fM_{bend}}{\varepsilon_a b\left(E_{panel}+E_{beam}k_b\right)H}$$

If the defining deformations are torsion ones then the defining load is twist moment M_{tors}. As only the skin panels work under the torsion loads, then from section blance condition: $fM_{tors}=\tau W_p$, where τ – is shear stress $W_p=\frac{4H^2t}{3}$ – section modulus of thin-walled wing-box, H – wing-box height.

Then:

$$t=\frac{3fM_{tors}}{4\tau H^2}$$

Mass-per-unit-length of section: $m_l=\rho_{panel}tb+\rho_{beam}k_btb$

If the density of material is equal to $\rho_{panel}=\rho_{beam}=\rho$, then

$$m_l(z)=\frac{\rho fM_{bend}}{\sigma_{panel}H}\frac{1+k_b}{1+\frac{E_{beam}}{E_{panel}}k_b}$$

In our case k_b is equal to 1. On Figure 3 the sample cross-section of wing-box of an airplane is shown. Wing-box properties are calculated following the approach that is described above to meet static strength criterions.

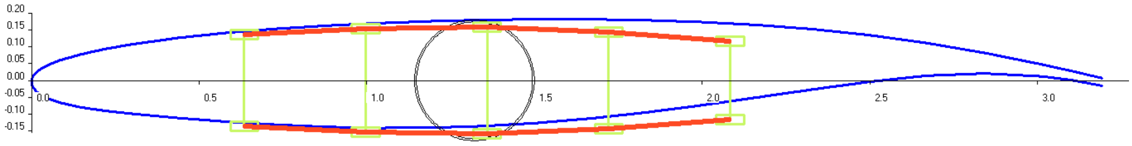

Figure 3 - Wing-box cross-section.

3.4 Rational jig-shape determination criterions

Aside from lift-to-drag ratio for given flight regime the other chosen parameters can be used, such as flight range at given fixed starting mass of the airplane or fuel consumption rate.

The flight range of an airplane with turbo-jet engine is determined by using Breguet formula [12]:

$$L = 3.6 \int_{G_2}^{G_1} \frac{aMK}{C_e} \frac{dG}{G}.$$

If the flight speed and turbine turn rate is constant, then: $L = 3.6 \frac{aKM}{C_e} \ln \frac{G_1}{G_2}$,

Where: G_1 – takeoff weight; G_2 – zero fuel weight; C_e – thrust-specific fuel consumption; a – speed of sound; M – Mach number.

G_1 is a weight of airplane with standard payload and maximum fuel weight, G_2 – landing weight with a standard payload and with minimum fuel residue. Then the formula for required fuel weight at fixed flight range (fuel consumption) can be written as follows:

$$\Delta G = G_2 \left(e^{\left(\frac{L C_e}{3.6 aKM}\right)} - 1\right).$$

4. Jig-shape definition of long range aircraft

This study is devoted to the methodology of rational choice of jig-shape of typical long-range airplane with the transonic flight regime.

4.1 Computation model

For the study preliminary computational model was created, with both aerodynamic and elastic mass properties represented. It consists of composition of thin flexible surfaces, which have the shape of median surfaces of real airplane's aggregates. It is assumed that aerodynamic disturbances and elastic displacements are small enough. In addition to that, it is assumed that center of mass position and plan-shape could be considered constant as airplane deforms. Longitudinal motion with symmetrical elastic deformations taken into account and lateral motion with antisymmetrical elastic deformations taken into account are considered separately. For computation of aerodynamic forces in quasi-static problems a linear panel method was used (also known as Woodward's method). Elastic-mass properties of the structure were modelled using Ritz's polynomial method.

4.2 Aerodynamic calculations

In preliminary model a cruise-shape was set. In ARGON software the aerodynamic model is broken into elementary panels as shown on Figure 4.

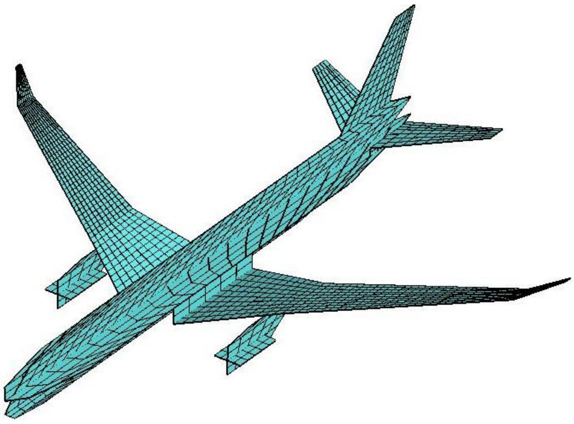

Figure 4 - LRA model for ARGON software.

The calculations were also performed using BLWF software. In this software aerodynamic forces are calculated using a fast-acting effective method based on the Euler equations with taking into account flow viscosity; the method has proved itself well in the tasks of aerodynamic design of aircraft with transonic flight regimes. The aerodynamic computational model for BLWF software is shown on Figure 5. It is created by interpolating of the mesh between sections perpendicular to the flow for fuselage and along the flow for all other surfaces.

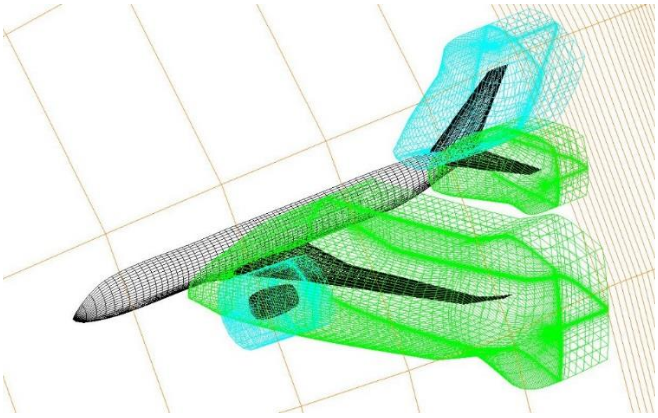

Figure 5 - LRA aerodynamic model for BLWF.

BLWF software package was used to calculate aerodynamic loads for several flight regimes. The differences and similarities between computation results of using BLWF and using ARGON's panel aerodynamics were analyzed. The analysis ultimately showed that for the range of flight regimes in question the precision of panel aerodynamics is sufficient. Load coefficients' C_L comparison for flight regimes' range close to the cruise flight is shown on Figure 6.

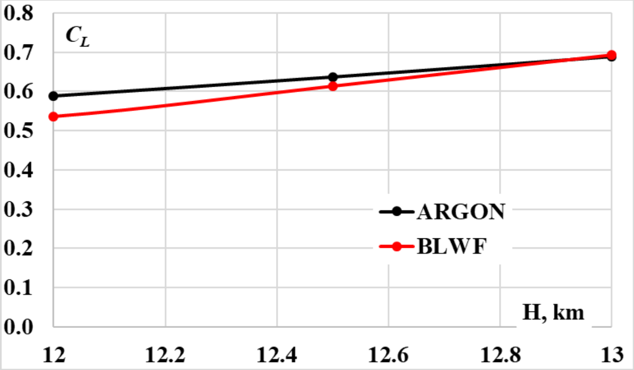

Figure 6 - C_L comparison for ARGON and BLWF.

Aerodynamic computation demonstrated the flow image that is specific to transonic regime. Pressure distributions in two sections are shown on Figures 7 and 8. The computational model variant used to get these images was optimized for a flight regime at 12.5 km altitude and 200 ton airplane weight. The calculations were performed for several flight altitudes. As can be seen on figures, the characteristic shock moves towards the tail end of the chord as the altitude increases. The pressure distributions, that were obtained from transonic aerodynamics' calculations were used to compute loads during iterative wing-box design phase and jig-shape re-calculation.

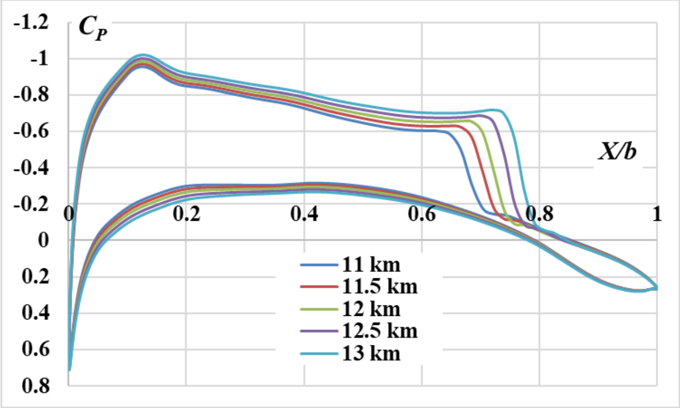

Figure 7 - Pressure distribution for the section located at 0.15 of wing span.

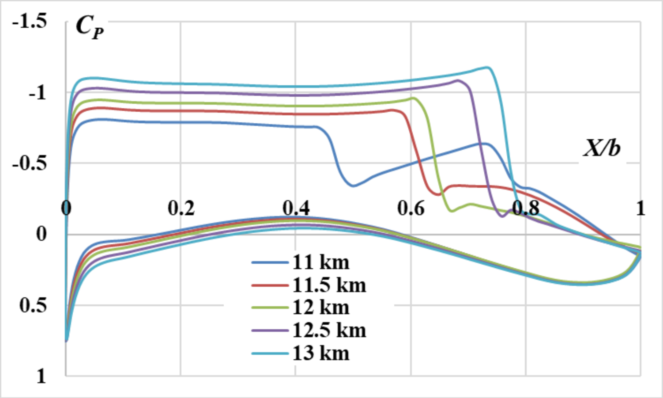

Figure 8 - Pressure distribution for the section located at 0.75 of wing span.

4.3 Wing-box and jig-shape optimization

For chosen typical load cases the basic parameters of wing-box structure were determined using iterative approach.

Strength and parameters of preliminary wing-box structure were obtained using loads of rigid plane for load cases that were representing quasi-static maneuvers. Several defining load cases for maneuver loads were chosen during preliminary research. ARGON software package tool for quasi-static loads' calculation was used to determine loads for chosen mathematical model of the aircraft. Flight parameters and control surfaces' deflections were taken into account. As described above, during the first step, rigid aircraft loads were calculated for LRA model, which was designed to meet strength's conditions based on integral loads' envelope. Required panel thickness were calculated for the model using the beam approximation. After the first iteration cruise shape was replaced by obtained jig-shape and the cycle of jig-shape calculation was repeated for modified model. Final stream-wise angles' distributions along the wing span that were obtained as a result of last iteration are shown on Figure 9.

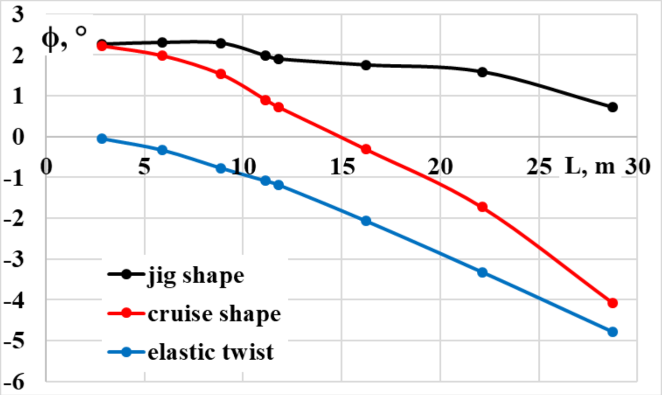

Figure 9 - Jig shape, cruise shape and elastic twist angle distributions along the wing span.

The wing-box of obtained model with proper jig-shape received a redesign, this time using the elastic aircraft loads. Resulting model had a wing-box optimized with elasticity of the aircraft taken into account. The structural weight of this wing-box is 4.15 ton.

As an example, there was determined how does the change of flight regime affects properties like maximum range with amount of fuel fixed and fuel consumption rate with maximum flight range fixed. The change of flight regimes was modeled as increasing and decreasing of flight altitude with respective changes of other parameters while the Mach number was fixed and was equal to 0.85.

4.4 Flight regime variation

For fuel-related calculations as G_1 was taken the aircraft weight with standard payload and maximum fuel weight. In this case G_1 was 229.3 ton. G_2 – was chosen to be equal to landing weight with a standard payload and with minimum fuel residue, in this case 157.4 ton. Required fuel weight at fixed flight range (fuel consumption) was then found using the Breguet formula for cruise flight at 12.5 km altitude and Mach number of 0.85. Using the mentioned before algorithm the structure and jig-shape of LRA wing-box was optimized for this load case. Lift-to-drag ratio values were calculated for this flight regime as well as for several others with different flight altitudes. The result was an estimate of flight range for chosen load cases, such as cruise regime, for which the jig-shape was optimized, as well as flight regimes with different altitudes. These are shown in Table 1. At this stage of parameters that are present in a Breguet's formula for different flight regimes the values of G_1 и G_2 stay the same. This is due to wing-box is the same for all cases. The conditions at the beginning of the flight and that of at its end remain constant too.

Table 1. The dependency of flight's maximum range from flight altitude at fixed fuel capacity.

H, km	L/D	L_{max}, km
11	18.37	12010
11.5	18.75	12255
12	19.05	12448
12.5	19.23	12566
13	19.31	12616

Aside from this the impact of correct choice of jig shape on lift-to-drag ratio and hence maximum flight range was analyzed. For this reason, with the help of aforementioned algorithm the computational models were created with different masses. The jig-shape of these models was optimized for cruise regimes of the aircraft with these different masses. The standard model was optimized for cruise flight regime with mass of 200 tons. There were also considered models optimized both for a mass of 166 tons (which represents an airplane with the standard payload and 5 tons of fuel) as well as optimized for 245 tons (maximum take-off weight without 5 tons of fuel). The effect of jig-shape optimization for particular flight weight and wing-box structure optimization for this jig-shape on maximum flight range was studied. In this research not only lift-to-drag ratio was different due to the change of jig-shape and load cases but also a structural weight of a wing-box. This wing-box weight change was small and still affected the outcome if only slightly.

The result of running the aforementioned algorithm for computational models, optimized for chosen weights was structural weights of wing-box. The structural weight of wing-box optimized for 166-ton aircraft was 124 kg lighter (per console) than baseline wing box optimized for 200-ton aircraft. The structural weight of wing-box, optimized for 245-ton aircraft, was 36 kg heavier (per console) than the baseline.

The Figure 10 shows the difference between loads (bending moment) distributed along the wing of the LRA variants with the wing-boxes and jig-shapes optimized for different weights of the aircraft at the cruise regime. These differences are shown between load values of baseline variant and two others.

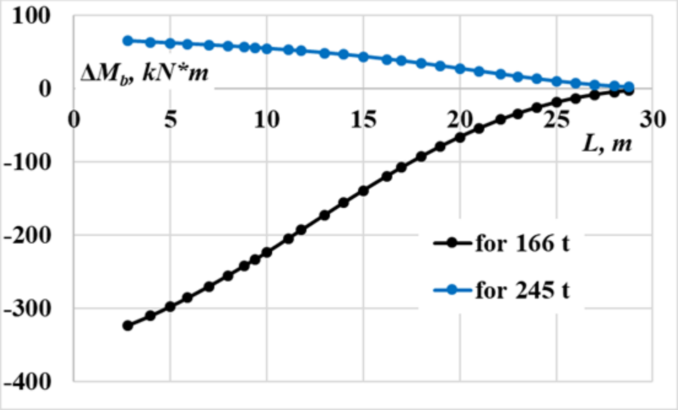

Figure 10 - The difference between loads of baseline variant and two others, distributed along the wing-span.

The next distribution (Figure 11) is the difference between thicknesses of structural elements along the wing of models optimized for 166 tons, 245 tons compared to that of optimized for 200 tons.

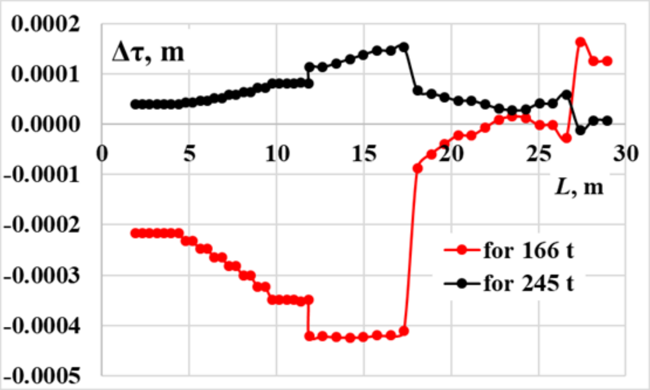

Figure 11 - The difference between thicknesses of baseline variant and two others, distributed along the wing-span.

Figure 12 demonstrates the distributions of jig-shape angles along the wing span, optimized for three different flight weights.

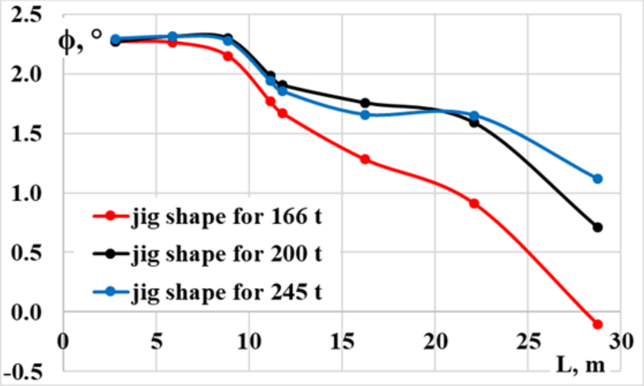

Figure 12 - Jig-shape angles, optimized for different aircraft weights.

The values for maximum flight ranges were calculated while taking into account G_1 and G_2 corrections due to wing-box weight changes that were caused by optimization for different jig-shapes. The dependencies of these maximum flight ranges from flight altitudes for the models used are shown on Figure 13. The dependencies for lift-to-drag values look almost the same. That is due to the fact that the range is directly proportional to lift-to-drag ratio. The differences are caused by changes of wing-box structural weights, which has some effect on the maximum range. The difference between lift-to-drag ratios is directly caused by difference between jig-shape angle distributions. For a fixed flight range of 12000 km there was a value of required mass of the fuel ΔG calculated and the dependencies are shown on Figure14.

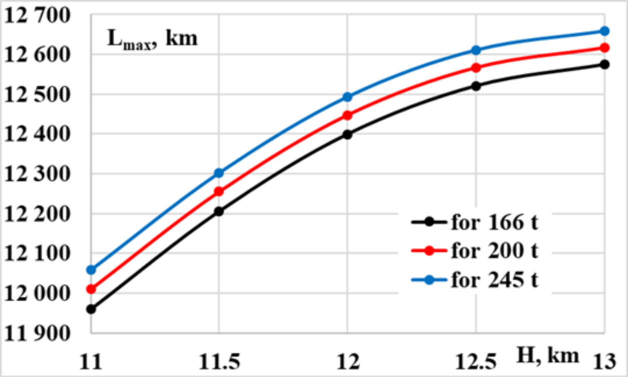

Figure 13 - Dependencies of maximum range from flight altitude for LRA with wing, optimized for different flight weights.

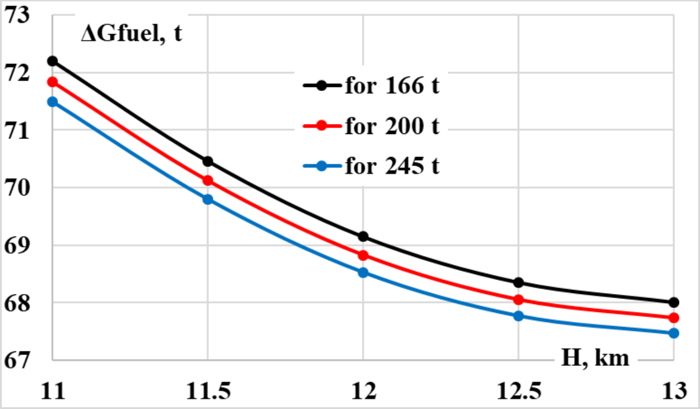

Figure 14 - Dependencies of fuel consumption rates from flight altitude for LRA with wing, optimized for different flight weights.

The computed data shows that if the jig-shape is optimized for the aircraft weight that is close to the take-off weight then maximum range increases. Optimization of the jig-shape for same load cases, but with the weight that is closer to landing one, leads to decreased maximum range compared to model, optimized for average flight weight. The flight at higher altitudes at same Mach number allows to increase maximum range somewhat.

5. Conclusion

The algorithm of the jig-shape rational choice for the long range aircraft with the transonic flight regime was developed. This includes a wing-box structural weight optimization. Computational models of LRA in question for ARGON and BLWF software packages were created.

With the help of developed algorithm, the parametric research was performed on typical LRA, that is devoted to studying jig-shape influence on maximum flight range and required fuel capacity.

In addition to that the effect of aircraft weight, for which the optimization of jig-shape is performed, on jig-shape angles themselves as well as on the maximum range.

It was found, that jig-shape optimization for aircraft weight close to maximum take-off weight slightly increases the max range (~0.4%), and for the weight, that is close to a landing weight, it decreases the range (~-0.4%) when compared to its value for average flight weight.

The off-design regimes' influence on maximum range was also analyzed. For this LRA, the flight at altitudes higher than chosen cruise one at the same Mach number allows to slightly increase the maximum range (~0.4%). Lowering the altitude by 1.5 km leads to maximum flight range decrease by 4.5%. There are still altitude limitations caused by the engine parameters among others, that were not considered in this work.

The results of computational studies demonstrated that the influence of jig-shape variation on fuel consumption rates, maximum flight range and structural weight is not very big, reaching several percent at its max. Still, if the long time-span of the flight and LRA's big fuel capacity, using these recommendations as guidelines may result in significant profit. Thus, taking these factors into account, while not being defining, still should be considered when choosing jig-shape and estimated load cases' combinations for LRA.

6. Contact Author Email Address

mailto: stataer@tsagi.ru

7. Copyright Statement

References

[1] Melih Papila, Raphael T. Haftka, William H. Mason and Rafael Alves “Tailoring Wing Structures for Reduced Drag Penalty in Off-design Flight Conditions” *AIAA* Paper No 2004 4637

[2] D.L. Rodriguez, M.J. Aftosmis, M. Nemec, G.R. Anderson. Optimized off-design performance of flexible wings with continuous trailing-edge flaps. *AIAA* Paper No 2015-1409

[3] R. Kolonay, F. Eastep, and B. Sanders “Optimal Scheduling of Control Surfaces on a Flexible Wing to Reduce Induced Drag” *AIAA* 2004-4362

[4] Martin Kruse, Tobias Wunderlich and Lars Heinrich. A Conceptual Study of a Transonic NLF Transport Aircraft with Forward Swept Wings. *AIAA* Paper No 2012-3208

[5] Kenway GKW, Martins JRRA, Kennedy. Aerostructural optimization of the Common Research Model configuration. *AIAA* Paper No. 2014-3274

[6] Juntao Xiong, Nhan Nguyen, Jason Fugate. “Jig Twist Optimization of Mach 0.745 Transonic Truss-Braced Wing Aircraft and High-Fidelity CFD Validation”, Paper *AIAA* 2020-0451

[7] Nhan Nguyen, Khanh Trinh, Kevin Reynolds, James Kless, Michael Aftosmis, James Urnes, Sr., Corey Ippolito. “Elastically Shaped Wing Optimization and Aircraft Concept for Improved Cruise Efficiency”. Conference *51st AIAA Aerospace Sciences Meeting*, Grapevine, TX, January 2013

[8] Gaetan K.W. Kenway, Joaquim R. R. A. Martins «Adjoint-based Aerodynamic and Aerostructural Optimization of Transport Aircraft*» Advanced Modeling and Simulation Seminar Series NASA Ames Research Center*, February 7, 2017

[9] Jan E.K. Hoogervorst “Wing aerostructural optimization using the Individual Discipline Feasible architecture”. *Thesis Report for Master of Science degree in Aerospace Engineering at the Delft University of Technology*, 2015

[10] Ishmuratov F.Z., Chedrik V.V. ARGON Code: Structural Aeroelastic Analysis and Optimization. In: *International Forum on Aeroelasticity and Structural Dynamics (IFASD)*, Amsterdam, 2003.

[11] Kuzmina S., Karas O., Ishmuratov F., Zichenkov M., Chedrik V. Analysis of static and dynamic aeroelastic characteristics of airplane in transonic flow. *28th Congress of the International Council of the Aeronautical Sciences 2012*, ICAS-2012.

[12] Z.S. Spakovsky *“Thermodynamics and propulsion”* MIT Turbines, 2002.