ICAS 2021 SHANGHAI
International Council of the Aeronautical Sciences
32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

Experimental Research of Cooperative Navigation Method for Cluster Aircrafts based on Mutual Distance Difference Model

Rong Wang, Junnan Du, Zhi Xiong, Jianye Liu, Yu Rui, Tingyu Nie

Navigation Research Center, College of Automation Engineering,
Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

Abstract:

Aircraft cluster flight technology has been paid more and more attention because of its wide search range, high efficiency and high reliability. In this paper, a cluster cooperation method based on mutual distance difference model is proposed by using cooperative navigation technology. By using the positioning information of other aircraft and the distance difference between them, the positioning accuracy of the aircraft with lower positioning accuracy is improved, and the simulation platform of cluster Aircraft Cooperative navigation based on relative measurement information is built.

Keywords: Cluster flight; Collaborative navigation; Mutual Distance Difference Model; Chan algorithm; Kalman filtering

1. Introduction

Aircraft cluster flight technology is receiving more and more attention, and aircraft cluster flight is one of the future development directions of aircraft technology. Aircraft cluster flight could expand the search range, improve the overall task efficiency, perform multiple tasks, and have high system reliability, which has significant advantages compared to single aircraft[1].

Cooperative navigation technology is one of the key technologies to improve the overall navigation performance of the cluster aircraft. The cooperative navigation technology utilizes the sensor information between multiple aircrafts to optimize the navigation performance of multiple aircraft[2]. When the difference in accuracy of the navigation device in the flight of the aircraft cluster is large, the difference in positioning accuracy makes it impossible for each aircraft in the cluster to accurately reach the established flight array position at the same time, which affects the effect of the cluster flight task[3]. The traditional cooperative navigation algorithm needs to simultaneously measure the relative distance and angle vector information between the aircrafts to correct the navigation accuracy of the low-precision aircraft. The method has high requirements on the measurement accuracy and clock synchronization of the airborne relative measurement sensors of each aircraft in the cluster, and the navigation performance is limited when the aircraft equipment is used with relatively low performance of the measurement sensor.

In recent years, researches on collaborative navigation including the use of radio sensors and vision sensors to design collaborative navigation algorithms, design of the MEMS-IMU/GNSS integrated navigation system, monocular vision and optic-flow navigation system, MEMS-IMU/LADAR navigation system, aerodynamics model aided navigation and nonlinear filtering technique researches etc[4,5].

In this paper, a cooperative approach for clustered aircraft based on mutual distance difference model is proposed. The positioning information of other aircrafts in the cluster flight and the distance difference information between them are used to improve the positioning accuracy of the aircraft with lower positioning accuracy in the cluster. Based on the analysis of TDOA (Time Difference of Arrival) equations, the high-precision aircraft in the cluster flight system is used as the reference aircraft, and the remaining aircraft are used as the aircraft to be positioned to establish a cooperative navigation model. Then, according to the established collaborative navigation model, the algorithm for solving the cooperative navigation model of the cluster aircraft is designed. The collaborative navigation model solving algorithm is based on the Chan algorithm and is corrected by Taylor iterative algorithm. The Kalman filter is used to fuse the results of the above algorithm and the airborne navigation information of the low-precision aircraft to estimate the inertial navigation error of the low-precision aircraft, so as to improve the onboard navigation accuracy of the low-precision aircraft.

2 Cooperative Navigation based on Mutual Distance Difference

2.1 Cooperative Navigation Scheme

According to the characteristics of the cooperative navigation model based on the mutual distance information established above, this section designs the cooperative positioning algorithm based on Chan-Taylor joint iterative algorithm and cooperative location algorithm based on spherical interpolation method respectively for different mutual distance measurement error environments. The scheme of cooperative navigation algorithm based on mutual distance information is shown in Figure 1.

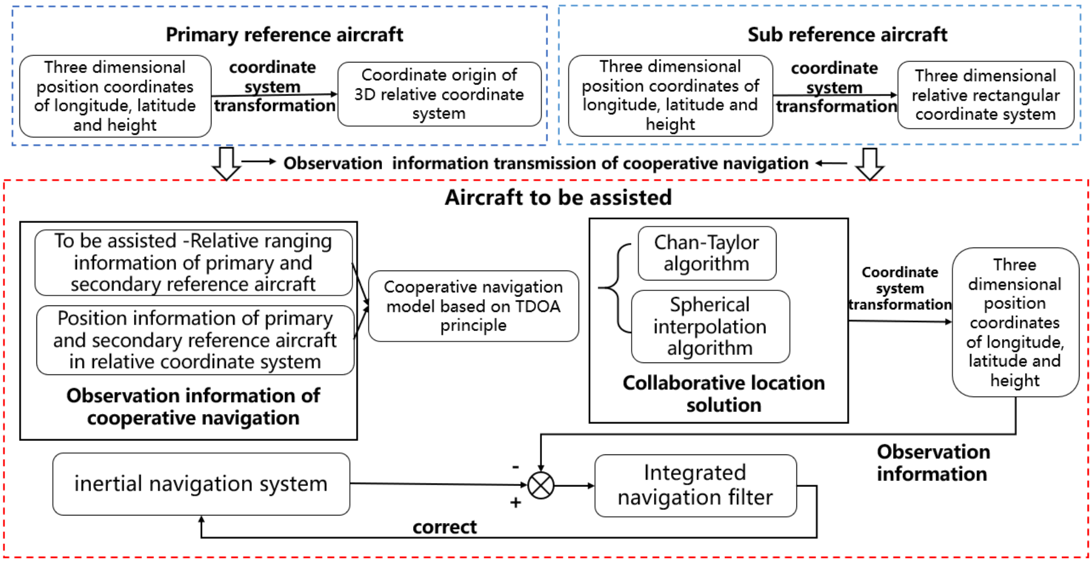


Figure 1- Cooperative navigation scheme based on mutual distance information.

The algorithm based on mutual distance information is mainly divided into two steps. Firstly, in this section, Chan-Taylor joint iterative algorithm and spherical interpolation algorithm are designed respectively for Gaussian error ranging environment and non Gaussian ranging error environment. Then, according to the position results of the aircraft to be assisted in the relative coordinate system calculated by the above algorithm, it is converted into the position coordinates of longitude and latitude through coordinate transformation, and it is used as the observation value to establish the Kalman filter model to correct the position information of the auxiliary aircraft's Airborne inertial navigation system.

2.2 Cooperative Navigation Model based on Mutual Distance Measurement

On the basis of the above hierarchical cooperative navigation structure, the cluster aircraft cooperative positioning algorithm based on the mutual distance information establishes the cooperative navigation model by acquiring the mutual distance information between the aircraft to be assisted and each reference aircraft to modify the navigation accuracy of the aircraft to be assisted. The schematic diagram of cluster aircraft cooperative positioning is shown in Figure 2.

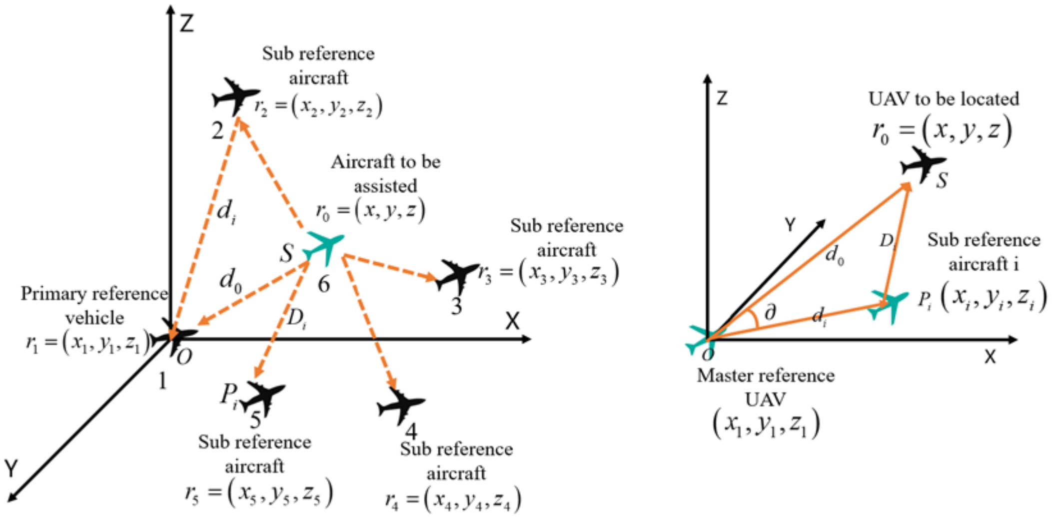


Figure 2 -Cooperative positioning of cluster aircraft based on mutual distance information

According to the proposed hierarchical cooperative navigation structure, one aircraft in the reference aircraft layer is taken as the primary reference aircraft (aircraft 1 in the figure), and the other aircraft in the reference aircraft layer is taken as the secondary reference aircraft, in which the three-dimensional relative position vector of the reference aircraft i is $\boldsymbol{r}_i=(x_i,y_i,z_i)$,taken a certain aircraft (aircraft 6 in the figure) in the assisted aircraft layer as the aircraft to be assisted, and the three-dimensional relative position coordinates of the aircraft to be assisted are $\boldsymbol{r}_0=(x,y,z)$,the cooperative navigation model is established by acquiring the distance information between the aircraft to be assisted and each reference aircraft. Finally, the position information of the aircraft to be assisted is calculated to correct the accuracy of the airborne navigation equipment. The distance difference between the auxiliary aircraft and the primary and secondary reference aircraft is β_i , the relative distance between the auxiliary vehicle and the primary reference vehicle is d_0 , the relative distance between the secondary reference vehicle and the primary reference vehicle is d_i , the relative distance data between the auxiliary vehicle and the sub reference vehicle is D_i , among which $i=2,3\cdots n$, n is the total number of reference aircraft of cluster aircraft system.

The longitude, latitude and altitude coordinates of each aircraft in the cluster system are transformed into the earth coordinate system （Earth-Centered, Earth Fixed, ECEF）， the conversion formula is shown below.

$$\begin{cases} x_i = \left(R_N + H_i\right)\cos L_i \cos\lambda_i \\ y_i = (R_N + H_i)\cos L_i \sin\lambda_i \\ z_i = R_N\left(1-f\right)^2 \sin L_i \end{cases} \tag{1}$$

Above which, R_N is the curvature radius of the earth's prime circle, f is the oblateness of the earth, λ_i is the longitude of the aircraft i ， L_i is the latitude of the aircraft i ， H_i is the height of the aircraft i . According to the above equation, the position coordinates of each aircraft in the three-dimensional earth coordinate system are calculated as x_i 、 y_i 、 z_i $\left(1 \le i \le n\right)$ ， n is the total number of vehicles in the cluster system. As shown in Figure 2.2, the relative coordinate system is established with the primary reference aircraft (aircraft 1) as the coordinate origin. The three-dimensional coordinate axis is parallel to the ECEF coordinate system, the Z axis is parallel to the earth rotation axis, the X axis intersects the zero meridian in the equatorial plane, and the Y axis forms the right-handed rectangular coordinate system with the X axis and Z axis. The coordinates of each aircraft in the relative coordinate system are as follows:

$$\begin{cases} x_i^r = x_i - x_1 \\ y_i^r = y_i - y_1 \qquad \left(i = 2,3\cdots n\right) \\ z_i^r = z_i - z_1 \end{cases} \tag{2}$$

Because the relative coordinate system is established with the primary reference aircraft as the coordinate origin, the relative coordinates of the primary reference aircraft in the relative coordinate system are as follows: $x_1^r = 0$ 、 $y_i^r = 0$ 、 $z_i^r = 0$.

According to the distance relationship between the aircraft to be assisted and the primary and secondary reference aircraft in the above model, it can be concluded as follows:

$$\overrightarrow{OS} = \overrightarrow{OP_i} + \overrightarrow{P_iS} \tag{3}$$

Among them, the point O is the primary reference aircraft, the point S is the auxiliary aircraft, and the point P_i is the secondary reference aircraft, the construction of distance difference data is as follows:

$$\beta_i = d_0 - D_i \tag{4}$$

According to the position coordinates of each aircraft in the relative coordinate system established above and the relative distance difference information between the aircraft to be assisted and each reference aircraft, the cooperative navigation model based on TDOA (time difference of arrival) principle is established as follows:

$$\begin{cases} \sqrt{\left(x_2^r - x\right)^2 + \left(y_2^r - y\right)^2 + \left(z_2^r - z\right)^2} - \sqrt{\left(x_1^r - x\right)^2 + \left(y_1^r - y\right)^2 + \left(z_1^r - z\right)^2} = \beta_2 \\ \sqrt{\left(x_3^r - x\right)^2 + \left(y_3^r - y\right)^2 + \left(z_3^r - z\right)^2} - \sqrt{\left(x_1^r - x\right)^2 + \left(y_1^r - y\right)^2 + \left(z_1^r - z\right)^2} = \beta_3 \\ \qquad\qquad\qquad\vdots \\ \sqrt{\left(x_i^r - x\right)^2 + \left(y_i^r - y\right)^2 + \left(z_i^r - z\right)^2} - \sqrt{\left(x_1^r - x\right)^2 + \left(y_1^r - y\right)^2 + \left(z_1^r - z\right)^2} = \beta_i \end{cases} \tag{5}$$

Among them, (x,y,z) is the position coordinates of the aircraft to be assisted in the relative coordinate system, (x_i^r, y_i^r, z_i^r) is the position coordinates of the reference aircraft in the relative coordinate system, and β_i is the distance difference data between the aircraft to be assisted and the primary and secondary reference aircraft. Convert the above formula (5) to a matrix representation as follows:

$$\begin{pmatrix} \beta_2^2 \\ \beta_3^2 \\ \vdots \\ \beta_i^2 \end{pmatrix} + 2 \begin{pmatrix} \beta_2 \\ \beta_3 \\ \vdots \\ \beta_i \end{pmatrix} * d_0 = -2 * \begin{pmatrix} X_{2,1} & Y_{2,1} & Z_{2,1} \\ X_{3,1} & Y_{3,1} & Z_{3,1} \\ \vdots & \vdots & \vdots \\ X_{i,1} & Y_{i,1} & Z_{i,1} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} K_2 - K_1 \\ K_3 - K_1 \\ \vdots \\ K_i - K_1 \end{pmatrix} \tag{6}$$

Among which, $X_{i,1} = x_i^r - x_1^r$, $Y_{i,1} = y_i^r - y_i^r$, $Z_{i,1} = z_i^r - z_i^r$; $K_i = (x_i^r)^2 + (y_i^r)^2 + (z_i^r)^2$, d_0 is the mutual distance information from the auxiliary aircraft to the primary reference aircraft, $i = 2,3 \cdots n$.

2.3 Cooperative Navigation Algorithm based on Mutual Distance Information

2.3.1Collaborative Localization Algorithm based on Chan-Taylor Joint Iteration

According to the characteristics of the above-mentioned cooperative navigation model, by analyzing the characteristics of the above-mentioned cooperative navigation model and the range error environment obeying Gaussian distribution, this section proposes a Chan- Taylor joint iterative algorithm for solving the cooperative navigation model. Chan algorithm uses non recursive method to solve TDOA hyperbolic equation. The algorithm uses weighted least square, (Weighted Least Square, WLS) to solve the positioning equations to obtain non iterative closed form solution. Taylor iterative algorithm needs to first estimate the position coordinates of the aircraft to be assisted in the relative coordinate system, then use the principle of square error to modify the pre designed position coordinates, and reduce the error of position coordinates estimation by iterative method. The algorithm is highly dependent on the selection of initial values. In this section, according to the characteristics of the two algorithms, a Chan Taylor based collaborative navigation model algorithm is proposed. The Chan algorithm solves the result of the collaborative navigation model through the double least squares algorithm, that is, the position coordinates of the aircraft to be assisted, and takes the solution result as the initial value of the Taylor iterative algorithm, and sets the corresponding iterative threshold for algorithm iteration, Finally, the position coordinates of the vehicle to be assisted in the relative coordinate system with high accuracy are obtained. The algorithm has high accuracy when the ranging error obeys Gaussian distribution.

Chan algorithm uses double weighted least square to make full use of redundant information. The initial solution of the algorithm is obtained by the first least square algorithm, and then the final solution is obtained by the second least square operation according to the initial solution and constraints of the first algorithm.

According to the above model formula, Let $\boldsymbol{z} = (x, y, z, R_1)$ be the unknown vector, where (x,y,z) are the position coordinates of the aircraft to be assisted in the relative coordinate system, and establish the error vector equation as follows:

$$\boldsymbol{\psi} = \boldsymbol{h} - \boldsymbol{G}\boldsymbol{z} \tag{7}$$

Among which,

$$\boldsymbol{h}=\frac{1}{2}\begin{pmatrix}\beta_2^2-K_2+K_1\\ \beta_3^2-K_3+K_1\\ \vdots\\ \beta_i^2-K_i+K_1\end{pmatrix}\qquad \boldsymbol{G}=\begin{pmatrix}X_{2,1} & Y_{2,1} & Z_{2,1} & \beta_2\\ X_{3,1} & Y_{3,1} & Z_{3,1} & \beta_3\\ \vdots & \vdots & \vdots & \vdots\\ X_{i,1} & Y_{i,1} & Z_{i,1} & \beta_i\end{pmatrix}$$

After the first WLS calculation of the error vector equation of the above equation (7), the estimated value of the position coordinates can be obtained as follows [6]:

$$\boldsymbol{z}=\arg\min\left\{\left(\boldsymbol{h}-\boldsymbol{G}\boldsymbol{z}\right)^{T}\boldsymbol{\psi}^{-1}\left(\boldsymbol{h}-\boldsymbol{G}\boldsymbol{z}\right)\right\}=\left(\boldsymbol{G}^{T}\boldsymbol{\psi}^{-1}\boldsymbol{G}\right)^{-1}\left(\boldsymbol{G}^{T}\boldsymbol{\psi}^{-1}\boldsymbol{h}\right)\tag{8}$$

Where, ψ is the covariance matrix of the error vector, and its calculation method is as follows:

$$\boldsymbol{\psi}=c\boldsymbol{BQB}\tag{9}$$

Among them, c is the radio transmission speed, $\boldsymbol{B}=diag\left\{D_2,D_3,\cdots D_n\right\}$, $\boldsymbol{Q}=\mathrm{diag}\left\{\sigma_2,\sigma_3,\cdots\sigma_n\right\}$, D_i is the relative distance between the sub reference aircraft and the aircraft to be assisted, and σ_i is the measurement error of distance difference data.

According to the first WLS calculation result $\boldsymbol{z}$ of the above formula (8) a new error vector is constructed as[7]

$$\boldsymbol{\psi}'=\boldsymbol{h}'-\boldsymbol{G}'\boldsymbol{z}'\tag{10}$$

Among which:

$$\boldsymbol{h}'=\begin{bmatrix}\left(\boldsymbol{z}(1)-x_1^r\right)^2\\ \left(\boldsymbol{z}(2)-y_1^r\right)^2\\ \left(\boldsymbol{z}(3)-z_1^r\right)^2\\ \boldsymbol{z}(4)^2\end{bmatrix}\quad \boldsymbol{G}'=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 1 & 1\end{bmatrix}\quad \boldsymbol{z}'=\begin{bmatrix}\left(x-x_1^r\right)^2\\ \left(y-y_1^r\right)^2\\ \left(z-z_1^r\right)^2\end{bmatrix}$$

$\boldsymbol{z}(1)$, $\boldsymbol{z}(2)$, $\boldsymbol{z}(3)$, $\boldsymbol{z}(4)$ is the first four values of the matrix $\boldsymbol{z}$.

The covariance matrix of the error vector ψ' is shown below [8]:

$$\boldsymbol{B}'=diag\left\{\left(\boldsymbol{z}(1)-x_1^r\right),\left(\boldsymbol{z}(2)-y_1^r\right),\left(\boldsymbol{z}(3)-z_1^r\right),\boldsymbol{z}(4)\right\}\tag{11}$$

$$\boldsymbol{\psi}'=4\boldsymbol{B}'\,\mathrm{cov}(\boldsymbol{z})\boldsymbol{B}'\tag{12}$$

The second WLS algorithm is used for estimation, and the solution equation is as follows:

$$\boldsymbol{z}'=\left(\boldsymbol{G}'^{T}\boldsymbol{\psi}'^{-1}\boldsymbol{G}'\right)^{-1}\left(\boldsymbol{G}'^{T}\boldsymbol{\psi}'^{-1}\boldsymbol{h}'\right)\tag{13}$$

Finally, the position coordinates of the vehicle to be assisted in the relative coordinate system are as follows[9]:

$$\boldsymbol{z}_p = \pm\sqrt{\boldsymbol{z'}} + \begin{pmatrix} x_1^r \\ y_1^r \\ z_1^r \end{pmatrix} \tag{14}$$

Among them, $\left(x_1^r, y_1^r, z_1^r\right)$ is the position coordinates of the main reference aircraft in relative coordinates. Finally, according to the distance between the calculation and the primary reference aircraft, the position coordinates with smaller distance are selected to obtain the initial value of the position $\left(x_v, y_v, z_v\right)$ of the auxiliary aircraft in the relative coordinate system by Chan algorithm.

According to the initial value solved by Chan algorithm, it is taken as the initial value of Taylor iterative algorithm, and the iterative threshold ε' of Taylor algorithm is set. The iterative process is repeated until the algorithm threshold is reached, and the position data of the aircraft to be assisted with high accuracy is obtained.

The Taylor iterative algorithm first establishes the distance difference function $f_i\left(x, y, z\right)$, $i = 2,3\cdots n$ of the aircraft, and the expression of the distance difference function of the aircraft is as follows:

$$f_i\left(x, y, z\right) = \hat{\beta}_i + \varepsilon_i \tag{15}$$

Among them, $\hat{\beta}_i$ is the distance difference data and ε_i is the distance difference data error. According to the position coordinates calculated by Chan algorithm, the position true value of the aircraft to be assisted is established as follows:

$$\begin{cases} x = x_v + \delta_x \\ y = y_v + \delta_y \\ z = z_v + \delta_z \end{cases} \tag{16}$$

Among them, $\left(x_v, y_v, z_v\right)$ is the initial value calculated by Chan algorithm, $\left(\delta_x, \delta_y, \delta_z\right)$ is the error of position truth value calculated by the algorithm, and $\left(x, y, z\right)$ is the position truth value of the aircraft to be assisted in the relative coordinate system.

Taylor expansion of the function f_i, and take the first two terms, as follows[10]:

$$f_{i,v} + \alpha_{i,1}\delta_x + \alpha_{i,2}\delta_y + \alpha_{i,3}\delta_z \approx D_i + \varepsilon_{i,1} \tag{17}$$

$$\begin{cases} f_{i,v} = f_i\left(x_v, y_v, z_v\right) \\ \alpha_{i,1} = \left.\dfrac{\partial f_i}{\partial x}\right|_{x_v, y_v, z_v} = \dfrac{x_1 - x_v}{d_0} - \dfrac{x_i - x_v}{d_0} \\ \alpha_{i,2} = \left.\dfrac{\partial f_i}{\partial y}\right|_{x_v, y_v, z_v} = \dfrac{x_1 - x_v}{d_0} - \dfrac{x_i - x_v}{D_i} \\ \alpha_{i,3} = \left.\dfrac{\partial f_i}{\partial z}\right|_{x_v, y_v, z_v} = \dfrac{y_1 - y_v}{d_0} - \dfrac{y_i - y_v}{D_i} \\ R_1 = \sqrt{\left(x_v - x_i^r\right)^2 + \left(y_v - y_i^r\right)^2 + \left(z_v - z_i^r\right)^2} \end{cases}$$

Among them, d_0 is the relative distance between the auxiliary aircraft and the primary reference aircraft, D_i is the relative distance between the auxiliary aircraft and the secondary reference aircraft.

Rewrite formula (17) into a matrix expression as follows:

$$\boldsymbol{A\delta} = \boldsymbol{D} + \boldsymbol{e} \tag{18}$$

Among which, $\boldsymbol{A}=\begin{pmatrix} \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} \\ \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} \\ \vdots & \vdots & \vdots \\ \alpha_{i,1} & \alpha_{i,2} & \alpha_{i,3} \end{pmatrix}$, $\boldsymbol{\delta}=\begin{pmatrix} \delta_x \\ \delta_y \\ \delta_z \end{pmatrix}$, $\boldsymbol{D}=\begin{pmatrix} \beta_2 - f_{2,v} \\ \beta_3 - f_{3,v} \\ \vdots \\ \beta_i - f_{i,v} \end{pmatrix}$, $\boldsymbol{e}=\begin{pmatrix} \varepsilon_{2,1} \\ \varepsilon_{3,1} \\ \vdots \\ \varepsilon_{i,1} \end{pmatrix}$

Use WLS to solve the formula (18) as follows:

$$\boldsymbol{\delta}=\left(\boldsymbol{A}^T\boldsymbol{Q}^{-1}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{Q}^{-1}\boldsymbol{D} \tag{19}$$

Where, $\boldsymbol{Q}$ is the covariance matrix of TDOA measurement error.

By constantly comparing the calculated $\boldsymbol{\delta}$ value with the set threshold ε' until it is lower than the threshold, the position coordinates of the aircraft to be assisted in the relative coordinate system are obtained as follows:

$$\begin{cases} x' = x_v + \delta_x \\ y' = y_v + \delta_y \\ z' = z_v + \delta_z \end{cases} \tag{20}$$

Among them, (x_v, y_v, z_v) is the initial value of the vehicle to be assisted calculated by Chan algorithm in relative coordinate system, $(\delta_x, \delta_y, \delta_z)$ is the position error of the vehicle to be assisted which jumps out of threshold threshold in the cyclic iteration, and (x', y', z') is the position coordinate of the target vehicle in the relative coordinate system obtained by the cyclic iteration.

2.3.2 Cooperative Localization Algorithm based on Spherical Interpolation

The above algorithm based on Chan-Taylor is mainly aimed at the Los environment, and the positioning accuracy is higher when the ranging error is mainly Gaussian noise. However, there are non line of sight (NLOS) errors in the measurement of mutual distance due to the position relationship of cluster aircraft. In the non line of sight (NLOS) error and other interference environment, the ranging error of each aircraft in the cluster aircraft system does not obey Gaussian distribution. In view of this situation, this section proposes a cooperative localization algorithm based on spherical interpolation algorithm, which can improve the accuracy of cooperative localization in non Gaussian error measurement environment to a certain extent.

The spherical interpolation algorithm only needs simple matrix operation to calculate the position of the aircraft to be assisted, so it needs less calculation, does not need a lot of iteration and search, and has high efficiency. In this algorithm, the distance equation error is introduced into the measurement equation to replace the traditional measurement error, and it is used as the optimization basis. The nonlinear constraint problem is transformed into a linear constraint problem. Finally, the double least square algorithm is used to solve the target position. This method can effectively improve the accuracy of target position solution in non Gaussian error environment.

The cooperative navigation model solved by spherical interpolation algorithm is based on TDOA principle. According to the position coordinates and relative relationship of each aircraft in Figure 2.2. The relative distance equation and distance difference data β_i between the auxiliary aircraft and the primary and secondary reference aircraft can be obtained.

The relative distance D_i between the aircraft to be assisted and the sub reference aircraft is calculated by the aircraft position coordinates in the relative coordinate system, as follows:

$$D_i = \left\| \boldsymbol{r}_0 - \boldsymbol{r}_i \right\| \tag{21}$$

The expansion of equation(21) is as follows:

$$D_i^2 = \left\| \boldsymbol{r}_0 \right\|^2 - 2\left\langle \boldsymbol{r}_0, \boldsymbol{r}_i \right\rangle + \left\| \boldsymbol{r}_i \right\|^2 = d_0^2 - 2\boldsymbol{r}_i^T \boldsymbol{r}_0 + d_i^2 \tag{22}$$

Substitute formula(4) into formula(22) and expand:

$$d_i^2 - \beta_i^2 - 2d_0\beta_i - 2\boldsymbol{r}_i^T \boldsymbol{r}_0 = 0 \quad i = 2,3\cdots n \tag{23}$$

Considering the actual measurement environment, each distance data has measurement error, and the above equation(23) has error. The spherical interpolation algorithm replaces the measurement error by introducing the error ε_i of distance difference equation, as follows:

$$\varepsilon_i = d_i^2 - \beta_i^2 - 2d_0\beta_i - 2\boldsymbol{r}_i^T \boldsymbol{r}_0 \tag{24}$$

The above formula (24) is transformed into a matrix expression as follows:

$$\boldsymbol{\varepsilon} = \boldsymbol{\alpha} - 2d_0\boldsymbol{\beta} - 2\boldsymbol{S}\boldsymbol{r}_0 \tag{25}$$

Among which:

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ \varepsilon_n \end{bmatrix} 、 \boldsymbol{\alpha} = \begin{bmatrix} d_2^2 - \beta_2^2 \\ d_3^2 - \beta_3^2 \\ \vdots \\ d_n^2 - \beta_n^2 \end{bmatrix} 、 \boldsymbol{\beta} = \begin{bmatrix} \beta_2 \\ \beta_3 \\ \vdots \\ \beta_n \end{bmatrix} 、 \boldsymbol{S} = \begin{bmatrix} \boldsymbol{r}_2^T \\ \boldsymbol{r}_3^T \\ \vdots \\ \boldsymbol{r}_n^T \end{bmatrix}$$

In the above matrix, the position $\boldsymbol{r}_0$ coordinates of the relative coordinate system of the auxiliary aircraft need to be solved. We can set the parameters d_0 known, and calculate the position coordinates of the relative coordinate system of the auxiliary aircraft by the WLS, as follows:

$$\hat{\boldsymbol{r}}_0 = \arg\min\{\boldsymbol{\varepsilon}\} = \frac{1}{2}\left(\boldsymbol{S}^T\boldsymbol{S}\right)^{-1}\boldsymbol{S}^T\left(\boldsymbol{\alpha} - 2d_0\boldsymbol{\beta}\right) \tag{26}$$

Through verification, the above results meet the nonlinear constraints

$$\hat{\boldsymbol{r}}_0^T \hat{\boldsymbol{r}}_0 = d_0^2 \tag{27}$$

The above results are substituted into equation (25) as follows:

$$\boldsymbol{\varepsilon}' = \boldsymbol{\alpha} - 2d_0\boldsymbol{\beta} - \boldsymbol{S}\boldsymbol{S}^T\left(\boldsymbol{S}^T\boldsymbol{S}\right)^{-1}\left(\boldsymbol{\alpha} - 2d_0\boldsymbol{\beta}\right) \tag{28}$$

WLS is used to solve the formula (28), as follows:

$$\hat{d}_0 = \arg\min\{\boldsymbol{\varepsilon}'\} = \frac{1}{2}\frac{\boldsymbol{\beta}^T\boldsymbol{E}^T\boldsymbol{E}\boldsymbol{\alpha}}{\boldsymbol{\beta}^T\boldsymbol{E}^T\boldsymbol{E}\boldsymbol{\beta}} \tag{29}$$

Substituting the calculated data $\hat{d}_0$ into equation (26), the three-dimensional position coordinates of the final aircraft to be assisted can be obtained as follows:

$$\hat{\boldsymbol{r}}_0 = \frac{1}{2}\left(\boldsymbol{S}^T\boldsymbol{S}\right)^{-1}\boldsymbol{S}^T\left(\boldsymbol{\alpha} - 2\boldsymbol{\beta}\frac{\boldsymbol{\beta}^T\boldsymbol{E}^T\boldsymbol{E}\boldsymbol{\alpha}}{\boldsymbol{\beta}^T\boldsymbol{E}^T\boldsymbol{E}\boldsymbol{\beta}}\right) \tag{30}$$

Among which, $\boldsymbol{E} = \boldsymbol{I} - \boldsymbol{S}\left(\boldsymbol{S}^T\boldsymbol{S}\right)^{-1}\boldsymbol{S}^T$ 。

$$\begin{cases} x' = \hat{\boldsymbol{r}}_0(1) \\ y' = \hat{\boldsymbol{r}}_0(2) \\ z' = \hat{\boldsymbol{r}}_0(3) \end{cases} \tag{31}$$

The position coordinates (x', y', z') of the aircraft to be assisted in the relative coordinate system are obtained by the above algorithm。

3 Experimental Research of Cooperative Navigation

3.1 Experimental Research Scheme

In order to verify the validity and reliability of the collaborative navigation algorithm proposed in this paper, a hardware simulation platform for collaborative navigation algorithm is established. The simulation and verification platform of the cluster flight cooperative navigation algorithm is mainly composed of STM32 module, UWB module, GNSS module and digital transmission module. By using the UWB module to measure the distance data between the aircraft; using the low-cost GNSS module to simulate the navigation information of the aircraft on-board; using the data transmission module to simultaneously transmit the data of multiple aircraft nodes to the terminal computer, all the above modules are connected to form an aircraft module node on the STM32 microcontroller module.

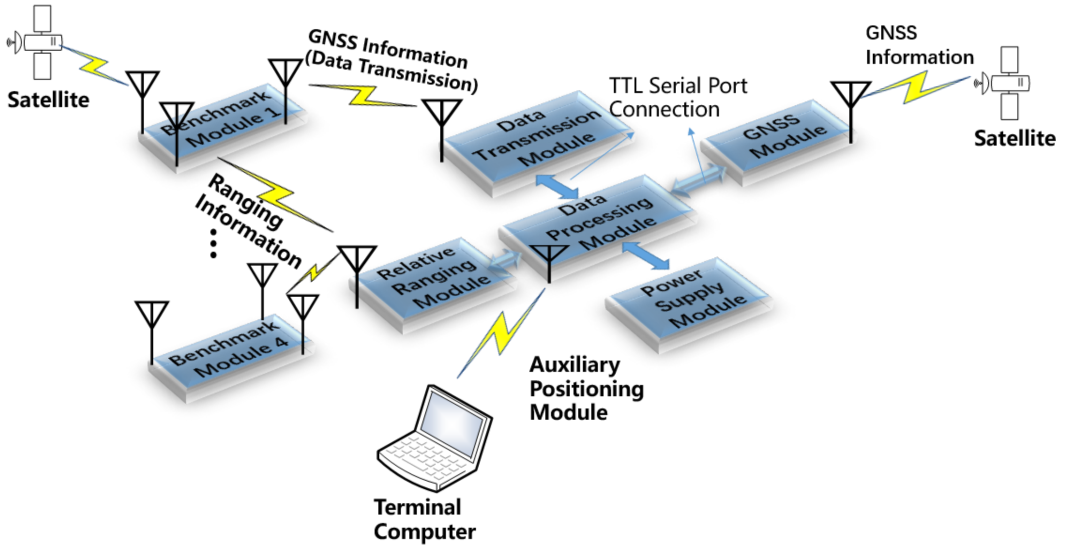


Figure3 Overall hardware module configuration diagram of cooperative navigation

3.2 Performance of Cooperative Navigation Simulation

In the actual verification process, following the principle of static first dynamic, based on a small number of nodes, and setting the aircraft node to be located to be static, the performance and error of the designed algorithm are analyzed by collecting data. Then, the target node to be located is placed on the quadrotor, and the aircraft node to be positioned is dynamically driven by manipulating the quadrotor, and at the same time, the sensor data transmitted during the flight is received by the terminal computer to analyze and verify the collaborative navigation algorithm designed in this paper. Finally, the reference aircraft module node is also placed on the quadrotor aircraft, and the reference aircraft module node and the aircraft module node to be located are set to simultaneously fly in the cluster, and the sensor data generated during the above flight process is collected to verify and analyze the data to evaluate algorithm performance.

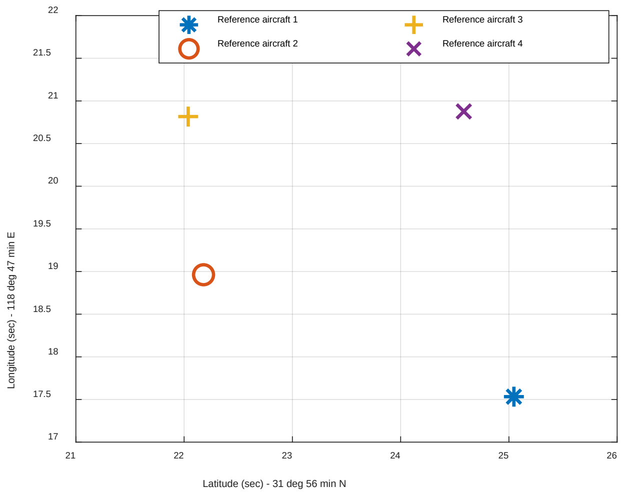


Figure 4-Initial position of reference aircraft

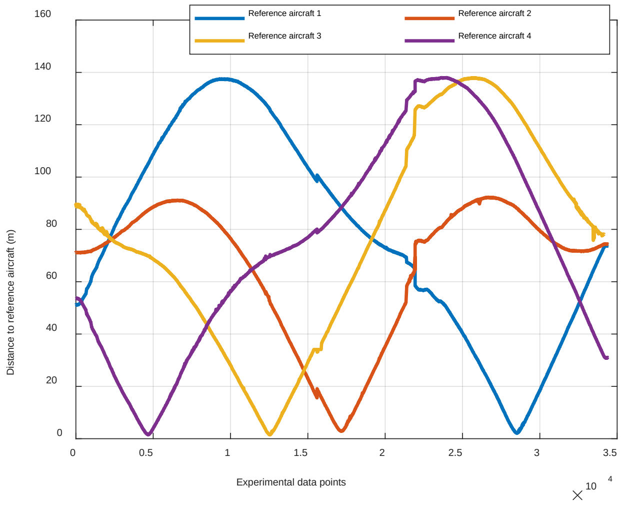
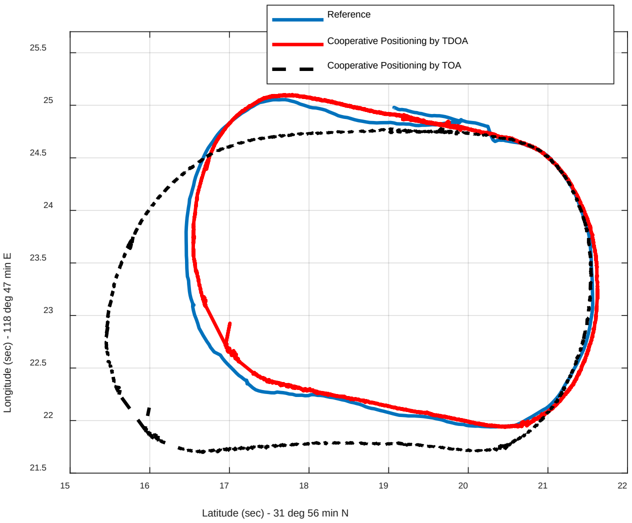


Figure 5-Relative distance between aircraft to be located and reference aircraft



Fugure6 Cooperative navigation algorithm and standard mobile path

From the above results, because the reference aircraft module and the aircraft module to be located move at the same time, some abnormal data points appear in the process of relative distance data acquisition, resulting in some abnormal track points in the track solution curve and irregular solution track; At the same time, it also increases the relative distance measurement error, which leads to the increase of the solution error of the cooperative navigation algorithm.

In order to intuitively reflect the error of the collaborative navigation algorithm, the abscissa and ordinate position in this paper is accurate to the unit of seconds. From the coincidence results of the above calculated track and the standard GNSS track, the position error of the calculated track and the standard track is less than 0.5 seconds; Therefore, the cooperative positioning algorithm based on the relative distance information designed in this paper also has good accuracy in the environment of simultaneous movement of the reference aircraft and the aircraft to be located, and the accuracy can meet the navigation and positioning requirements of the low accuracy aircraft to be located in the aircraft cluster environment. The simulation results further verify the accuracy, effectiveness and reliability of the proposed algorithm in the field test environment.

To sum up, the cooperative navigation algorithm of cluster aircraft based on relative information designed in this paper is verified in two cases: the relative position of the reference aircraft module is fixed when the aircraft module to be located is moving, and the relative position of the reference aircraft module is fixed, and the reference aircraft module and the aircraft module to be located are moving at the same time. From the analysis results of the measured data, the collaborative positioning algorithm based on the relative distance information studied in this paper has high accuracy, which can meet the requirements of aircraft navigation and positioning with low accuracy in the actual cluster flight, and effectively verify the effectiveness and practicability of the algorithm designed in this paper in engineering.

The software simulation and the above-mentioned aircraft cluster flight test results show that the cluster aircraft cooperative navigation method based on the distance difference model designed in this paper can effectively utilize the sensor data in the cluster flight system to improve the overall cluster system navigation accuracy.

4 Conclusions

In this paper, a platform for cooperative navigation of cluster aircraft based on relative measurement information is built, which is mainly composed of a hardware simulation system for cooperative navigation based on relative distance information and a software simulation system for cooperative navigation based on relative measurement information. Firstly, the software modules of sensor parameter setting, cluster aircraft integrated display, navigation algorithm calculation error display and aircraft flight status display are designed and developed in the software simulation system. Through the simulation of relative measurement data, the verification of collaborative navigation algorithm based on relative measurement information is effectively supported. On this basis, this paper builds a hardware test and verification system composed of relative distance module, independent module, data transmission module and data processing module, and verifies the effectiveness and reliability of the cooperative navigation algorithm based on relative distance information through measured data acquisition and processing.

5 Acknowledgements

This work was partially supported by the National Natural Science Foundation of China (Grant No. 62073163, 61703208, 61873125), the Foundation Research Project of Jiangsu Province (The Natural Science Foundation of Jiangsu Province, Grant No. BK20170815, BK20170767, BK20181291), the Fundamental Research Funds for the Central Universities(Grant No. NZ2020004, NZ2019007), the 111 Project(B20007), Supported by Shanghai Aerospace Science and Technology Innovation Fund(SAST2020-073, SAST2019-085), the National Key Research and Development Program of China (Grant No. 2019YFA0706003), Introduction plan of high end experts(G20200010142), the Science and Technology Innovation Project for the Selected Returned Overseas Chinese Scholars in Nanjing, the Foundation of Key Laboratory of Navigation, Guidance and Health-Management Technologies of Advanced Aircraft (Nanjing University of Aeronautics and Astronautics)， Ministry of Industry and Information Technology, the Foundation of Engineering Research Center of Autonomous Control Technology of Aircraft (Nanjing University of Aeronautics and Astronautics), Ministry of Education, Jiangsu Key Laboratory “Internet of Things and Control Technologies”& the Priority Academic Program Development of Jiangsu Higher Education Institutions.

6 Contact Author Email Address

rongwang@nuaa.edu.cn， dujunnan@nuaa.edu.cn

7 Copyright Statement

References

[1] Qu Y, Zhang Y. Cooperative localization against GPS signal loss in multiple UAVs flight[J]. *Journal of Systems Engineering and Electronics*, 2011,22(1):103–112.

[2] Zhou D. Zhang H, Pan Q, et al. An improved range parameterized square root cubature information filter algorithm for multi-UAV cooperative passive location[C]. IEEE International Conference on Information and Automation IEEE. 2015:1079-1084.

[3] Liu Y, Zhu Y. A collaborative integrity monitor algorithm for low space aviation under

limited number of navigation satellites[C]. 2013 International Conference on Connected Vehicles and Expo (ICCVE), Las Vegas, USA, Dec 2-6, 2013, pp. 910-915.

[4] Tang. S, Qin Z and Xin J. Collaborative Task Assignment Scheme for Multi-UAV Based on Cluster Structure, *2010 Second International Conference on Intelligent Human-Machine Systems and Cybernetics*, Nanjing, China, 2010, pp. 285-289.

[5] Wang R, Du J, Xiong Z, Chen X, Liu J. Hierarchical Collaborative Navigation Method for UAV Swarm. *Journal of Aerospace Engineering*, 2021, 34(1): 04020097

[6] Li A, Luan F, An improved localization algorithm based on chan with high positioning accuracy in NLOS-WGN environment[C]. *2018 10th International Conference on Intelligent Human-Machine Systems and Cybernetics*, 2018:332-335.

[7] Zhang J, Yu C, Tang B, et al. Chan Location Algorithm Application in 3-Dimension Space Location[C]. *2008 ISECS International Colloquium on Computing, Communication, Control, and Management*, 2008:622~626.

[8] Canclini A, Antonacci F, Sarti A, et,al. Acoustic source localization with distributed asynchronous microphone networks[J]. *IEEE Transactions on Audio, Speech and Language Processing*, IEEE 2013, 21(2):439~443.

[9] Chang X, Sheng Y, Jiang Y, et al. Three-dimensional positioning of wireless communication base station[C]. *2017 IEEE 2nd Advanced Information Technology, Electronic and Automation Control Conference (IAEAC)*. IEEE 2017:2727-2732.

[10] Fang S, Lin T. Indoor location system basedon discriminant-adaptive neural network in IEEE 802.11 environments[J] *IEEE Transactions on Neural Nerworks* 2008.19(11):1973-1978.