ICAS 2020 SHANGHAI
International Council of the Aeronautical Sciences
32nd Congress of the International Council of the Aeronautical Sciences
September 6-10, 2021
Pudong Shangri-La, Shanghai, China

RESEARCH OF LABYRINTH SEALS MEASUREMENT-BASED PROBABILITISTIC ANALYSIS METHOD FOR TURBOFAN INTERNAL AIR SYSTEM

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Abstract

Random uncertainty factors widely existing in the manufactory and operation of turbofan may lead to uncertainty on labyrinth seals clearance, and then affects the function of internal air system. In order to quantitatively analyze and promote the robustness and reliability of internal air system, and to figure out the determining labyrinth seals, this article set up a labyrinth seals measurement-based probabilistic analysis method for internal air system. The case study shows that, based on measured data of labyrinth seals clearance, this method could effectively evaluate the robustness and reliability of an internal air system by means of quantitatively analyzes its function parameters' probability distribution and failure probability. This method could also recognize the key labyrinth seals, so as to support the optimization in terms of robustness and reliability of internal air system.

Keywords: probabilistic analysis, internal air system, labyrinth seals clearance

1. Introduction

Labyrinth seal is one of the most important effective seals on turbofan. Random uncertainty factors, such as machining and assembling error, relative displacement between rotor and stator, deformation and abrasion of the labyrinth seals， exist widely in machining, assembling and operation of turbofan [1]， and may lead to uncertainty on labyrinth seals clearance. Internal air system plays the role of turbofan components and accessories cooling, bearing chamber sealing, control of bearing axial loads and so on. The uncertainty of labyrinth seals clearance would distract the function of internal air system from the designed values, making the parameters related to internal air system function present as a probability distribution. This may affect the operation security and service life of turbofan.

Previous researches usually establish theoretical mathematical model for the uncertainty of labyrinth seals clearance to carry out probabilistic analysis for internal air system, as in theses [2]-[3]. In this paper, a labyrinth seals measurement-based probabilistic analysis method is built, on purpose of probabilistic analysis (i.e., the determination of the robustness and reliability of an internal air system in consideration of the uncertainty of labyrinth seals clearances) and sensitivity analysis (i.e., the determination of how the uncertainty of labyrinth seals clearances contributes to the variation of parameters of an internal air system's functions) .The effect of this method is exhibited by an application.

2. Methodology

The labyrinth seals measurement-based probabilistic analysis method, as illustrated in Figure 1, is built on the basis of Monte Carlo simulation method [4] through literature research and engineering practice.

In the part of uncertainty analysis and discretization of labyrinth seals clearances, the labyrinth seals to be studied are determined and their characteristics of uncertainty are analyzed based on measured data. Then the labyrinth seals clearances are discretized to execute the probabilistic analysis of internal air system. After the conversion of clearances samples from cold to work condition and simulations of internal air system, the probability distribution of concerned parameter to internal air system function can be defined and the key labyrinth seals whose clearance variation affect the internal air system remarkably can be recognized

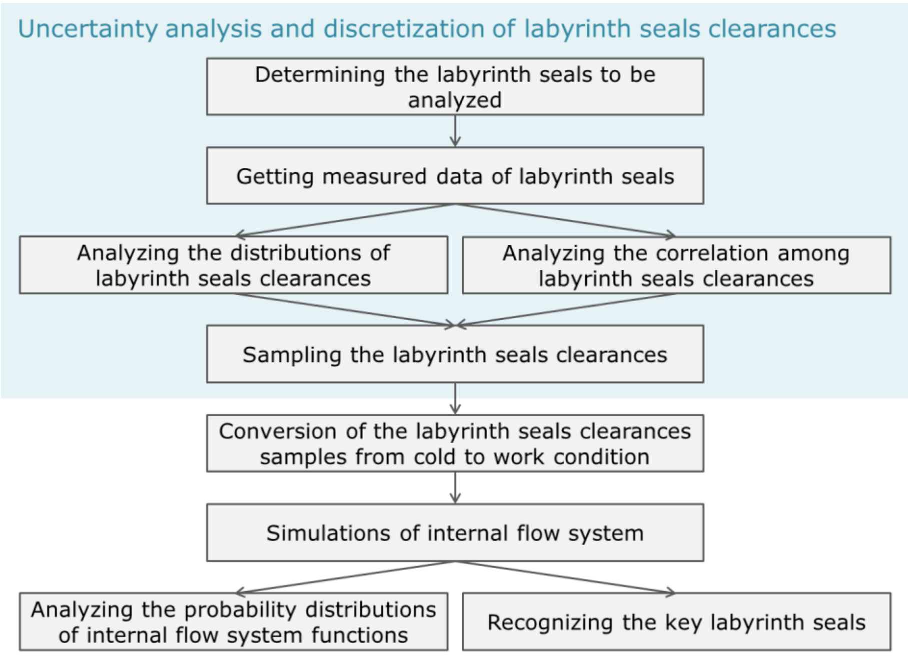

Figure 1 – Workflow of labyrinth seals measurement-based probabilistic analysis method.

2.1 Analysis of the labyrinth seals clearances distribution

The analysis of the labyrinth seals clearances distribution is a hypothesis test problem. The distribution of labyrinth seals clearances measured data can be determined by Kolmogorov-Smirnov test or kernel density estimation. The procedure of distribution analysis is as follows:

- Obtain the empirical distribution function $F_n(x)$ of the measured data

$$F_n(x)=\begin{cases}0 & x<x_{(1)}\\ \dfrac{n_1+n_2+\cdots+n_k}{n} & x_{(k)}\le x<x_{(k+1)};k=1,2,\cdots,r-1\\ 1 & x\ge x_{(r)}\end{cases} \quad (1)$$

  where n_k is the frequency of data $x_{(k)}$.

- Obtain the cumulative distribution function $F(x)$ by fitting the distribution parameters based on measured data according to common distribution form, such as normal distribution, log normal distribution, Poisson's distribution, Weibull distribution, gamma distribution and so on.

- Construct KS statistics for every contribution form

$$KS=\sup_{-\infty<x<\infty}(|F_n(x)-F(x)|)\,. \quad (2)$$

- Calculate the $KS(\alpha)$ by checking against the table at confidence level of α.

- If $KS<KS(\alpha)$ for more than one distribution forms, select the distribution whose KS is the biggest. If $KS\ge KS(\alpha)$ for all the distribution forms, the estimated kernel density function can be an alternative.

2.2 Analysis of the correlation among labyrinth seals clearances

The correlation among a pair of labyrinth seals clearances X and Y can be analyzed by Pearson correlation coefficient, which is defined as

$$r = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E^2(X)}\sqrt{E(Y^2) - E^2(Y)}} \ . \tag{3}$$

Correlation coefficient varies from -1 to 1, which means the correlation of a pair of labyrinth seals clearances varies from highly negative correlation to highly positive correlation. When the correlation coefficient approach to 0, the clearances of two labyrinth seals tend to uncorrelated. All the correlation coefficients compose correlation matrix.

2.3 Sampling of labyrinth seals clearances

Within the probability distribution and correlation matrix, the labyrinth seals clearances can be discretized. Latin Hypercube Sampling (LHS) method [5] has the advantage of both homogeneity and randomness, and could obtain dependent sample rapidly [6]. Probabilistic analysis method use LHS method to carry out labyrinth seals clearances sampling.

The brief procedure of producing a LHS sample of size N is as follows [6]: Suppose the joint distribution of the labyrinth seals clearances $\mathbf{X}$ to be given by F, and define $P = (p_{ij})$ to be an $N \times D$ matrix, where D is defined by the number of labyrinth seals , and each column of P is an independent random permutation of {1,2,…, N }. Then let $\xi_{ij}(i = 1, \cdots, N; J = 1, \cdots, D)$ be ND iid $U[0,1]$ random variables independent of P. The ij-th LHS sample is defined by

$$X_{ij} = F_j^{-1}[(P_{ij} - 1 + \xi_{ij})/N], (i = 1, \cdots, N; J = 1, \cdots, D) \tag{4}$$

LHS method cannot obtain sample which is neither normal distribution nor uniform distribution. If the distribution of labyrinth seals clearance is not the above two, the LHS method should be used to obtain dependent normal sample, and then orthogonal transformed to certain distribution. Orthogonal transformation may import error to correlation matrix of LHS sample, and the error is positively associated with the variable coefficient ($\delta = \sigma / \mu$) [7]. According to existing measured data, the variable coefficient of labyrinth seals clearance usually stays below 0.15, which stands for a weak degree of variation. So the correlation matrix error related to orthogonal transformation is usually acceptable.

2.4 Analysis of the probability distributions of internal air system functions

After the conversion of clearances samples and N times simulations of internal air system, the robustness and reliability of the internal air system can be assessed by the standard deviation σ_y and failure probability P_f of function parameters, respectively.

$$\begin{aligned} \mu_y &= \frac{1}{N}\sum_{I=1}^{N} y_i \\ \sigma_y &= \sqrt{\frac{1}{N-1}\sum_{I=1}^{N}(y_i - \mu_y)^2} \\ P_f &= \frac{1}{N}\sum_{I=1}^{N} I_i \begin{cases} I_i = 1, & y_i \le 0 \\ I_i = 0, & y_i > 0 \end{cases} \end{aligned} \tag{5}$$

Furthermore, the probability density function $f(y)$ and cumulative distribution function $F(y)$ of any function parameter $y_i(i = 1, 2, \cdots, N)$ of internal air system can be obtained by hypothesis test described above. Based on this, the probability of any function parameter falls in any interval of can be easily got.

2.5 Recognition of the key labyrinth seals

The rank of Sobol's indices [8] (also called variance-based sensitivity indices) is employed to recognize the key labyrinth seals whose clearance variation affect the internal air system remarkably.

The definition of Sobol's indices is related to the High-dimensional model representation (HDMR) of system response function. According to HDMR, system response function can be expanded as [9]

$$Y = g_0 + \sum_{i=1}^{n} g_i(X_i) + \sum_{i \neq J} g_{ij}(X_i, X_j) + \cdots + g_{12\cdots n}(X_i, X_j, \cdots, X_n) \quad (6)$$

The Sobol's main indices is defined as

$$S_i = \frac{V_i}{Var(Y)} = \frac{Var[E(Y \mid X_i)]}{Var(Y)}, \quad (7)$$

which means the reduction of output variance by fasten X_i; The Sobol's total indices is defined as

$$S_i^T = \frac{V_i^T}{Var(Y)} = 1 - \frac{Var[E(Y \mid X_{\sim i})]}{Var(Y)} = \frac{E[Var(Y \mid X_{\sim i})]}{Var(Y)}, \quad (8)$$

Where $X_{\sim i}$ means all the input variables but X_i. S_i^T means the remnant output variance when $X_{\sim i}$ is fastened.

The above equations of Sobol's indices are suitable for independent input variables. As to dependent input variables, the modified Sobol's indices method mentioned in [10] is practicable.

3. The deterministic model of internal air system

The internal air system of turbofan usually consist of turbine blade cooling system, turbine components cooling and gas sealing system, turbine blade tip clearance control system, compressor components temperature control system, bearing chamber cooling and sealing system, bearing axial loads control system and so on. The flow path of these branch systems connect with one other in series and parallel. The boundary nodes represent position where the energy exchange between internal air system and main & external duct stream takes place. The internal nodes represent the cavity of internal air system. The throttle unit stands for the structure in the internal air system, such as orifice, prewhirler, labyrinth seals, impact flow, forced flow and so on.

Nodes and units form the unified one – dimensional computing network of internal air system [11], as illustrated in Figure 2.

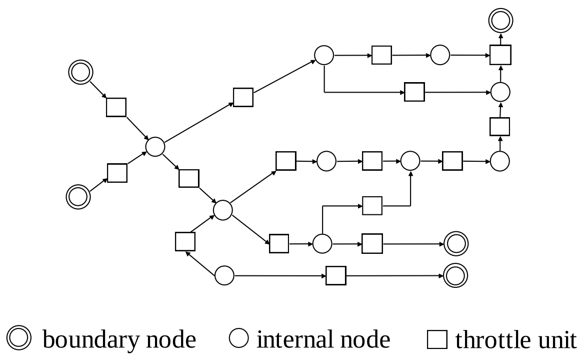

Figure 2 – Sketch map of one-dimensional computing network of internal air system.

Every internal node should satisfy the three laws of conservation on steady state

$$\begin{gathered} \sum G_j = 0 \\ P_1^* - P_2^* = K_1 \frac{\rho_2 V_2^2}{2} \\ \sum G_i (H(T_{i1}) - H(T_{i2})) + Q_i = 0 \end{gathered} \quad (9)$$

The approach to solving the internal air system is to solve these equations in all the nodes simultaneously when the structure parameters, network topotaxy, boundary aerodynamic and thermal parameters of the internal air system are known conditions. By doing this, the flow distribution on all the internal flow stream and the pressure and temperature on all the cavities are determined.

4. Application

The number of function parameters of an internal air system is no less than 10. Here we take bearing axial load as an example to exhibit the effect of the measurement-based probabilistic analysis method.

For a certain turbofan, there are 10 labyrinth seals in the internal air system denoted by *a-j*. As illustrated in Figure 3, the Kolmogorov-Smirnov test shows that the clearance measured data are normal distribution for all of these labyrinth seals. And the absolute value all the coefficients in Pearson correlation matrix (Table 1) are below 0.4, which means the input variables (i.e., labyrinth seals clearances) tend to be independent.

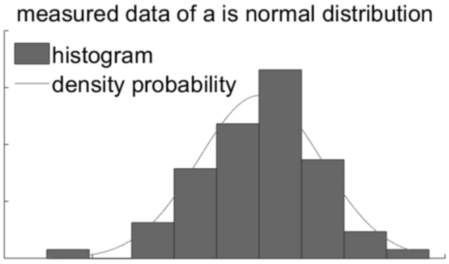

Figure 3 – The Kolmogorov-Smirnov tested distribution of clearance measured data for one of the ten labyrinth seals.

Table 1 – The Pearson correlation matrix of clearance measured data of labyrinth seals.

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*
a	1.00	0.02	-0.12	0.03	-0.08	-0.20	0.01	-0.09	-0.03	0.00
b	0.02	1.00	0.20	0.31	0.14	0.07	0.36	0.14	0.29	0.24
c	-0.12	0.20	1.00	0.35	0.00	0.09	0.26	-0.23	0.13	0.00
d	0.03	0.31	0.35	1.00	-0.06	-0.11	0.14	-0.07	0.24	0.00
e	-0.08	0.14	0.00	-0.06	1.00	0.27	0.18	0.02	0.08	0.15
f	-0.20	0.07	0.09	-0.11	0.27	1.00	-0.02	-0.18	-0.08	-0.05
g	0.01	0.36	0.26	0.14	0.18	-0.02	1.00	0.19	0.18	0.15
h	-0.09	0.14	-0.23	-0.07	0.02	-0.18	0.19	1.00	0.30	0.08
i	-0.03	0.29	0.13	0.24	0.08	-0.08	0.18	0.30	1.00	0.02
j	0.00	0.24	0.00	0.00	0.15	-0.05	0.15	0.08	0.02	1.00

After LHS sampling with size of 1000 and corresponding solving of the internal air system, the 1000 bearing axial loads are obtained. Its standard deviation σ_y is 0.138 (non-dimensional disposed); its failure probability P_f is 0.86%. By hypothesis test, the distribution of bearing axial load is obtained as gamma distribution. Based on this, the probability of bearing axial load falls in the interval illustrated in Figure 4 by rectangular box is calculated as 88.69%.

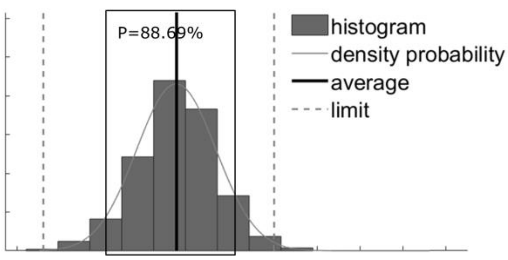

Figure 4 –The distribution of the bearing axial load.

The Sobol's main indices and total indices are ranked in Figure 5 separately. It is obvious that the key labyrinth seals to the bearing axial load are *b* and *e*.

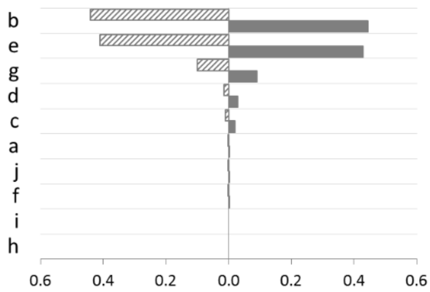

Figure 5 –The rank of Sobol's indices.

5. Conclusion & Outlook

In this paper, a labyrinth seals measurement-based probabilistic analysis method is built on the basis of hypothetic test, correlation analysis, LHS sampling, Sobol's indices and so on. Application exhibits the ability of this method to 1) assess the robustness and reliability of an internal air system and 2) recognize the key labyrinth seals.

Limited by the calculated amount of solving internal air system, the LHS sample size is constrained, which results in insufficient accuracy of probabilistic analysis. The introducing of surrogate model into probabilistic analysis method would be the nest research direction.

References

[1] Paniagua G, Denos R and Almeida S. Effect of the hub endwall cavity flow on the flow-field of a transonic high-pressure turbine. *Journal of Turbomachinery*, Vol. 126, No. 4, pp 578-586, 2004.

[2] Brach S and Muller Y. Probabilistic analysis of the secondary air system of a low-pressure turbine. *Transactions of the ASME*, Vol. 137, 022602, 2015.

[3] Cloud D and Stearns E. Probabilistic analysis of a turbofan secondary flow system. *Proceedings of ASME Turbo Expo 2004*, Vienna, Austria, Power for land, sea and air, GT2004-53197, 2004.

[4] Marala A. Sample size requirement for Monte Carlo simulations using Latin hypercube sampling. *Independent Research Projects in Applied Mathematics*, pp 1-24, 2008.

[5] McKay M D, Bechman R J and Conover W J. A comparison of three methods for selection values of input variables in the analysis of output from a computer code. *Technometrics*, Vol. 21, No. 2, pp 239-245, 1979.

[6] Stein M. Large sample properties of simulations using Latin hypercube sampling. *Technometrics*, Vol. 29, No. 2, pp 143-151, 1987.

[7] Wu S, Zhang K and Li D. The influence of correlation transformation on reliability when the correlation non-normal variables are transformed. *Engineering Journal of Wuhan University*, Vol. 44, No. 2, pp 151-155, 2011.

[8] Sobol I M. Sensitivity analysis for non-linear mathematical models. *Mathematical Modelling and Computational Experiment*, No. 1, pp 407-414, 1993.

[9] Sobol I M. Theorems and examples on high dimensional model representation. *Reliability Engineering and System Safety*, Vol. 79, No. 2, pp 187-193, 2003.

[10] Xiao D, Ferlauto M, Song L and Li J. Sensitivity analysis for model with dependent inputs using sparse polynomial chaos expansions. *14th World Congress in Computational Mechanics (WCCM) ECCOMAS Congress*, 2020.

[11] Lu H, Yang Y, Wang M. Numerical simulation of aero-engine internal air system character. *Aeroengine*, Vol. 23, No. 1, pp 6-13, 1997.

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