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MODELING AND DYNAMICS STABILITY ANALYSIS FOR A BLENDED-WING-BODY AIRCRAFT

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Abstract

Dynamic characteristics of a Blended-Wing-Body (BWB) aircraft are analyzed in this paper. Based on the aerodynamic and geometry characteristics of BWB aircraft, the small disturbance assumptions are determined, and the Full Order Equations of Motion (FOEOM) with longitudinal-lateral-directional coupling are derived. The conventional EOM and FOEOM of a BWB aircraft are solved and analyzed. The results indicate that FOEOM can reflect the longitudinal-lateral-directional coupling characteristics of BWB aircraft, which are reflected in eigenvectors.

Keywords: Blended-Wing-Body, full order equations of motion, longitudinal-lateral-directional coupling, mode characteristic

1. Introduction

In order to achieve higher stealth, aerodynamic, and structure performances, Innovative Configuration Aircraft, represented by Blended-Wing-Body (BWB) aircraft, as shown in Figure 1, have been increasingly utilized [1][2]. Compared with the conventional aircraft, BWB aircraft has no vertical tail, and the longitudinal and directional static stability are insufficient or even unstable.

Figure 1 – Typical BWB aircraft.

Innovative control effectors (ICEs), such as split drag rudder, spoiler slot deflectors, all moving tips, etc., are widely utilized on BWB aircraft[3][4][5][6]. The rolling, pitching and yawing moments are generated by changing the local lift and drag, as shown in Figure 2.
Since there is no coventional horizontal and vertical tail, the moment arms of ICEs are much shorter than conventional elevator and rudder. In order to generate sufficient control moments, ICEs are large in size and quantity, resulting in strong couping between aerodynamic forces and moments. Besides, the inertia mass distribution of BWB aircraft is different from that of conventional aircraft [7], and the influence of inertia product can no longer be ignored. These differences make the mode characteristics of BWB aircraft significantly different from that of conventional aircraft.
The mode characteristics of aircraft are the basis of flying quality evaluation and flight control law design. At present, the dynamic stability analysis of BWB aircraft is still based on the linear small

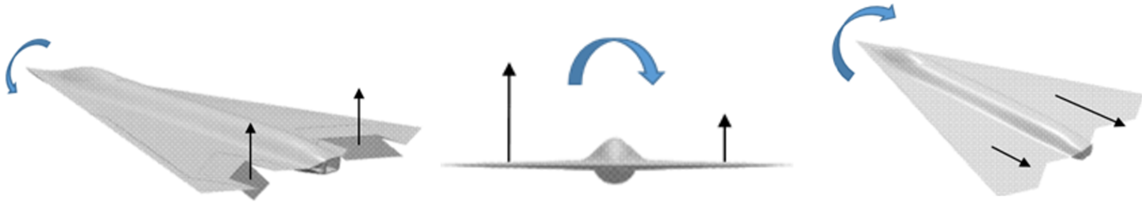

Figure 2 – Mechanism of innovative control effectors (ICEs).

disturbance EOM with longitudinal and lateral/directional decoupled [8][9][10], which can not fully reflect the dynamic characteristics of BWB aircraft.
In order to solve this problem, based on the aerodynamic and geometry characteristics of BWB aircraft, this paper analyzes the small disturbance assumptions, derives the FOEOM with longitudinal-lateral-directional coupling, and analyzes the dynamic characteristics of BWB aircraft.

2. Equation of Motion

Assuming that the aircraft is a rigid body with 6 DOF, and neglecting the Earth Curvature and Rotation, the EOM are as follows[11]:

$$\begin{cases} F_x = m(\dot{u}+qw-rv) \\ F_y = m(\dot{v}+ru-pw) \\ F_z = m(\dot{w}+pv-qu) \end{cases} \tag{1}$$

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (I_z-I_y)qr+(I_{xz}q-I_{xy}r)p+I_{yz}(q^2-r^2)+L \\ (I_z-I_x)pr+(I_{xy}r-I_{yz}p)q+I_{xz}(r^2-p^2)+M \\ (I_x-I_y)pq+(I_{yz}q-I_{xz}q)r+I_{xy}(p^2-q^2)+N \end{bmatrix} \tag{2}$$

$$\begin{cases} \dot{\phi} = p+tan\theta(qsin\phi+rcos\phi) \\ \dot{\theta} = qcos\phi-rsin\phi \\ \dot{\psi} = sec\theta(qsin\phi+rcos\phi) \end{cases} \tag{3}$$

Since ψ has no effect on other parameters, the correlation equation can be removed, and the EOM is of order 8.

2.1 EOM for conventional aircraft

It can be considered that the conventional aircraft satisfy the following assumptions[11][12]:

1. All the motion parameters consist of small deviations from a reference condition of steady flight;
2. The instantaneous aerodynamic forces and moments depend on the instantaneous motion parameters;
3. The aerodynamic forces and moments vary linearly with the motion parameters and control surface deflections;
4. The longitudinal aerodynamic forces and moments, including F_x, F_z and M are only affected by the longitudinal variable, including the forward velocity u, angle of attack α and pitch rate q;
5. The lateral aerodynamic forces and moments, including F_z, L and N are only affected by the lateral-directional variables, inlcuding the sideslip β, roll rate p and yaw rate r.

Based on these assumptions, 2 sets of 4^{th} order EOM can be obtained:

$$\dot{x} = Ax+Bu \tag{4}$$

For longitudinal, $x_{long} = [\Delta u \quad \alpha \quad q \quad \Delta\theta]^T$; for lateral-directional, $x_{lat} = [\beta \quad p \quad r \quad \phi]^T$.

2.2 Small Disturbance Assumption for BWB aircraft

For BWB aircraft, Assumption 1, 2 and 3 are still satisfied. However, the influence of lateral motion on longitudinal forces and moments is significantly greater than that of conventional aircraft:

- ICEs generates moments mainly by changing the local lift and drag. Because of the short arm, these control surfaces are large in size and quantity, which affect longitudinal forces and moments significantly.
- For conventional aircraft, it is usually $I_z > I_y > I_x$, while for BWB aircraft, it is usually $I_x > I_z > I_y$.
- The inertia product can no longer be ignored, which will make the longitudinal variables affect the lateral/directional force and moments.

Therefore, for BWB aircraft, Assumption 4 and 5 are no longer satisfied.

2.2.1 Moment Equations

Moment equations under small disturbace theory ($p = \Delta p, q = \Delta q, r = \Delta r$) are[13]:

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (I_z - I_y)\Delta q \Delta r + (I_{xz}\Delta q - I_{xy}\Delta r)\Delta p + I_{yz}(\Delta q^2 - \Delta r^2) + \Delta L \\ (I_z - I_x)\Delta p \Delta r + (I_{xy}\Delta r - I_{yz}\Delta p)\Delta q + I_{xz}(\Delta r^2 - \Delta p^2) + \Delta M \\ (I_x - I_y)\Delta p \Delta q + (I_{yz}\Delta q - I_{xz}\Delta q)\Delta r + I_{xy}(\Delta p^2 - \Delta q^2) + \Delta N \end{bmatrix} \tag{5}$$

For BWB aircraft, $I_z > I_x > I_y$, $|I_y - I_z|$ and $|I_x - I_y|$ are large, while $|I_z - I_x|$ are small. For high maneuverability aircraft, $\Delta p, \Delta q$ and Δr can not be regarded as a small quantity. Therefore, $(I_y - I_z)\Delta q \Delta r$ and $(I_x - I_y)\Delta q \Delta r$ can not be ignored. The moment equations can be simplified as:

$$\begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} (I_z - I_y)qr + L \\ M \\ (I_x - I_y)pq + N \end{bmatrix} \tag{6}$$

For low maneuverability aircraft, since $\Delta p, \Delta q$ and Δr are small, $(I_y - I_z)\Delta q \Delta r$ and $(I_x - I_y)\Delta p \Delta q$ can be ignored like conventional aircraft.

2.2.2 Angular Velocity Equations

Neglecting the high order terms, the angular velocity equations under small-disturbance condition ($\theta = \theta_0 + \Delta\theta, \phi = \Delta\phi$) can be simplified as:

$$\begin{cases} \dot{\phi} = p + tan\theta_0 r \\ \Delta\dot{\theta} = q \end{cases} \tag{7}$$

2.2.3 Force Equations

Neglecting high order terms, and converting V, W into α, β, the velocity equation under small disturbance condition ($u = u_0 + \Delta u, v = \Delta v, w = \Delta w$) can be simplified as:

$$\begin{cases} \Delta\dot{u} = -g(sin\theta_0 + \Delta\theta cos\theta_0) + \Delta X/m \\ \dot{\beta} = -r + gcos\theta_0 \cdot \phi/u_0 + \Delta Y/mu_0 \\ \dot{\alpha} = q + g(cos\theta_0 - \Delta\theta sin\theta_0) + \Delta Z/mu_0 \end{cases} \tag{8}$$

2.3 Full Order EOM

From Eqs.(6), (7) and (8), and expressing the aerodynamic forces and moments in the form of Taylor series, the FOEOM can be obtained:

$$E\dot{x} = Fx + Gu \tag{9}$$

where, $x = [\Delta u \quad \alpha \quad q \quad \Delta\theta \quad \beta \quad p \quad r \quad \phi]^T$, $u = [\delta_1 \ \cdots \ \delta_n]^T$

$$E=\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_y & 0 & 0 & -I_{xy} & -I_{yz} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -I_{xy} & 0 & 0 & I_x & -I_{xz} & 0 \\ 0 & 0 & I_{yz} & 0 & 0 & -I_{xz} & -I_z & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right], \quad G=\left[\begin{array}{ccc} X_{\delta_1}/mu_0 & \cdots & X_{\delta_n}/mu_0 \\ Z_{\delta_1}/mu_0 & \cdots & Z_{\delta_n}/mu_0 \\ M_{\delta_1} & \cdots & M_{\delta_n} \\ 0 & \cdots & 0 \\ \hline Y_{\delta_1}/mu_0 & \cdots & Y_{\delta_n}/mu_0 \\ L_{\delta_1} & \cdots & L_{\delta_n} \\ N_{\delta_1} & \cdots & N_{\delta_n} \\ 0 & \cdots & 0 \end{array}\right]$$

$$F=\left[\begin{array}{cccc|cccc} X_u/mu_0 & X_\alpha/mu_0 & X_q/mu_0 & -gcos\theta_0/mu_0 & X_\beta/mu_0 & X_p/mu_0 & X_r/mu_0 & 0 \\ Z_u/mu_0 & Z_\alpha/mu_0 & Z_q/mu_0 & -gsin\theta_0/mu_0 & Z_\beta/mu_0 & Z_p/mu_0 & Z_r/mu_0 & 0 \\ M_u & M_\alpha & M_a & 0 & M_\beta & M_p & M_r & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & Y_\beta/mu_0 & Y_p/mu_0 & Y_r/mu_0 & 0 \\ 0 & 0 & 0 & 0 & L_\beta & L_p & L_r & 0 \\ 0 & 0 & 0 & 0 & N_\beta & N_p & N_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

3. Mode Chacteristic Analysis

For dynamic stability analysis, $x_0 \neq 0, u=0$, the EOM has a general solution:

$$\dot{x} = x_0 e^{\lambda t} \tag{10}$$

Where, λ is the eigenvalue, and x_0 is the eigenvector[12]. The eigenvalue can be a real root, corresponding to a real vector; it can also be a pair of conjugate complex roots corresponding to a pair of conjugate eigenvectors.

3.1 Modes for Conventinal EOM

For conventional aircraft, based on the 6DOF EOM, small disturbance theory and longitudinal lateral/directional decoupling, the EOM can be described by 2 sets of 4^{th} order state space equation:

$$\dot{x}_{long} = A_{long}x_{long} + B_{long}u_{long} \tag{11}$$

$$\dot{x}_{lat} = A_{lat}x_{lat} + B_{lat}u_{lat} \tag{12}$$

The dynamic stability can be characterized by eigenvalues of matrices A_{long} and A_{lat}. For conventional aircraft, the solutions of longitudinal EOM are usually 2 pairs of conjugate complex roots, corresponding to 2 pairs of 4^{th} order conjugate eigenvectors; while the solutions of lateral/directional EOM are 2 real roots and 1 pair of conjugate complex roots, corresponding to 2 4^{th} order real eigenvectors and 1 pair of 4^{th} order conjugate eigenvectors. Therefore, a conventional aircraft usually has 5 modes, which are the Short-Period, Phugoid, Roll, Dutch-Roll and Spiral modes:

$$V_{long} = \underbrace{\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix} \begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}}_{Short-Period}, \underbrace{\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix} \begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}}_{Phugoid} \tag{13}$$

$$V_{lat} = \underbrace{\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}}_{Roll}, \underbrace{\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}}_{Spiral}, \underbrace{\begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix} \begin{bmatrix} \times \\ \times \\ \times \\ \times \end{bmatrix}}_{Dutch-Roll} \tag{14}$$

3.2 Modes for Full Order EOM

The full order equation with longitudinal/lateral/directional coupling is of order 8, which has 8 eigenvalues, corresponding to eight 8^{th} order eigenvectors.

$$V_{full} = \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix}, \begin{bmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{bmatrix} \tag{15}$$

The number of eigenvalues, i.e., the number of modes, depends on the aerodynamic configuration of the aircraft.

3.3 Typical Example

In this paper, the mode characteristics of a statically stable BWB aircraft with V-tail are solved by using conventional EOM and FOEOM respectively. The eigenvalues and eigenvectors based on the conventional EOM are as follows:

$$\lambda_{long} = [-1.3428 \pm 0.0382i, \quad -0.0026 \pm 0.0155i]^T$$

$$V_{long} = \begin{bmatrix} 0.0028 \pm 0.0007i \\ -0.9161 \\ 0.3197 \mp 0.0349i \\ -0.2386 \pm 0.0192i \end{bmatrix}, \begin{bmatrix} -0.9348 \\ 0.0745 \mp 0.0016i \\ -0.0054 \pm 0.0002i \\ 0.0673 \pm 0.3405i \end{bmatrix}$$

$$\lambda_{lat} = [-1.5652, \quad -0.0304 \pm 0.8668i, \quad 0.0026]^T$$

$$V_{lat} = \begin{bmatrix} -0.0354 \\ -0.8417 \\ -0.0316 \\ 0.5378 \end{bmatrix}, \begin{bmatrix} -0.6586 \\ -0.3120 \pm 0.1144i \\ 0.0067 \mp 0.5559i \\ 0.1444 \pm 0.3548i \end{bmatrix}, \begin{bmatrix} 0.0041 \\ 0.0026 \\ 0.0423 \\ 0.9991 \end{bmatrix}$$

The eigenvalues based on FOEOM:

$$\lambda_{full} = [-1.3428 \pm 0.0382i \quad -0.0026 \pm 0.0155i \quad -1.5652 \quad -0.0313 \pm 0.8669i \quad 0.0044]^T$$

It can be seen that the eigenvalues λ_{long} and λ_{lat} obtained by solving FOEOM are almost the same as the eigenvalues λ_{full} obtained by conventional EOM.
The eigenvectors based on FOEOM:

$$V_{full} = \begin{bmatrix} 0.0028 \mp 0.0007i \\ 0.9161 \\ -0.3197 \pm 0.0349i \\ 0.2386 \mp 0.0192i \\ \hline 2.5 \cdot 10^{-5} \pm 2 \cdot 10^{-6}i \\ 5.7 \cdot 10^{-5} \mp 10^{-5}i \\ -3.5 \cdot 10^{-5} \pm 4 \cdot 10^{-6}i \\ -4.2 \cdot 10^{-5} \pm 10^{-5}i \end{bmatrix}, \begin{bmatrix} -0.9348 \\ 0.0745 \mp 0.0016i \\ -0.0054 \pm 0.0002i \\ 0.0673 \pm 0.3405i \\ \hline -4.5 \cdot 10^{-8} \pm 1.8 \cdot 10^{-8}i \\ -2 \cdot 10^{-10} \pm 9 \cdot 10^{-9}i \\ -2.2 \cdot 10^{-8} \pm 2 \cdot 10^{-9}i \\ -5.4 \cdot 10^{-7} \pm 1.6 \cdot 10^{-7}i \end{bmatrix}, \begin{bmatrix} -5 \cdot 10^{-8} \\ -0.0003 \\ 0.0001 \\ -0.0001 \\ \hline -0.0354 \\ -0.8413 \\ -0.0315 \\ 0.5384 \end{bmatrix},$$

$$\begin{bmatrix} 6.4 \cdot 10^{-6} \mp 4.7 \cdot 10^{-6}i \\ 5.8 \cdot 10^{-5} \mp 0.0001i \\ 0.0002 \mp 0.0001i \\ 7.2 \cdot 10^{-5} \pm 0.0001i \\ \hline 0.6609 \\ -0.3132 \pm 0.1149i \\ 0.0061 \mp 0.5577i \\ 0.1166 \pm 0.3558i \end{bmatrix}, \begin{bmatrix} 9.6 \cdot 10^{-6} \\ 8.2 \cdot 10^{-7}i \\ 10^{-8} \\ 2.3 \cdot 10^{-6} \\ \hline 0.0042 \\ 0.0025 \\ 0.0423 \\ 0.9991 \end{bmatrix}$$

It can be seen that, the first 2 pairs of conjugate eigenvectors of λ_{full} correspond to the first 2 pairs of conjugate eigenvalues of V_{full}, and the elements of the first 4 rows are almost the same as V_{long}. Elements of the last 4 rows reflect the influence of longitudinal disturbance on lateral/directional parameters, which is about 3-5 orders of magnitude smaller than the elements of the first 4 rows.
The last 4 eigenvectors correspond to the last 4 eigenvalues of λ_{full}, and elements of the last 4 rows are almost the same as V_{lat}. Elements of the first 4 rows reflect the influence of lateral/directional disturbance on longitudinal parameters, which is about 3-5 orders of magnitude smaller than elements of the last 4 rows.

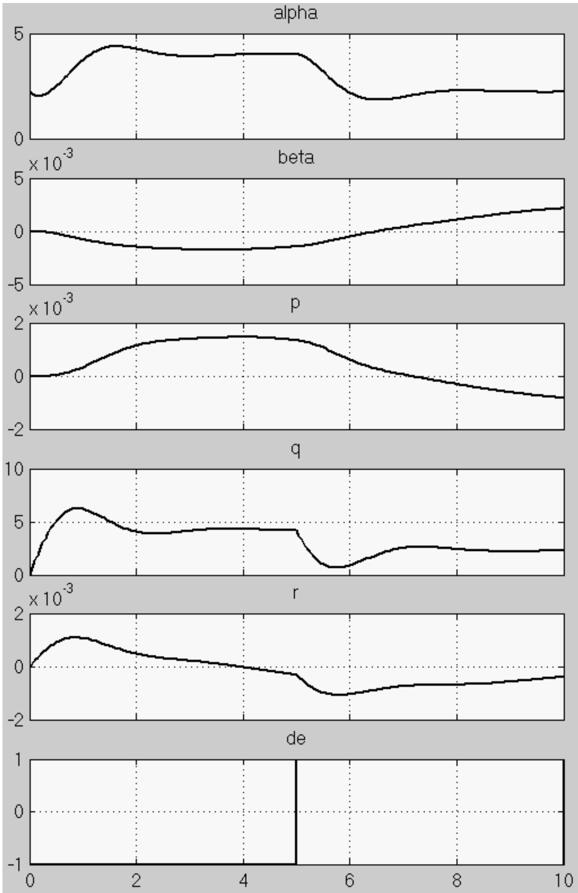

Figure 3 – Response of unit step elevator input.

Figure 3 shows the time response of positive and negative unit step elevator deflection. It can be seen that under the elevator step input, the responses of AOA α and pitch rate q are typical second-order system response. The peak time T_p and overshoot O_s of response correspond to the first pair of eigenvalues of λ_{full}.
The response characteristics of BWB aircraft are different from those of conventional aircraft. In the case of only longitudinal control, the response of β, p and r are not 0, but have a small response of 3 orders of magnitude lower than that of α and q.

4. Conclusion

For BWB aircraft, due to the short control surface arm, strong coupling between aerodynamic forces and moments, unconventional distribution of moment of inertia, and non-negligible inertia product, longitudinal and lateral/directional motion can no longer be decoupled.
By comparing the eigenvalues and eigenvectors of the conventional EOM and FOEOM, it can be seen that the eigenvalues of these two EOM are indistinguishable, and the differences is mainly reflected in the eigenvectors:

- For conventional EOM, the eigenvectors of longitudinal and lateral/directional motion are both 4 groups, all of which are of order 4, and have no influence on each other.

- For FOEOM, the eigenvectors of longitudinal-lateral-directional motion are 8 groups, all of which are of order 8. The coupling characteristics of longitudinal and lateral/directional are mainly reflected in the eigenvectors, and the coupling terms is usually 3-5 orders of magnitude smaller than the normal terms.

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6. Copyright Statement

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