OPTIMAL TRAJECTORY DEPENDENCE ON THE ATMOSPHERE MODEL FOR THE SUN-POWERED AIRPLANE

Sergey V. Serokhvostov[1],Tatiana E. Churkina[2]

[1] Moscow Institute of Physics and Technology (National Research University)
[2] Moscow Aviation Institute (National Research University)

Abstract

The problem of optimal flight altitude for the airplane with the electrical powerplant and solar cells in the long-time flight is investigated for the problem of accumulator energy maximization at the end of the flight. Atmosphere density models and solar radiation models as functions of altitude and day time are formed and investigated. Optimal altitude for the fixed-altitude mission and altitude as function of time for nonfixed altitude mission are obtained. Necessary condition for the optimal trajectory is checked. The optimized function sensitivity to the deviation of flight parameters from the optimal ones is investigated. The influence of airplane parameters on the results is analyzed.

Keywords: sun-powered airplane, optimization, atmosphere model

1. Introduction

By this time a set of ”heavy” solar-powered (SP) airplanes was designed and built (such as ”Pathfinder”, ”Centurion”, ”Helios”, ”Solar Impuls”, ”SoLong”, ”Zephyr” and others, see Figures 1-3). Also, information about a set of small-sized solar planes can be found (some of them are presented in Figures 4–5).

Figure 1 – “Helios” [1].

Figure 2 – “Zephyr” [2].

Figure 3 – “Solar Impulse 2” [3].

Figure 4 – Solar Wonder Aircraft [4].

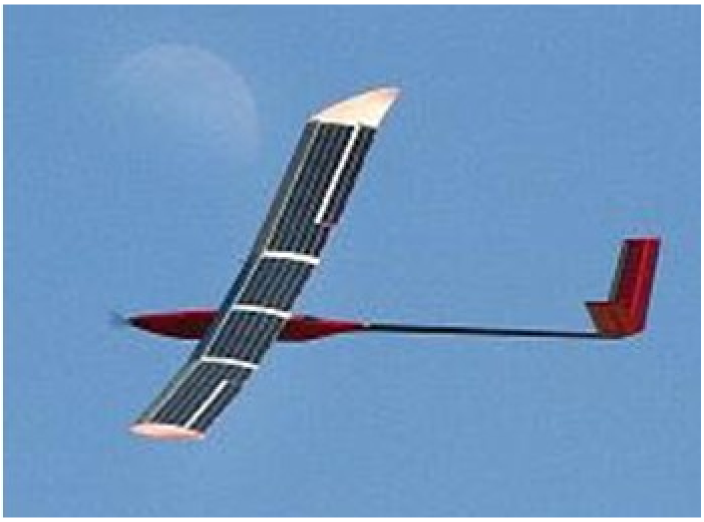

Figure 5 – Sky-Sailor [5].

But for the present state of the art the design of the SP airplane for the long-endurance mission is still a serious problem because of the moderate value of the solar radiation intensity, rather low efficiency and rather high density of the solar cells, insufficient energy density of the onboard energy storage and some other factors. Some questions concerning the optimal multi-day flight were investigated in [6, 7, 8]. First of all, the optimal altitude for the flight at constant altitude of the ”heavy” airplane at high altitudes was found. But in these investigations rather simple model of solar radiation was used and the analysis was restricted by the altitudes higher than 11 km. On the other hand, the flight altitudes of Solar Impulse were less than 10 km.

In the present research all the altitudes from 0 km to 20 km for the airplane and more precise density and radiation models are investigated.

2. Optimal Flight Altitude. Problem Statement

Suppose that the airplane must fly at constant altitude (that can be chosen before the flight) with constant velocity V for all its longtime mission with the minimum value of the total energy E consumed from accumulator. For this case the equations of motion are

$$W\eta - \left(C_{D0} + A \cdot C_L^{\,2}\right)\rho \frac{V^3}{2} S = 0\,, \tag{1}$$

$$C_L \rho \frac{V^2}{2} S - mg = 0\,, \tag{2}$$

$$\dot{E} = W - W_{PV}\,,$$

where m is an aircraft mass, W — power consumed by the engine, η — powerplant efficiency (assumed to be constant), C_{D0} — drag coefficient at zero lift, $A = 1/(\pi e \lambda)$, λ — aspect ratio of the wing, e – Oswald coefficient, C_L — lift coefficient, $\rho = \rho(h)$ — air density depending on flight altitude h, S — wing area, g — acceleration of gravity, W_{PV} — power of solar cells.

For this problem all these characteristics except W_{PV} and E are assumed to be constant during the flight. Power W_{PV} depends on the flight altitude, day time, aircraft orientation with respect to the sun location. But for the first step of our investigations we can assume that W_{PV} is the mean value and depends only on the altitude:

$$W_{PV} = I(h)S\beta$$

where I is a part of solar radiation intensity normal to the solar panel, β is the ratio of the solar cells area to the wing area. So, from the mathematical point of view, it is required to find the minimum of

$$W - I(h)S\beta \tag{3}$$

at conditions (1)–(2).
The solution of this problem for the common case is the system of equations

$$C_L = \sqrt{\frac{3C_{D0}}{A}}, \qquad V = -\sqrt{\frac{3}{AC_{D0}}}\frac{\eta\rho}{2mg\frac{\partial\rho}{\partial h}}\frac{\partial I}{\partial h}S\beta, \qquad C_{D0}V^3S\frac{\partial\rho}{\partial h} = -\frac{\partial I}{\partial h}\eta S\beta \tag{4}$$

One can see that for the final formulas it is required to know the atmosphere properties — the dependence of air density and solar radiation intensity on the altitude and day time.

3. Atmosphere Model

For the atmosphere density it is worth using International Standard Atmosphere (ISA) model. Within this model, the atmosphere's temperature decreases linearly with altitude from 0 km to about 10 km altitude, then from about 11 km to 20 km the atmosphere is isothermal. So, according to these data, the expression for atmosphere density for 0–10 km can be derived and written as

$$\rho = \rho_0\left(1 - \frac{\alpha h}{T_0}\right)^{\left(\frac{\mu g}{\alpha R} - 1\right)}$$

where ρ_0 — density at sea level (1.225 kg/m^3), α≈6.5·10^{-3} K/m (found by authors), T_0 — temperature at sea level (288 ^{0}K), μ — molar mass of the air (0.029 kg/mole), R = 8.31 Joule/(mole·K).
For these data

$$\rho = \rho_0\left(1 - \frac{\alpha h}{T_0}\right)^{4.38}.$$

For the isothermal part of the atmosphere the formula for air density can be written as

$$\rho=\rho_{01}\exp(-(h-11000)/h_0)$$

where h_0 = 6 374 m, ρ_{01} is the air density at 11 000 m altitude.
Analysis of the data from [9] shows that the intensity of the solar radiation can be expressed as the multiplication of two functions, one of them depends only on the altitude and another depends only on the day time.
As for the dependency of solar intensity on the altitude, let's make some assumptions. First of all, assume that the composition of atmosphere (percentage of the gases) is the same at all altitudes. Second, assume that in atmosphere there is no pollutions that can absorb the solar light. So, one can use the Bouguer's law written in the form

$$\frac{dI}{dh} = B\rho(h)I \tag{5}$$

where B — coefficient of proportionality that depends on the atmosphere composition. Assume that the composition is constant at all the altitudes, so B is constant. The analysis of the data from [9] proves this assumption at least for the altitudes of 11–20 km. The value of B obtained by authors on

the basis of [9] is B=5.7 10^{-5} m^2 /kg.
For low atmosphere, the above formula turns into

$$\frac{dI}{dh} = IB\rho_0 \left(1 - \frac{\alpha h}{T_0}\right)^{4.38}$$

which gives

$$I = I_0 \exp\left(\frac{B\rho_0 T_0}{5.38\alpha}\left[1 - \left(1 - \frac{\alpha h}{T_0}\right)^{5.38}\right]\right).$$

This formula can be used for the numerical investigation but is rather difficult for the analytical ones. To simplify the formula it is useful to analyze the experimental data for the dependency of solar radiation on the altitude. It is known that above the atmosphere the solar intensity is about 1.3 kW/m^2, and at the sea level it is about 1 kW/m^2. On the other hand, the air density at sea level and at the altitude of 10 km differs nearly 3 times. So, the main effect of different absorption at different altitudes is due to the air density. The idea is to assume that the intensity does not differ significantly, and

$$\frac{dI}{dh} = I_0 B\rho_0 \left(1 - \frac{\alpha h}{T_0}\right)^{4.38}.$$

So,

$$I = I_0 \left(\frac{B\rho_0 T_0}{5.38\alpha}\left[1 - \left(1 - \frac{\alpha h}{T_0}\right)^{5.38}\right] + 1\right). \qquad (6)$$

The ability to use all above mentioned assumptions must be proved by the comparison of precise solution with the approximate one.
For the isothermal part of the atmosphere

$$\frac{dI}{dh} = B\rho_{01} \exp\left(-\frac{h}{h_0}\right) I$$

and

$$I = I_{01} \exp\left[h_0 B\rho_{01}\left(1 - \exp\left(-\frac{h}{h_0}\right)\right)\right]$$

where I_{01} is the solar intensity at 11 km.
The simplification gives

$$\frac{dI}{dh} = B\rho(h) I_{01}$$

and

$$I = I_{01}\left(1 + h_0 B\rho_{01}\left(1 - \exp\left(-\frac{h}{h_0}\right)\right)\right). \qquad (7)$$

The dependency of solar intensity on the day time at fixed altitude for the first steps of analysis (assuming that day is 12 hours and the night is 12 hours) can be expressed [6] as constant zero function for the night time and the sine function with the zero time moment at the start of sunrise with the period of 24 hours for the day time.

4. Optimal Altitude Problem. Solution and Analysis

System (4) under assumption (5) can be resolved for the variable h in the common case as

$$\frac{\rho(h)^5 \left(I(h)\right)^2}{\left(\dfrac{\partial\rho(h)}{\partial h}\right)^2} B^2 = \left(\frac{2mg}{S}\right)^3 \frac{1}{(\eta\beta)^2} \sqrt{\frac{A^3 C_{D0}}{27}}. \tag{8}$$

For the low atmosphere it gives

$$\left(1-\frac{\alpha h}{T_0}\right)^{15.14} \left(\exp\left(\frac{B\rho_0 T_0}{5.38\alpha}\left[1-\left(1-\frac{\alpha h}{T_0}\right)^{5.38}\right]\right)\right)^2 = \left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{(\eta\beta B I_0)^2} \sqrt{\frac{A^3 C_{D0}}{27}} \left(4.38\frac{\alpha}{T_0}\right)^2. \tag{9}$$

The equation (8) for the common case of air density

$$\rho=\rho_0\left(1-\frac{\alpha h}{T_0}\right)^{\gamma}$$

where γ is some number, gives

$$\left(1-\frac{\alpha h}{T_0}\right)^{(\gamma\cdot 3+2)} \left(\exp\left(\frac{B\rho_0 T_0}{(\gamma+1)\alpha}\left[1-\left(1-\frac{\alpha h}{T_0}\right)^{(\gamma+1)}\right]\right)\right)^2 = \left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{(\eta\beta B I_0)^2} \sqrt{\frac{A^3 C_{D0}}{27}} \left(\gamma\frac{\alpha}{T_0}\right)^2. \tag{9a}$$

For the isothermal part of atmosphere

$$\exp\left(-\frac{3h}{h_0}\right)\left(\exp\left[h_0 B\rho_{01}\left(1-\exp\left(-\frac{h}{h_0}\right)\right)\right]\right)^2 = \left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{(\eta\beta B h_0 I_{01})^2} \sqrt{\frac{A^3 C_{D0}}{27}}. \tag{10}$$

Equations (9), (10) can't be solved analytically. To solve the problem numerically some data about aircraft are required. Let's use the data as for "Cirrus" glider upgraded with solar cells by authors (see Figure 6). Assume that the maximum value of solar radiation at the zero altitude is 825 W/m^2 . The main data for the airplane are m = 2 kg, C_{D0} = 0.02, aspect ratio λ = 15, S = 0.72 m^2 , area covered by cells is S/2 (i.e. β = 0.5), powerplant efficiency — 50%, solar cell efficiency — 20%.

The flight altitude as function of solar intensity is given in Figure 7. One can see that the graphs for the low atmosphere and isothermal models do not cross at the altitudes of 10–11 km as can be expected. This can be explained by the fact that at the altitudes between 10 km and 11 km the atmosphere changes the properties, the graph for low atmosphere is valid for h < 10 km, graph for isothermal atmosphere is valid for h > 11 km, and from 10 to 11 km these graphs must be interpolated by continuous and smooth line from one graph to another.

To solve the problem of optimal altitude for 24h mission one must find the "mean" value of I for 24 hours I_{mean}. Using the sine dependency of intensity on the day time, one can find that

$$I_{mean} = I_{max} \cdot 0.318,$$

where I_{max} is maximal solar intensity (at midday).

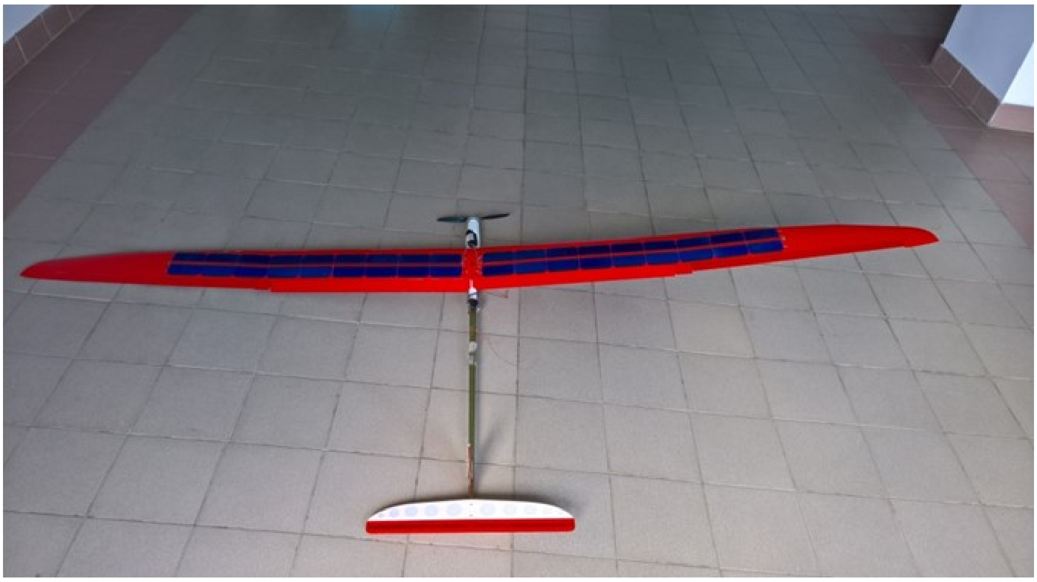

Figure 6 – Experimental aircraft made by authors.

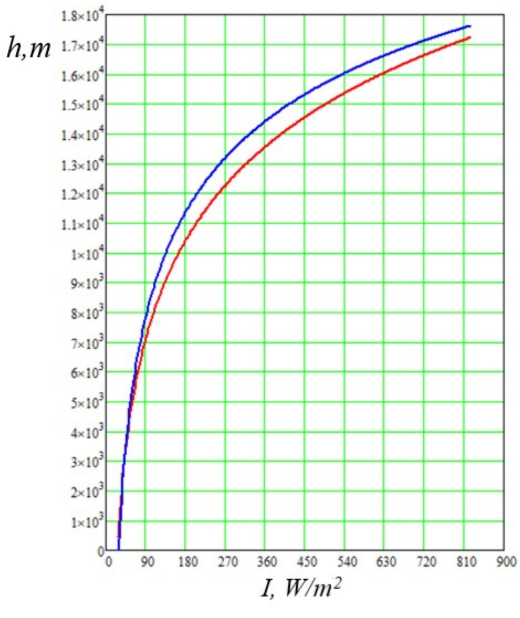

Figure 7 – Altitude as function of solar intensity. Blue line — low atmosphere model (9), red line — isothermal model (10)

For these data the flight altitude calculated using the formula for low atmosphere gives the value of 9 812 m, the calculation with the formula for the isothermal atmosphere gives 8 907m. But it should be mentioned that these formulas are for the equator conditions. For the 45^0 latitude (for example, Torino conditions) the low atmosphere formula gives 7854 m, isothermal formula gives 7 107 m. The both cases give the result corresponding to the low atmosphere for the both models, the conclusion is that the low atmosphere model must be used for the precise results. Now let's estimate the disadvantage for the flight at not optimal altitude. Figure 8 shows the mean electrical power incoming to the accumulator as function of altitude for the case of 45^0 latitude conditions for the airplane analyzed. One can see that the disadvantage at the altitude difference of 1 km with respect to the optimal one is about 0.2% of the accumulator incoming power and practically the same value of 0.2% with respect to the power consumed by the motor. This means that it is not necessary to maintain the flight altitude precisely for the best value of accumulator incoming power. Also, if necessary, the model of isothermal atmosphere can be used in low altitudes as the maximal difference between these two graphs at $h < 10$ km is less than 1 km.

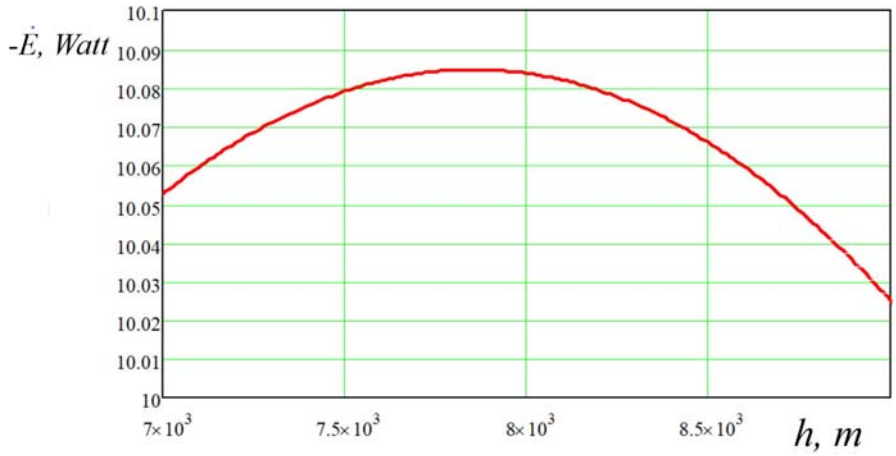

Figure 8 – Accumulator incoming power as function of altitude near the maximum

Now let's investigate the accuracy of the simplified formula (6). Figure 9 shows the results of calculations. The deviations does not exceed 1 km, so the more simple formula can be used. As for the accuracy of formula (7), the calculations show that the results obtained with formula (7) practically coincide with precise ones.

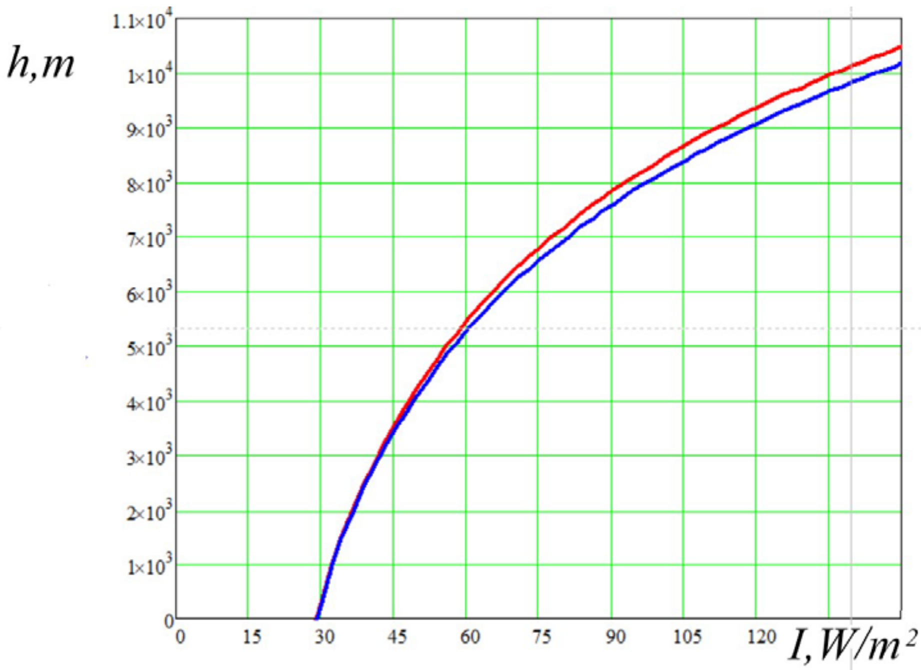

Figure 9 – Results of calculation for various models. Red — precise, blue — model (6)

Some words should be told about the formula (8) itself. One can see that the right part deals only with parameters of aircraft and atmosphere, so it is constant for the defined aircraft for the defined latitude of flight, and in the left part there are no aircraft parameters. For any shape of density and intensity functions it is possible to analyze the sensitivity of the result to the aircraft characteristics.

First of all, one can see that the main influence gives the wing loading (*mg/S*), as it is in the third power. Also the powerplant efficiency and cells area to wing area ratio can influence significantly. As for the drag coefficient at zero lift C_{D0}, its influence is small enough.
Another conclusion can be also made. As there is no aircraft parameters in the left part, one can find the altitude (for example, numerically) as the function of the magnitude of right part, and it will be the universal solution for all the possible aircraft. As a consequence, two different planes with the same wing loading *mg/S*, the same wing shape, powerplant efficiencies and percentage of wing covered by solar cells have the same optimal flight altitude.
Moreover, the graph from Figure 7 can be used for the calculation of optimal altitude for any airplane. For example, let's evaluate the altitude for the solar plane similar to the "Helios" with the characteristics: m=700 kg, S=180 m^2, β=1, λ=30. Comparing these data with those of "Cirrus" (used above) and formulas (9) - (10) one can conclude that the right parts of these equations coincide for both aircraft in the case that for the "Cirrus" the equivalent solar intensity is about two times higher. So, according to Figure 7, the optimal flight altitude of "Helios" is in isothermal part of atmosphere and is about 12.5 km from the Earth surface (It should be mentioned that the previous investigations [6], [7] give the same value).
It is mentioned above that formulas (9) and (10) can't be solved analytically. On the other hand, the values of $B\rho_0 T_0/(5.38\alpha)$ and $h_0 B\rho_{01}$ are small enough so formulas (9)-(10) can be simplified as

$$\left(1-\frac{\alpha h}{T_0}\right)^{15.14} = \left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{\left(\eta\beta B I_0\right)^2}\sqrt{\frac{A^3 C_{D0}}{27}}\left(4.38\frac{\alpha}{T_0}\right)^2, \qquad (11)$$

$$\exp\left(-\frac{3h}{h_0}\right) = \left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{\left(\eta\beta B h_0 I_{01}\right)^2}\sqrt{\frac{A^3 C_{D0}}{27}}. \qquad (12)$$

These formulas enable to find less precise but analytical solution of the altitude as function of parameters.
For low atmosphere

$$h \approx \frac{T_0}{\alpha}\left\{1-\left[\left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{\left(\eta\beta B I_0\right)^2}\sqrt{\frac{A^3 C_{D0}}{27}}\left(4.38\frac{\alpha}{T_0}\right)^2\right]^{\frac{1}{15.4}}\right\}.$$

For isothermal atmosphere

$$h \approx h_0\left[\frac{2}{3}\ln\left(\eta\beta B h_0 I_{01}\right) - \ln\left(\frac{2mg}{S\rho_0}\right) - \frac{1}{6}\ln\left(C_{D0}\right) + \frac{1}{2}\ln\left(\pi e\lambda\right)\right].$$

Using these formulas, one can estimate once more the optimal flight altitude of "Helios" using data for "Cirrus". Calculations give the deviation of less than 1 km.
It is also useful to have the expression for the power consumed from the accumulator. For the low atmosphere

$$P \approx \beta B I_0 \rho_0 S\frac{T_0}{\alpha}\left[\frac{1}{4.38\cdot 5.38}\left(\frac{2mg}{S\rho_0}\right)^3 \frac{1}{\left(\eta\beta B I_0\right)^2}\sqrt{\frac{A^3 C_{D0}}{27}}\left(4.38\frac{\alpha}{T_0}\right)^2\right]^{\frac{5.38}{15.14}} - I_0 S\beta\left(1+\frac{B\rho_0 T_0}{5.38\alpha}\right) \qquad (13)$$

For the isothermal atmosphere

$$P \approx 6mg\frac{\left(C_{D0}\right)^{\frac{1}{6}}}{\eta^{\frac{2}{3}}\left(\beta B h_0 I_0\right)^{\frac{1}{3}}\sqrt{3\pi e\lambda}} - I_0 S\beta\left(1+h_0 B\rho_{01}\right) \tag{14}$$

Note that the indexes (exponents) for the parameters m, I_0, λ, ρ_0 etc. in (13) and (14) do not coincide but rather close to each other. This enables to analyze the influence of these parameters on P using the formula (14).

Other interesting thing can be noted. The constructions $1/(B\rho_0)$ and $1/(B\rho_{01})$ can be defined as characteristic lengths of solar intensity difference in low and isothermic atmosphere respectively, T_0/α and h_0 are characteristic lengths of density difference, so $B\rho_0 T_0/(5.38\alpha)$ and $h_0 B\rho_{01}$ are the ratios of these characteristic values (density length/solar intensity length). For the Earth's atmosphere

$$B\rho_0 T_0/(5.38\alpha) \approx 0.6$$

$$h_0 B\rho_{01} \approx 0.12$$

5. Flight Path Optimization

For a set of flight tasks the flight altitude can vary with time. In this case the aircraft can change the altitude during the day to gather the maximum of the energy available. The background of this effect is that at lower altitudes the power consumption by the powerplant is lower and at higher altitudes the solar radiation is higher. So, at the day time it is better to fly higher, at the night time it is better to fly lower. This task in general case was analyzed previously in [6]. So, it is better to shortly repeat the problem statement and the result in common case and implement them to the problem investigated here. The problem solved is as follows: the airplane with solar panels can fly at non-fixed altitude. There are no restrictions on accumulator power (charging and discharging) and capacity, vertical velocity of the plane, maximal accelerations along all the axes. There is minimal altitude restriction (Earth's surface or some others). Only the motion in vertical plane is analyzed. The goal is to have maximal available energy in the accumulators at the end of the flight.

The problem was solved using Pontryagin's maximum principle. Hamilton function was

$$\mathrm{H} = \frac{2P_Z}{m}\left(W\eta - \left(C_{D0} + A\left(\frac{2mg}{\rho Z S}\right)^2\right)\rho\frac{Z^{3/2}}{2}S - mgV_Y\right) + P_H V_Y + \left(W - I(h,t)S\right),$$

where $Z=V^2$, control variables are W and vertical component of velocity V_Y,

$$\dot{P}_Z = -\frac{\partial \mathrm{H}}{\partial Z} = -\left(-\frac{2P_Z}{m}\left(\frac{3}{2}C_{D0}\rho\frac{Z^{1/2}}{2}S - \frac{1}{2}\frac{A\left(2mg\right)^2}{2Z^{3/2}\rho S}\right)\right),$$

$$\dot{P}_H = -\frac{\partial \mathrm{H}}{\partial h} = -\left(-\frac{2P_Z}{m}\left(C_{D0}\frac{Z^{3/2}}{2}S\frac{\partial\rho}{\partial h} - \frac{A\left(2mg\right)^2}{2Z^{1/2}\rho^2 S}\frac{\partial\rho}{\partial h}\right) + \left(-\frac{\partial I}{\partial h}\right)S\right).$$

For this problem the possible parts of the trajectory are available

1. Maximal power mode.
2. Minimal power mode.
3. Flight along the restriction(s).
4. Cruise mode.

The main part of flight is cruise mode and is described by equations

$$\frac{2\eta P_Z}{m}+1=0,$$
$$2P_Z g+P_H=0,$$
$$\frac{2\eta \dot{P}_Z}{m}=0,$$
$$\frac{-1}{\eta}\left(C_{D0}\frac{Z^{3/2}}{2}S\frac{\partial\rho}{\partial h}-\frac{A(2mg)^2}{2Z^{1/2}\rho^2 S}\frac{\partial\rho}{\partial h}\right)+\left(\frac{\partial I}{\partial h}\right)S=0.$$

The solution shows that the altitude, velocity and lift coefficient are defined by the same formulas as for the task of optimal altitude (4), but for the solar intensity value not the mean but the values of intensity at the each moment of time must be used. As an example, the altitude as the function of time for the above mentioned airplane and equator conditions for the day time is shown in Fig. 10. Minimal and maximal power modes are used at the beginning (to reach cruise mode from initial conditions) and at the end of flight (to reach final conditions). One can see that the altitude difference during the day time is high enough in both models and is about 14 km. The most intensive altitude change (and, consequently, vertical velocity) is at the time near sunrise and sunset. These parts require more thorough investigation (it is planned in the future work). Also, as in the previous chapter, the sensitivity of the deviations is low enough, so the altitude error of 1 km from the optimal one does not give valuable disadvantage. So, for the preliminary analysis one can use only one model in all the range of altitudes.

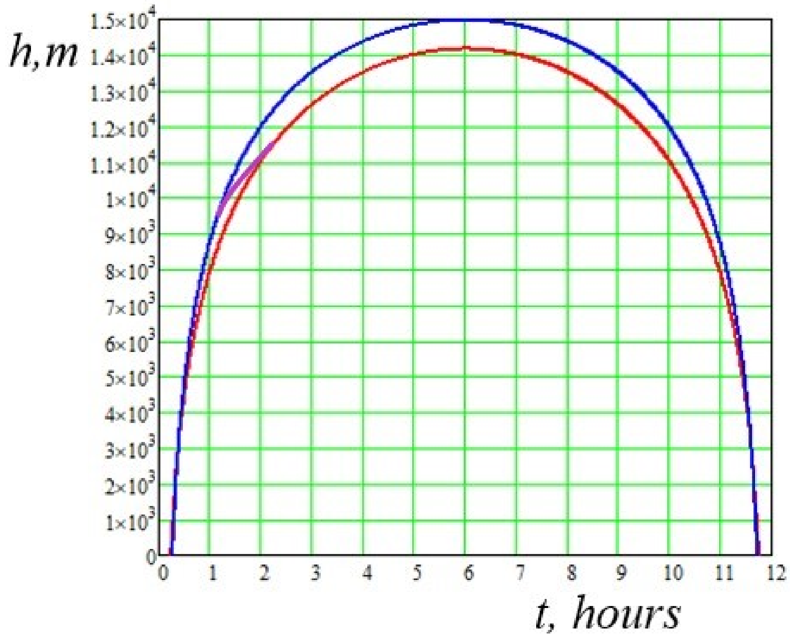

Figure 9 – Altitude as function of time for the airplane and conditions investigated. Blue line — low atmosphere model (9), red line — isothermal model (10), purple — interpolation between the models.

It is also required to check this solution for the necessary condition of extremum in the form of [10]

$$\sum_{m,s=1}^{2}\frac{\partial}{\partial u_m}\left(\frac{d^2}{dt^2}\left(\frac{\partial H}{\partial u_s}\right)\right)\eta_m\eta_s\le 0,$$

where u_i – control variables, η_i – some small values.
The investigation of this question is rather bulky and is not given here but the results show that for the both atmosphere models this condition is valid.
At the end it is worth saying that all the results obtained can be implemented not only to the flight in the atmosphere of the Earth, but also for the flight in the atmosphere of any planet which atmosphere model can be described by the model mentioned above.

6. Conclusion

1. The problem of optimal altitude for the solar powered airplane was stated and analyzed. It was shown that the mathematical models of the air density and solar radiation as function of time are necessary for this task.
2. Models of density and solar radiation were proposed. Some simplifications in the expressions were proposed and analyzed for the applicability and accuracy.
3. Formula for the defining the optimal flight altitude was obtained in the common case. Formulas for the cases of low atmosphere and isothermal atmosphere were derived.
4. The optimal fixed altitudes were obtained for some test airplane. The method of optimal altitude definition for any solar airplane was proposed.
5. The simplified formulas for the analytical investigation of optimal altitude were proposed and analyzed. Formulas for the corresponding accumulator power were obtained.
5. For the case of non-fixed altitude the optimal flight altitude as a function of time was found numerically for the test airplane. Necessary optimum conditions were checked.
6. The influence of airplane parameters on the optimal altitude was investigated. It was shown that the main influence gives the wing loading and the smallest influence gives drag coefficient at zero lift.
7. Sensitivity of optimized functions to the altitude deviations was investigated. It was shown that the altitude deviation from optimum up to 1 km gives practically no disadvantage.

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8. Contact Author Email Address

mailto: serokhvostov@phystech.edu

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