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Decoupling Control Law Design for Oblique-wing Aircraft based on Eigenstructure Assignment and Model Following

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Abstract

Decoupling control law for oblique-wing aircraft (OWA) is studied. To this end, the coupling characteristicss of OWA is briefly introduced and the need to improve its flying quality through flight control is analyzed. Then a linearized flight dynamics model is built. Methods of eigenstructure assignment and model following are applied to achieve modal decoupling and dynamic decoupling respectively. Finally, F-8 OWRA (Oblique Wing Research Aircraft) is taken as a case study to demonstrate the theories built in this paper. The results indicate that both control law design methods are applicable to decouple OWA and improve its flying quality in a certain degree.

Keywords: oblique-wing aircraft; decoupling control law; eigenstructure assignment; model following

1. Introduction

An oblique wing aircraft can arbitrarily change the angle of the wing to meet the requirements of different flight conditions. Compared with the traditional variable sweep wing configuration, the oblique wing can more effectively reduce the supersonic wave drag and has no disadvantages such as backward shifting of aerodynamic center and the complicated wing shaft [1]. However, when the wing is inclined, it is structurally divided into a forward-swept front wing and a backward-swept rear wing and thus the aircraft does not have a longitudinal symmetry plane and shows differently from the conventional ones. Aerodynamic coupling, inertial coupling and the aeroelastic problems cause the aircraft's modal characteristic and control response fail to meet the requirements of flying quality specification [2]. Normal control and flight can only be achieved through flight control.

The key to the flight control of oblique-wing aircraft is decoupling control. Aircraft decoupling control methods are generally divided into four categories, namely, force and moment decoupling, modal decoupling, static decoupling and dynamic decoupling [3]. Decoupling of force and moment can make sure that a pilot's single manipulation could only generate force or moment in one direction. Modal decoupling, which is also called semi-dynamic decoupling, can make the change of some motion parameters independent of specific modes. Static decoupling can realize that a certain command of the pilot only changes the steady-state value of a single motion parameter. Dynamic decoupling can ensure that a pilot's control command only causes a single motion parameter to change, including the dynamic response process and steady-state value.

This paper focuses on the study of modal decoupling and dynamic decoupling, which both have greater impact on the flight quality of oblique-wing aircraft. The corresponding control law design methods are eigenstructure assignment and model following.

2. Decoupling Control Law Design for Oblique-wing Aircraft

2.1 Small-perturbation linear flight dynamics model for Oblique-wing Aircraft

Before carrying out the control law design based on linear system theory, the linear flight dynamics model of the aircraft should be established first and the plant characteristics should be analyzed. Since the oblique-wing aircraft is different from aircrafts of conventional configurations, the traditional separated longitudinal and lateral linear models are no longer applicable. The coupled flight dynamics model for the oblique wing aircraft based on six-degree-of-freedom model and the small-perturbation linearization theory is as follows:

$$\begin{bmatrix} C_{lon-lon} & C_{lon-lat} \\ C_{lat-lon} & C_{lat-lat} \end{bmatrix} \begin{bmatrix} \dot{x}_{lon} \\ \dot{x}_{lat} \end{bmatrix} = \begin{bmatrix} A_{lon-lon} & A_{lon-lat} \\ A_{lat-lon} & A_{lat-lat} \end{bmatrix} \begin{bmatrix} x_{lon} \\ x_{lat} \end{bmatrix} + Bu \tag{1}$$

where $x_{lon} = (\Delta V, \Delta\alpha, q, \Delta\theta)^T$, $x_{lat} = (\Delta\beta, p, r, \phi)^T$, $u = (\Delta\delta_{eL}, \Delta\delta_{eR}, \delta_{aL}, \delta_{aR}, \Delta\delta_r)^T$.

Due to space limitations, the modeling process will not be discussed here in detail, and the specific expressions of each matrix element in the model can be found in Reference [4].

2.2 Decoupling Control Law Design for Oblique-wing Aircraft based on Eigenstructure Assignment

The dynamic response of a linear system is not only related to the eigenvalues, but also depends on the eigenvectors. Eigenstructure assignment is to achieve modal decoupling by simultaneously selecting the eigenvalues and eigenvectors of the closed-loop system, thereby improving the flight dynamics characteristics of the system. By selecting certain motion parameters of the system eigenvectors as zero, this method can achieve modal decoupling. In 1976, Moore first figured out that in a multiple-input multiple-output system, in addition to setting the eigenvalues of the system, additional degrees of freedom can be used to configure the eigenvectors of the closed-loop system [5]. Later, with the efforts of Andary and other scholars, this method has been successfully applied to the design of lateral control law of Boeing 767, Airbus A320 and B-2 [6].

For controllable and observable linear time-invariant systems,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{2}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{r\times n}$, and they are constant matrices, rank(B)=m, rank(C)=r; x is n-dimensional state variable, u is m-dimensional control input, y is r-dimensional output.

The equation of the full state feedback control law is

$$u = Kx + v \tag{3}$$

where $K \in R^{m\times n}$ is the feedback matrix, v is the reference input, then the corresponding closed loop system can be described as

$$\begin{aligned} \dot{x} &= (A + BK)x + Bv \\ y &= Cx \end{aligned} \tag{4}$$

The problem of realizing eigenstructure assignment through full state feedback can be described as: a given self-conjugate (if the conjugate of a complex variable from the set is also in this set, then this set is called a self-conjugate set) scalar set $\{\lambda_i\}_{i=1}^n$ and corresponding self-conjugate n-dimensional vector set $\{v_i\}_{i=1}^n$, find a m×n-dimensional real matrix K to make that the eigenvalue of the closed-loop system matrix A+BK is $\{\lambda_i\}_{i=1}^n$, the eigenvector is $\{v_i\}_{i=1}^n$ [5].

Assume that $\{\lambda_i\}_{i=1}^n$ is a set of self-conjugate complex eigenvalues and each value is different, $\{v_i\}_{i=1}^n$ is a set of self-conjugate vectors, if and only if for each i, the following conditions are satisfied,

(1) $\{v_i\}_{i=1}^n$ is a set of linear independent vectors in the complex domain Cn ;

(2) When $\lambda_i = \lambda_j^*$, $v_i = v_j^*$;

(3) $v_i \in span\{N_{\lambda_i}\}$;

Then there exists a m×n-dimensional real matrix, that $(A + BK)v_i = \lambda_i v_i$ is true, and when rank(B)=m, the matrix K is unique [5].

When a matrix K that satisfies the above conditions exists, then

$$K[v_1 \quad v_2 \quad \cdots \quad v_n] = [-M_{\lambda_1} z_1 \quad -M_{\lambda_2} z_2 \quad \cdots \quad -M_{\lambda_n} z_n] \tag{5}$$

If each λ_i is a real number, then v_i and z_i are real vectors, when the eigenvector matrix is not singular, then

$$K=\left[-M_{\lambda_1}z_1 \quad -M_{\lambda_2}z_2 \quad \cdots \quad -M_{\lambda_n}z_n\right]\left[v_1 \quad v_2 \quad \cdots \quad v_n\right]^{-1} \tag{6}$$

If λ_i is a complex number, the above formula needs to be adjusted slightly. Assume $\lambda_1=\lambda_2^*$, then $v_1=v_2^*$, $z_1=z_2^*$, thus

$$K\left[v_{1R}+jv_{1I} \quad v_{1R}-jv_{1I} \quad \cdots \quad v_n\right]=\left[w_{1R}+jw_{1I} \quad w_{1R}-jw_{1I} \quad \cdots \quad -M_{\lambda_n}z_n\right] \tag{7}$$

where $w_{1R}+jw_{1I}=-M_{\lambda_1}z_1$, $w_{1R}-jw_{1I}=-M_{\lambda_2}z_2$, the subscripts R and I respectively indicate the real and imaginary parts.

Right-multiply the following non-singular matrix at both ends of the above formula

$$\begin{bmatrix} 1/2 & -j1/2 & 0 \\ 1/2 & -j1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{8}$$

then

$$K=\left[w_{1R} \quad w_{1I} \quad w_3 \quad \cdots \quad -M_{\lambda_n}z_n\right]\left[v_{1R} \quad v_{1I} \quad v_3 \quad \cdots \quad v_n\right]^{-1} \tag{9}$$

Fixed eigenvalues and eigenvectors uniquely determine A+BK，thus rank(B)=m, the matrix K is unique.

2.3 Decoupling Control Law Design for Oblique-wing Aircraft based on Model Following

Model following technique is one of the important methods in the field of flight control design. Its essence is to make the aircraft fly like a reference model (or an ideal model) with the desired flight quality [7]. For decoupling control of oblique-wing aircraft, the key point is to select a reference model that is decoupled. The theory of model following is introduced as follows.

First, define the state space model of the continuous linear time-invariant controlled plant as

$$\begin{aligned} \dot{x}_p &= A_p x_p + B_p u_p \\ y_p &= C_p x_p \end{aligned} \tag{10}$$

where $A_p \in R^{n\times n}$, $B_p \in R^{n\times m}$, and they are constant matrices, xp is n-dimensional state variable, up is m-dimensional control input, yp is r-dimensional output.

Second, suppose the state space model of the reference model is

$$\begin{aligned} \dot{x}_m &= A_m x_m + B_m u_m \\ y_m &= C_m x_m \end{aligned} \tag{11}$$

where $A_m \in R^{n\times n}$, $B_m \in R^{n\times m}$, $C_m \in R^{r\times n}$, and they are constant matrices, xm is n-dimensional state variable, um is m-dimensional control input, ym is r-dimensional output.

Furthermore, a quadratic performance index function is introduced [8],

$$J=\frac{1}{2}\int_0^\infty \left[\left(y_p-y_m\right)^T Q\left(y_p-y_m\right)+u_p{}^T R u_p\right]dt \tag{12}$$

Generally, $C_p \equiv C_m = I$, then the above formula can be rewritten as

$$J=\frac{1}{2}\int_0^\infty \left[\left(x_p-x_m\right)^T Q\left(x_p-x_m\right)+u_p{}^T R u_p\right]dt \tag{13}$$

where the weighting matrix $Q\in R^{n\times n}$ is a positive semi-definite symmetric matrix, $R\in R^{m\times m}$ is a positive definite symmetric matrix.

Model following is to find a control input $u_p(\bullet)$ to minimize the performance index J. That is, for a reference input um, xp is infinitely close to xm, yp is infinitely close to ym, and at the same time the control energy consumption is the least. This method is also known as linear quadratic optimal tracking [9].

Considering different forms of control input $u_p(\bullet)$, model following can be divided into the explicit model following and the implicit model following techniques, as shown in Figure 1 and Figure 2 respectively.

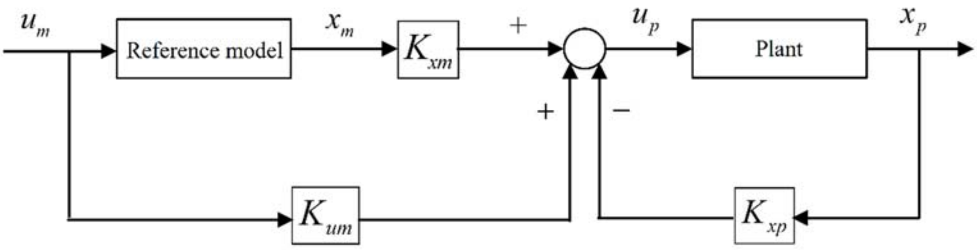

Figure 1 – Explicit model following

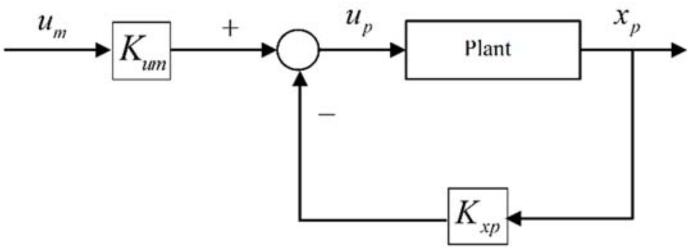

Figure 2 – Implicit model following

Explicit model following has fewer constraints on the reference model, but the selection of each element of the weighting matrix is more difficult, requiring certain engineering practical experience and iterations. Implicit model following has more restrictions on the reference model, however, it is simple to process and the error is relatively low. This paper focuses on decoupling control based on implicit model following. The control law structure does not include a reference model [3], described as below,

$$u_p = -K_{xp}x_p + K_{um}u_m \tag{14}$$

where K_{um} is the feedforward matrix, K_{xp} is the feedback matrix.

The implicit model following is difficult to transform into a kind of optimal regulator, so the solution is more complicated. A simplified solution is established [10].

Combining equations (10) and (14), the controlled closed-loop system model is given below.

$$\dot{x}_p = \left(A_p - B_pK_{xp}\right)x_p + B_pK_{um}u_m \tag{15}$$

Comparing equation (15) with equation (11), we can see that if there is

$$A_p - B_pK_{xp} = A_m \tag{16}$$

$$B_pK_{um} = B_m \tag{17}$$

the control matrix can be obtained from the following formula

$$K_{xp} = B_p^+\left(A_p - A_m\right) \tag{18}$$

$$K_{um} = B_p^+B_m \tag{19}$$

The optimal tracking that satisfies equations (16) and (17) is called complete implicit model following.

3. Demonstration

Take F-8 OWRA as an example to demonstrate the theories built before. The flight condition is that the wing inclined angle is 45 degree, Mach number is 0.8, and pressure altitude is 6096 m. The state is high subsonic speed, medium oblique angle, and medium altitude, which is quite representative. The flight dynamics model used in the form of section 2.1 is taken from reference [11].

First is the design of decoupling control law based on eigenstructure assignment. Table 1 shows a set of ideal eigenvalues that meet the requirements of the flight quality specification for typical modes. Compared with the eigenvalues of the aircraft, the damping of each mode and the frequency of periodic modes are improved.

Table 1 – Ideal eigenvalues (eigenstructure assignment)

NO.	Mode	Eigenvalues of aircraft	Ideal eigenvalues
1	(quasi) long period	- 0.0072±0.0503*i*	- 0.5±1.0*i*
2	(quasi) short period	- 1.0936±2.8360*i*	- 4±4.0*i*
3	(quasi) spiral	- 0.0047	- 0.8
4	(quasi) roll	- 2.6550	- 6.0
5	(quasi) Dutch roll	- 0.5722±3.4151*i*	- 5.0±6.0*i*

The feedback matrix K is calculated according to the principles in section 2.2, and the values of each element are shown in Table 2.

Table 2 – Feedback matrix K (eigenstructure assignment)

K_{ij}	ΔV	$\Delta\alpha$	q	$\Delta\theta$	$\Delta\beta$	p	r	φ
δ_{eL}	0.02	-20.20	0.06	6.75	2.15	0.04	-0.49	0.04
δ_{eR}	-0.01	11.97	0.49	-1.28	-2.42	0.00	0.49	-0.01
δ_{aL}	-0.29	61.42	2.97	-60.91	4.82	-0.16	0.09	-0.28
δ_{aR}	-0.26	18.10	2.46	-55.32	1.35	0.10	-0.04	0.07
δ_r	0.01	-8.37	-0.14	2.62	-6.01	-0.06	1.04	-0.06

The key to the design of the decoupling control law based on model following lies in the selection of reference matrix A_m and B_m. In addition to satisfying the longitudinal and lateral separation, it should also improve the aircraft's longitudinal static stability, yaw static stability, pitch damping, roll damping and yaw damping, while reducing the elevator's lateral control efficiency and increasing the aileron's lateral control efficiency. The control law is built according to Section 2.3, and the feedback matrix K_{xp}, the feedforward matrix K_{um} and the corresponding closed-loop system are obtained. Table 3 and Table 4 list the elements of K_{xp} and K_{um} respectively.

Table 3 – Elements of Kxp (model following)

K_{ij}	ΔV	$\Delta\alpha$	q	$\Delta\theta$	$\Delta\beta$	p	r	φ
δ_{eL}	-0.0020	-1.3460	-0.0591	0.0001	0.6170	-0.0255	0.1820	0.1055
δ_{eR}	0.0012	0.4681	-0.2082	-0.0001	-0.3843	-0.0029	-0.1226	-0.0718
δ_{aL}	0.0044	-0.7411	0.3654	-0.0001	-3.8027	0.0270	-0.6858	0.3362
δ_{aR}	-0.0004	-6.3160	0.6016	0.0002	-1.8508	-0.1010	0.2058	0.6607
δ_r	-0.0007	-0.0568	0.0374	0.0000	0.4450	0.0050	-0.4786	0.0339

Table 4 – Elements of Kum (model following)

K_{ij}	δ_{eLm}	δ_{eRm}	δ_{aLm}	δ_{aRm}	δ_{rm}
δ_{eL}	0.7270	0.2662	0.1230	0.1222	-0.3888
δ_{eR}	0.0919	0.4946	-0.2331	0.0606	0.3142
δ_{aL}	0.1790	0.1040	2.1200	-0.6940	0.6278
δ_{aR}	0.1007	0.3000	0.7315	1.5288	-0.1470
δ_r	0.2021	-0.1083	0.2040	-0.1294	0.8733

The key indexes comparison of the closed-loop system after decoupling control is shown in Table 5. D_1 and D_2 are Type I and Type II coupling indicator, which respectively represent the degree of state-mode coupling and input-mode coupling. The specific calculation method is detailed in Reference [4]. K represents the feedback matrix of each control law, max(|K|) and || K || are the maximum element value and norm of matrix K respectively. $k_2(V)$ is the modal robustness.

Table 5 – Comparison of decoupling control laws

Method	D_1	D_2	max(|K|)	|| K ||	$k_2(V)$
Eigenstructure assignment	0.0000	1.0164	61.42	104	339
model following	0.5313	1.4195	6.32	7.04	1100

Before and after decoupling using the technique of model following, with the elevator deflecting symmetrically by 1 degree, the responses of the aircraft are shown in Figure 3 and Figure 4 respectively.

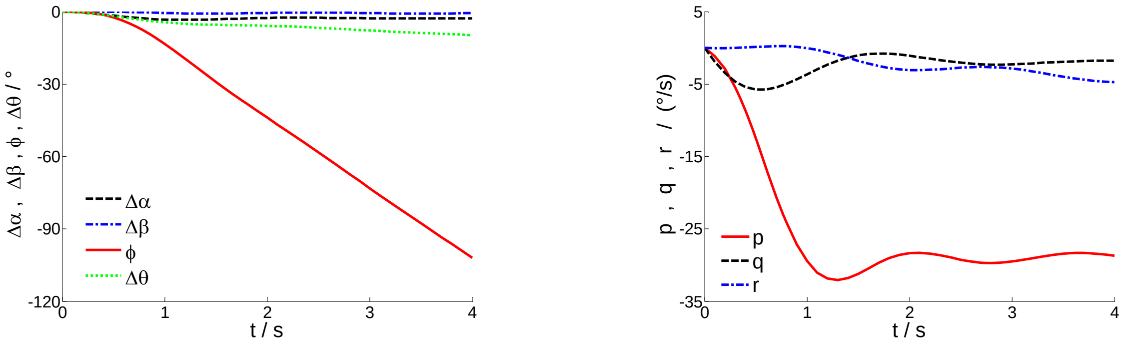

Figure 3 – Responses to elevator deflecting symmetrically by 1 degree (before decoupling)

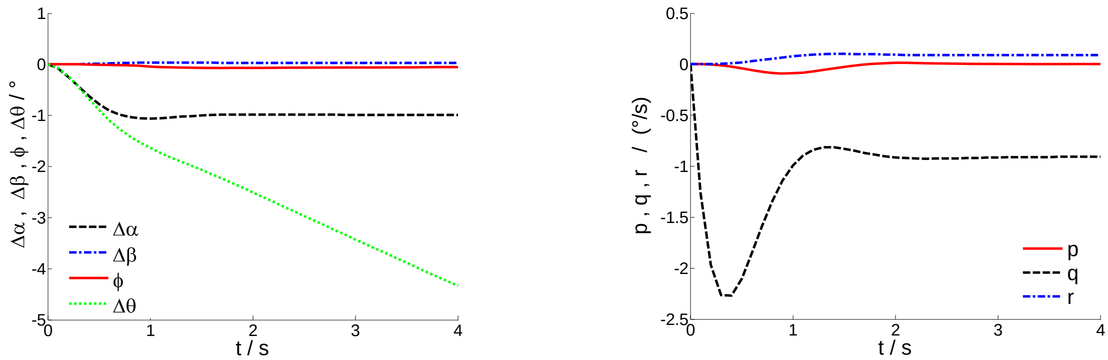

Figure 4 – Responses to elevator deflecting symmetrically by 1 degree (after decoupling)

Based on the above analysis, the following conclusions can be drawn:
(1) Eigenstructure assignment can accurately realize the longitudinal and lateral modal decoupling;
(2) Although the model following method does not completely realize modal decoupling, it basically eliminates the coupling of longitudinal-lateral motion, and especially to a large extent reduces the lateral motion caused by longitudinal control;
(3) The max(|K|) and || K || of the model following decoupling feedback matrix are far smaller than

the corresponding values of the eigenstructure assignment method, so the corresponding closed-loop system has lower energy consumption;

(4) The modal robustness of the eigenstructure assignment decoupling control is better than that of model following.

4. Conclusions

Considering the decoupling control demand of the oblique-wing aircraft, with the simplified small-perturbation linear flight dynamics model, this paper establishes modal decoupling control based on eigenstructure assignment and dynamic decoupling control based on model following and demonstrates the methods through a case study of F-8 OWRA with the wing inclined angle being 45 degree. The results show that the two decoupling control laws can achieve the expected goals and improve the flight quality of oblique-wing aircraft to a certain extent. Eigenstructure assignment can accurately realize the longitudinal and lateral modal decoupling. Model following could easily eliminate the coupling of longitudinal-lateral motion.

Furthermore, combination of eigenstructure assignment and model following shall be explored to simultaneously achieve decoupling and dynamic decoupling, to make oblique-wing aircraft better meet the requirements of the existing flight quality specifications.

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