# FUEL STARVATION CAUSED BY HUB CLOSURE 

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#### Abstract

Many airlines run their home base as a hub airport to enable the efficient transfer of connecting passengers, causing several inbound and outbound peaks during the day with the airport operating near its maximum capacity. Disturbances, such as thunderstorms or blocked runways, however, can greatly interfere with the schedules. In those situations, airport capacity reduces greatly, sometimes the entire airport has to be closed. The inbound traffic has to be diverted to nearby, usually smaller airports. If a disruption occurs during a heavy inbound situation, the diverted traffic may not be easily handled by the diversion airports due to their restricted capacity, causing possible fuel starvation onboard those aircraft that have to hold until a landing slot becomes available. This study will demonstrate an approach to determine such probability of fuel starvation and and show the effects of additional fuel that can be taken on board prior departure.


## 1 Setting

Large airports are typically run as hub airports with a large number of connecting passengers, resulting in several inbound and outbound traffics peaks during the day. In case of disruptions, the airport capacity can decrease greatly, causing significant delays. Disruptions during the operation of an airport can be caused by various factors. Severe weather situations, i. e. thunderstorms, especially if located in the final approach path or just above the airfield can close down an entire airport because no circumnavigation is possible. Incidents occurring at the airport cannot only shut down a single runway, e.g. in case of a runway excursion, but also an entire airport. If, for example, the fire department is fully involved in the recovery of an incident, flight operations have to be suspended.

In the case of an airport shutdown, the incoming traffic has to be diverted to nearby airports. This si-


Fig. 1 Distribution of allocated inbound slots at Munich Airport during one day [1]
tuations is more severe if the closed airport is a hub airport with flight-plan-defined inbound and outbound peaks and the closure occurs during an inbound peak. Figure 1 shows the planned arrivals of Munich airport (EDDM/MUC) during one day for the summer schedule of 2016. The horizontal scale shows the local time between 02:00 and 22:00. Night restrictions apply between $22: 30$ and 04:00. One can clearly see the five inbound peaks that are distributed over the day. During each of these peaks, the airport is operating at its maximum capacity of nearly 60 arrivals, as indicated by the red line. In addition, nearby airports, such as Nuremberg or Stuttgart, are typically smaller and have lower capacities. Stuttgart (EDDS/STR) with one runway, for instance, can handle up to 32 inbound aircraft per hour [2], which is approximately half of Munich. If a large number of aircraft diverts to one of these airports, congestions are likely to occur, resulting in aircraft entering holdings and additional fuel burn. Ultimately, the fuel on board might not be sufficient to sustain a safe flight until a landing slot becomes available.

A similar situation already occurred in 2012 when Madrid airport closed down due to thunderstorms. A large amount of inbound traffic was diverted to Valencia, resulting in two aircraft landing near their final reserve of 30 minutes of flight time and two aircraft


Fig. 2 Approach scenario
landing even below the final reserve that is required by law [3].

## 2 Objectives

The objective of this work is to develop a tool that is able to compute the probability $p_{\text {starvation }}$ of any aircraft running out of fuel if the destination airport is suddenly closed, with aircraft diverting to nearby alternates. A changeable parameter is the amount of extra fuel taken on board the aircraft. The tool, when fully developed, will thus provide safety managers the probability of fuel starvation connected with the information about how extra fuel, which always means extra costs, can reduce the risk.

## 3 Model

### 3.1 Approach scenario

Each approach scenario consists of a list of aircraft that are currently approaching the destination airport. The important properties of each aircraft include the position and the wake turbulence category. The interest radius is set to 200 km . We assume that, if the aircraft is located more than 200 km away from the destination airport and is informed about the closure, it will immediately land at nearby airports before reaching the area of the destination. From the initial point of the simulation, the aircraft will start flying to their respective alternates.

As shown in figure 2, the number of approaching aircraft is determined first, then the distance $r_{i}$ of each

|  | Preceeding aircraft |  |  |
| :--- | :--- | :--- | :--- |
| Succeeding A/C | Medium | Heavy | Super |
| Medium | 3 | 5 | 7 |
| Heavy | 3 | 4 | 6 |
| Super | 3 | 3 | 3 |

Table 1 Minimum wake turbulence separation for different categories [5] in nautical miles
aircraft and their respective azimuth angle $\psi_{i}$ to the destination (Dest) is created accordlingly for the initialization. The flying times to both the destination and alternate airports are determined using a method described in the following section 3.4.

### 3.2 Alternate selection

The selection of the alternate (Alt) for each aircraft is solely based on the remaining distance to it, i.e. each aircraft will choose the closest alternate to land. Other factors, such as the availability of crew or passenger handling facilities, are not taken into account.

### 3.3 Landing sequence

At each alternate airport, aircraft land following a first-come-first-served principle. If several aircraft arrive at the airport in a short period of time, the subceeding aircraft can only land if there is sufficient separation to the previous aircraft. The separation depends on the wake turbulence category of both aircraft. The minimum wake separation is defined by the ICAO and shown in table 1. It specifies the minimum distance between two aircraft on final approach, provided in nautical miles. It depends on both the wake turbulence category of the preceeding aircraft as well as the one of the following aircraft.

The wake turbulence category will thus heavily influence the airport's capacity and delays of the following aircraft that are approaching.

### 3.4 Approach modeling

Figure 3 shows 28 flights approaching Munich airport whose trajectories could be obtained from radar data.

When looking at the track distance vs. altitude plot, one can see that the last part on the final approach is identical for all flights. For some flights, the altitude stays constant for a certain period of time during the approach. However, some flights were able to perform a continuous descend approach. It becomes clear that approaches can vary heaviliy and the objective should


Fig. 3 Data of 28 real flights


Fig. 4 Dependency of the indicated airspeed over altitude
be to set up a reference that describes an average approach.

Common procedure during approach is to maintain a given indicated airspeed (IAS) which is changed just a few times by instruction of the air traffic controller. As the aircraft is descending, the true airspeed (TAS) continuously decreases as the air density becomes higher.

In our method, the approach is modeled as four different segments. Each segment $i$ is flown with a fixed IAS value which is referred to as $v_{i}$. The TAS then decreases continuously. The segments are defined by fixed altitude values $h_{i}$. When the altitude of the aircraft $h$ decreases below one of the defined altitudes $h_{i}$, the IAS is adjusted to a new given constant value $v_{i}$. In order to avoid a sudden discontinuous change of the aircraft's speed, the IAS over altitude function is not a step function, it is rather modeled using a tangens hyperbolicus function. On the one hand, the use of a tanh function better describes the aircraft's dynamics as the speed can only change continuously. On the other hand, it also provides a mathematically smooth model of the aircraft's kinematics. The IAS as a function of the altitude of the aircraft, which is shown in figure 4 can be mathematically described as:

$$
\begin{equation*}
v_{I A S}(h)=v_{0}+\sum_{i=1}^{n}\left(v_{i+1}-v_{i} \frac{\tanh \left[\alpha\left(h-h_{i}\right)+1\right]}{2}\right) \tag{1}
\end{equation*}
$$

$v_{i}$ describes the different possible discrete values for the IAS depending on the altitude, with $n$ being the total number of different values while $h_{i}$ are the given altitudes at which the changes of IAS occur. $\alpha$ is a parameter for the tanh function, it affects the smooth-
ness of the edges. The flight path angle is assumed to be constant.

The only constraint that the aircraft has to fulfill becomes relevant when the aircraft reaches the final approach. The very last part is almost identical among all flights with the aircaft being stabilized on the Instrument Landing System (ILS) approach, as one can see in figure 3. The IAS is therefore set to a fixed value of 140 knots ( $72 \frac{\mathrm{~m}}{\mathrm{~s}}$ ) if the aircraft descends below an altitude of 2300 feet ( 701 m ) above the airport elevation. The flight path angle is then also set to a fixed value of $3^{\circ}$, which is a standard value for an ILS approach.

### 3.5 Model parameters

As described in section 3.4, the approach trajectory of each flight is modeled using piece-wise constant IAS that only changes if certain given altitudes are passed. There are three different altitudes at which the IAS should change. Along with the flight path angle before reaching the final approach segment, it results in a total of seven model parameters that have to be determined using real data.

- Three altitudes at which the IAS changes occur $h_{1}, h_{2}$ and $h_{1}$, indicated in figure 4 as the altitudes at which the steps can be seen
- Three IAS values $V_{1}, V_{2}$ and $V_{3}$, also as indicated in figure 4
- Flight path angle before reaching the final approach $\gamma$

The final approach speed is set to a fixed value as well as the final flight path angle. The airport elevation is referred to as $h_{0}$.

The initial guesses $\bar{h}, \bar{v}$ and $\bar{\gamma}$ are chosen to be:

$$
\begin{gather*}
\bar{h}=\left[\begin{array}{c}
h_{0}+2300 \mathrm{ft} \\
6000 \mathrm{ft} \\
10000 \mathrm{ft}
\end{array}\right]=\left[\begin{array}{c}
h_{0}+701 \mathrm{~m} \\
1829 \mathrm{~m} \\
3048 \mathrm{~m}
\end{array}\right]  \tag{2}\\
\bar{v}=\left[\begin{array}{c}
170 \mathrm{kts} \\
250 \mathrm{kts} \\
290 \mathrm{kts}
\end{array}\right]=\left[\begin{array}{c}
87 \\
129 \\
149
\end{array}\right] \frac{\mathrm{m}}{\mathrm{~s}}  \tag{3}\\
\bar{\gamma}=2^{\circ} \tag{4}
\end{gather*}
$$

Model calibration is run using the 28 real approaches on Munich airport. For this particular purpose of determining the arrival time of each aircraft at their
respective alternate, the cost function $C$ will only use the difference between the simulated flight time $\tilde{t}_{i A c}$ and the real flight time $t_{i A c}$ for each aircraft $i_{A c} . \tilde{t}_{i A c}$ is obtained by simulating the entire approach with the parameters $v, h$ and $\gamma$ until touchdown of each aircraft. $n_{A c}$ is the total number of aircraft in the scenario.

$$
\begin{equation*}
C=\left\|\sum_{i_{A c}=1}^{n_{A c}}\left(\tilde{t}_{i A c}-t_{i A c}\right)\right\| \tag{5}
\end{equation*}
$$

Since a model with given parameters will always deliver the same flight trajectory for the same initial conditions as it is a deterministic model, it is impossible to fit the model such that is fits perfectly for each and every flight. A least square cost function is therefore not suitable. However, the cost function is designed so that the average flight time among all flights fits to the model.

The model calibration returns the following parameter values:

$$
\begin{gather*}
h_{\text {opt }}=\left[\begin{array}{c}
h_{0}+2490 \mathrm{ft} \\
6822 \mathrm{ft} \\
9846 \mathrm{ft}
\end{array}\right]=\left[\begin{array}{c}
h_{0}+759 \mathrm{~m} \\
2079 \mathrm{ft} \\
3001 \mathrm{ft}
\end{array}\right]  \tag{6}\\
V_{\text {opt }}=\left[\begin{array}{c}
173 \mathrm{kts} \\
255 \mathrm{kts} \\
295 \mathrm{kts}
\end{array}\right]=\left[\begin{array}{c}
89 \\
131 \\
152
\end{array}\right] \frac{\mathrm{m}}{\mathrm{~s}}  \tag{7}\\
\gamma_{\text {opt }}=2^{\circ} \tag{8}
\end{gather*}
$$

with $C$ being less than $10^{-12} \mathrm{~s}$. The obtained model parameters are close to the initial parameters and are located within a reasonable range. Figure 5 shows the 28 recorded flights along with the corresponding data generated by the model, for both the initial guesses as well as the calibrated parameters. The model with the calibrated parameters thus serve as a reference approach to this particular airport.

It should be mentioned that, as approaches to different airports can vary greatly, the calibrated model parameters should be used carefully and it should be kept in mind that these parameters might only be valid for the particular given airport for which data is available. It might be necessary to perform the calibration separately for each individual airport and subsequently obtaining different sets of parameters.

### 3.6 Fuel computation

When planning any flight, the amount of fuel that is taken on board before departure is computed according to the expected fuel consumption of the aircraft. The
fuel on board consists of many different parts, namely [6]:

Taxi fuel: Fuel required to taxi from the parking position to the take-off runway

Trip fuel: Fuel required from brake release until landing at the destination airport

Alternate fuel: Fuel required to fly from the missed approach point at destination to the planned alternate airport

Final reserve: Fuel required to fly for 30 minutes at 1500 ft above the alternate aerodrome.

Contingency: 5\% of trip fuel or 5 minutes of flying time, whichever is greater in order to compensate for unexpected conditions enroute

Extra fuel: Additional fuel that can be taken on board by decision of the flight crew

From the initial position of each aircraft at the beginning of the simulation, the flight time to both the destination as well as the alternate airport are obtained for each individual aircraft using the model described in section 3.4. We assume that each flight is performed as planned, i.e. the remaining amount of trip fuel onboard the aircraft in the beginning is exactly the fraction of the trip fuel that is required to fly to the destination airport from present position plus alternate fuel, contingency, final reserve and, if applicable, extra fuel.

## 4 Probability quantification

As the goal is to quantify the region that represents the area in which fuel starvations occur, the input distributions representing the airport operation has to be propagated through the model which was described above. From a mathematical perspective, the goal is to evaluate a multidimensional integral:

$$
\begin{equation*}
p_{\text {starvation }}=\iint_{\text {starvation }} p(\theta) \mathrm{d} \theta \tag{9}
\end{equation*}
$$

$\theta$ represents a vector of contributing factors that have influences on the outcome of whether fuel starvation occurs or not. $p(\theta)$ is the probability density function for a given set of contributing factors $\theta$. For now, a Monte Carlo simulation is used to propagate the distributions through the model. We can therefore obtain an estimate for the probability $\hat{p}_{\text {starvation }}$ by dividing the number of scenarios $n_{\text {starvation }}$ in which

| Parameter | Type | Parameters |
| :--- | :--- | :--- |
| \# of A/C | Weibull | $a=10, b=1.5$ <br> $\mu=9.0, \sigma=6.1$ |
| Azimuth mean <br> $\left[{ }^{\circ}\right]$ | Uniform | $[-180 ; 180)$ |
| Azimuth std <br> $\left[{ }^{\circ}\right]$ | Uniform | $[0 ; 180)$ |
| Distance mean <br> $[\mathrm{m}]$ | Weibull | $a=100000, b=1.5$ <br>  <br> Distance std <br> $[\mathrm{m}]$ Normal |
| $\mu=9.0 e 05, \sigma=6.1 e 05$ |  |  |

Table 2 Assumed input distributions for the simulation
fuel starvation occurs by the total number of considered scenarios $N$.

$$
\begin{equation*}
\hat{p}_{\text {starvation }}=\frac{n_{\text {starvation }}}{N} \tag{10}
\end{equation*}
$$

The input data consist of distributions of the traffic situation at the airport. Namely how the the aircraft are located around the airport at the begin of the situation, i.e. the time when the diversion to the alternate airports starts, expressed in distance and azimuth from the airport, both described by a mean and a standard deviation as well as the number of approaching aircraft.

## 5 Results

### 5.1 Input data

For the simulation to return useful results, real traffic data for the airport has to be obtained containing the distribution of approaching traffic over time. As these data is currently not fully available and this work aims at demonstrating the functionality of the method, the input distributions are based on assumption that are plausible. The results will, therefore, only refer to the assumed input data. However, real data can be inserted into the algorithm at any time in order to derive statements describing the real probabilities. The input distributions as well as their parameters and, if applicable, mean value and standard deviation used here are listed in table 2. The distributions and their parameters have been chosen to be plausible. Weibull distributions have been selected in several occasions since they offer a lower boundary, which is physically motivated for the number of aircraft and distance from destination, which cannot be smaller than zero for both.


Fig. 6 Buffer to final reserve - one alternate

Two different scenarios will be examined during this study. Both of them consider Munich airport (EDDM) as the destination hub airport that is to be closed. The first scenario calls for only one alternate airport to be available, which is Nuremberg airport (EDDN). The second scenario provides several alternates from which one has to be chosen for each flight as shown in table 3.

### 5.2 Propagation

Each scenario that is generated is simulated. Since the goal is to determine the probability of fuel starvation, the lowest remaining amount of fuel onboard over all aircraft is considered from each scenario. The flight time that is available by the amount of fuel on board at the begin of the simulation is compared to the actual flight time until the landing at the alternate airport, taking account waiting time due to traffic situation.

### 5.3 Scenarios

### 5.3.1 One alternate

In the first scenario, all approaching aircraft will be diverted to one single airport, which is Nuremberg. A total of 10000 approach scenarios based on the given input distributions were generated and simulated. The results for the simulated samples are shown in figure 6. It is displayed as time buffer to reaching a state of low fuel, which is set to be the legally defined final reserve of 30 minutes.

As indicated in figure 6, 16\% of the simulated samples contain flights that have already less fuel than the required final reserve at the time of touchdown. 11 samples have less than -1800 seconds ( -30 minutes),


Fig. 7 Influence of the number of approaching aircraft on the buffer to final reserve - one alternate

| Name | ICAO | Dist to EDDM [m] |
| :--- | :--- | :--- |
| Salzburg | LOWS | 1.11 e 5 |
| Nuremberg | EDDN | 1.37 e 5 |
| Linz | LOWL | 1.79 e 5 |
| Friedrichshafen | EDNY | 1.86 e 5 |
| Stuttgart | EDDS | 1.92 e 5 |
| Zurich | LSZH | 2.61 e 5 |

Table 3 Available alternate airports for scenario 2
which means that these flights have already run out of fuel after completely consuming the final reserve.

When looking at the influences of the contributing factors, one can see in figure 7 that, not surprisingly, the number of approaching aircraft has a great influence on the outcome of the approach scenario.

### 5.3.2 Several alternates

The available alternate airports are shown in table 3. The great circle distance to Munich airport is provided as well. In this scenario, each aircraft will choose the airport that is the closest to its current position as the alternate airport. Thus, the aircraft will be distributed to land at several airports instead of a single one.

The results for the scenario with several alternates available return higher buffer to final reserve values in general, which is consistent with the expectation, as seen in figure 8 . Only $4 \%$ of the samples contain flights that land below final reserve. Out of these, only one contain flights landing without any fuel at all.

The magnitude of the impact of the number of approaching aircraft onto the buffer to low fuel remains unchanged. However, one can see in figure 9 that the


Fig. 8 Buffer to final reserve - several alternates
spread has increased. This can be explained by the influence of the exact position of each aircraft. If more aircraft is added, the waiting time until landing will only increase if the aircraft are close to each other, resulting in choosing the same airport as alternate. If their initial position are far apart, they will not influence each other since different alternates are used. Taking the example shown before in figure 2, aircraft 1 will fly to alternate 3 while aircraft 2 will choose alternate 2 . If additional aircraft are added that are located close to aircraft 3 , they will most likely not affect the approach of aircraft 1 because they will most likely choose alternate 1 or 2 , explaning the larger spread in figure 9 .

### 5.4 Influences of additional fuel

In order to reduce the probability of low fuel, additional fuel can be taken on board prior departure. The decision is typically based on the forecast of the traffic situation and the weather forecast at the destination and alternate airport. For example, if thunderstorms are forecasted at arrival, delays will become more likely, typically additional fuel is then added. However, general company-specific recommendations about fuel policy can also be made regardless of the individual situation.

Based on the results obtained from both scenarios, the addition of extra fuel will simply shift the histograms in figure 6 and 8 towards the right. Assuming an extra fuel amount of 15 minutes for each aircraft, the number of samples with aircraft landing below final reserve would be reduced to $1.4 \%$ for the first scenario (one alternate) and $0.3 \%$ for the second scenario (several alternates).


Fig. 9 Influence of the number of approaching aircraft on the buffer to final reserve - several alternates

### 5.5 Discussion

$15 \%$ probability of having a fuel starvation for a single alternate and $4 \%$ for several alternates appear to be very large compared to the rare number of fuel starvation events actually happening. However, one has to keep in mind that the worst case scenarios is considered in the simulation. First, the simulation is run assuming that no extra fuel is taken on board each aircraft. This is rarely the case. Especially if disruptions during the approach is anticipated, e.g. due to bad weather, extra fuel is typically taken by the crew. As already shown in section 5.4 , the probability of landing below final reserve reduces to only $1.4 \%$ if each aircraft has 15 minutes of extra fuel. Second, the scenarios call for a complete closure of an airport over a fairly long amount of time so that landing is completely prohibited. This is a very rare event in the reality. Even heavy thunderstorms will sometimes only lead to a short closure of airports, sometimes a few approaches are still possible. Third, the scenario offering several alternate airports only considers six airports in the proximity, as shown in table 3 . However, depending on the situation and capability of the individual aircraft, there are several additional airports that can be chosen, thus further reducing the probability of any aircraft running low on fuel.

The real probabilities are thus much lower. However, real data have to be fed into the simulation in order to determine the real numbers. If the probabilities are small, Monte Carlo becomes increasingly inefficient as the computational costs is inverse proportional to the probability to be computed. Other methods, such as the subset simulation, can then be used as an
enhancement if necessary [7].

## 6 Conclusion and outlook

A method has been developed to analyze the traffic flow in case of a sudden closure of an airport with the approaching aircraft diverting to nearby airports. The method requires the distribution of traffic of this particular airport as input an returns the probability of any aircraft landing below the legally required final reserve by carrying out Monte Carlo simulations. The model that describes the movement of each individual aircraft during the approach was calibrated using available real data. The results can be used to derive recommendations to flight crews about taking additional fuel on board prior departure as indicated in section 5.4.

The next steps will contain the incorporation of real approach data containing information about the amount of traffic of a given airport over a certain period of time in order to use it as input for the algorithm.

Several factors have so far not been considered in the modeling. It is planned that their impact will be assessed and it will be decided whether it is necessary to extend the model. These factors include:

Incorporation of the weather situation: Currently the separation values are based on wake turbulence only. However, under certain adverse weather conditions with low visibility, the separation can be significantly increased, resulting to reduction of airport capacity. Weather data from each involved airport will be necessary.

## Approaching traffic at alternate airports:

Currently only the diverted aircraft will land at the alternate airports. However, these airports typically have regularly scheduled traffic as well that should be taken into account.

Suitability of airports: Not all alternate airports might be capable of accommodating any type of aircraft. Some airlines might have preferences concerning alternate airports due to operational reasons (e.g. passenger accomodation or maintenance facility). This will be an additional criteria for alternate search, which is currently solely based on the distance to the airports, if several alternates are available.

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