30th Congress of the International Council of the Aeronautical Sciences
DCC, Daejeon, Korea ; September 25-30, 2016

STRUCTURAL CONSTRAINTS INVESTIGATION IN WING AERODYNAMIC/STRUCTURAL OPTIMIZATION

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Keywords: *wing design, aero-structural optimization, multidisciplinary design optimization, KS function*

Abstract

For wing aero-structural optimization problem, a large number of stress constraints have to be considered and numerical difficulties in the optimization is caused. To overcome these difficulties and reduce the computation cost, this paper aims to find an efficient constraint aggregation method, in which the constraints are aggregated to one or few constraints. Four approaches are investigated, i.e., maximum constraint approach and constraint aggregation approaches using KS function (with constant parameter, adaptive parameter and aggregating constraints of different component respectively). An in-house surrogate-based optimization (SBO) toolbox called SurroOpt is used to tackle the problems associated with no-smooth constraint bound. The comparison of different approaches is carried out in both structural and aero-structural optimizations of a transport aircraft wing. The results show that, (1) adaptive constraint aggregation using KS function achieves the maximum weight reduction; (2) for constant aggregation method, more wing weight reduction can be achieved, when larger aggregation parameter is used; (3) the SBO is very efficient and robust, even when the curvature of the KS function where the active constraints intersect is very large

1 Introduction

In high-fidelity aero-structural optimization of wing, the structural model consists of thousands of shell elements, which means large numbers of stress constraints must be considered. For such a large-scale optimization problem, the computational cost of numerical optimization increases rapidly with the increase if number of constaints.

Surrogate modeling (such as Kriging model) is a promising method for wing optimizations [1]. Compared with the gradient-based algorithms, it can realize global search in the feasible region and it is more robust for complicated design space. However, when applied to wing optimization, its computational cost can be prohibitive if kriging models are built for the stress constraints of every structural element. Constraint aggregation by lumping large number of constraints into one might be an effective way to solve this problem. In Ref.[2], the constraint aggregation is applied to structural optimizations. In Ref.[3], different ways of lumping stress constraints were compared in the framework of a gradient-based algorithm and concluded that the proposed adaptive Kreisselmeier-Steinhauser (KS) function can obtain the minimum-weight design. This method was later applied in the aeroservoelastic design optimization of a flexible wing [4] and the multi-point high-fidelity aerostructural optimization of a transport wing [5].

This work aims to investigate how to introduce stress constraints into wing aero-structural optimization, based on the surrogate-based optimization framework. The remainder of the paper is organized as follows. In section 2, four approaches of aggregating stress constraints are presented, which are: (1) only the maximum stress is constrained [6]; (2) all of the stress constraints are lumped into one constraint via KS function [7] and then introduced into optimization; (3) lumping the constraints via the adaptive KS function proposed by Ref.[3]; (4) lumping the stress constraints w.r.t. the same component (i.e. the constraints of upper skin, lower skin, ribs, etc. are lumped respectively.). Section 3 presents the numerical analysis models as well as the optimization model and algorithm. The comparisons are made in both wing structural and aero-structural optimization problems. All of the optimizations are performed by using an in-house SBO toolbox, "SurroOpt" [1]. The last section gives the conclusions.

2 Constraint Aggregation

In this section, four approaches for stress constraint aggregation are introduced.

2.1 The Maximum Constraint Approach

The maximum constraint approach is the simplest method, in which only the most -violated stress constraint is considered, while the remained constraints are ignored. The optimization model is

$$\begin{aligned} &\min\ f(\mathbf{x}) \\ &\text{s.t. } \max\left\{\sigma_i(\mathbf{x}) \middle| i=1,\cdots,N\right\} \le [\sigma_b], \\ &\qquad \mathbf{g}(\mathbf{x}) \le 0 \end{aligned} \tag{1}$$

Where, the maximum among all stresses is constrained and **g**(**x**) denotes the other constraints for structure deformation and aerodynamic performance.

In gradient-based optimization, the method of considering the most violated constraint will lead to iteration oscillation, which is caused by a wrong search direction. Therefore, a gradient-free, surrogate-based optimization based on kriging model is used. However, parameter tuning for the kriging model of the most violated constraint probably will not improve fidelity of the corresponding constraint in the next iteration, which may increase the number of iterations.

2.2 Constraint Aggregation Using KS Function

The KS function was originally proposed by Kreisselmeier and Steinhauser in 1979 in [8]. It can produce an envelope surface that represents a conservative estimate of the maximum among the set of functions. When being used to aggregate constraints, the KS function can be defined as

$$KS\left[\mathbf{g}(\mathbf{x})\right] = \frac{1}{\rho}\ln\left[\sum_{j=1}^{NG} e^{\rho g_j(\mathbf{x})}\right], \tag{2}$$

where ρ is the aggregation parameter and **g** are the constraints. In order to prevent numerical overflow due to too large exponent, we use an alternative expression of

$$KS\left[\mathbf{g}(\mathbf{x})\right] = g_{\max}(\mathbf{x}) + \frac{1}{\rho}\ln\left[\sum_{j=1}^{NG} e^{\rho\left(g_j(\mathbf{x}) - g_{\max}(\mathbf{x})\right)}\right], \tag{3}$$

where $g_{\max}$ is the maximum among all the constraints evaluated at the current design point **x**.

For aero-structural optimization of wings, the optimization model based on KS aggregation method is defined as

$$\begin{aligned} &\min\ f(\mathbf{x}) \\ &\text{s.t. } KS\left\{\sigma_i(\mathbf{x}) \le [\sigma_b] \middle| i=1,\cdots,N\right\}, \\ &\qquad \mathbf{g}(\mathbf{x}) \le 0 \end{aligned} \tag{4}$$

where the KS function lumps all of stress constraints into a single constraint.

At a design point, the KS function value is bounded as

$$g_{\max}(\mathbf{x}) < KS\left[\mathbf{g}(\mathbf{x})\right] < g_{\max}(\mathbf{x}) + \frac{\ln \mathrm{NG}}{\rho}. \tag{5}$$

From Eq.(5), the following properties about KS function can be concluded.: (1) with the increase of ρ, the value of KS function goes closer to $g_{\max}$, the maximum among all the constraints. When ρ approaches infinity, the KS function becomes $g_{\max}$. (2) the KS function constructs a conservative estimation of the original active constraints as $g_{\max}$ must be less than 0 when KS[**g**(**x**)]<=0, which means a conservative optimum. The KS function defines a smaller feasible region than the true feasible region of using the original constraints.

The aggregation parameter, ρ, decides the approximation fidelity of the KS function, has impact one the convergence speed and the optimal results. If ρ is too small, the lumped constraint is farer from the original active constraint, which leads to a too conservative optimum. If ρ is too large, the curvature of the KS function where the active constraints intersect can be very large, which causes numerical difficulties in gradient-based optimizations due to an ill-conditioned Hessian matrix. However, this might not be a problem for the gradient-free SBO for the optimization does not rely on the gradient.

Fig. 1 gives an example of the KS function using constraint aggregation method. Three convex inequality constraints in Eq.(6), (7) and (8) are lumped and the KS function is shown as the blue curve in Fig. 1.

$$g_1(x) = 1 - x \quad (6)$$

$$g_2(x) = 2x^2 - 1 \quad (7)$$

$$g_3(x) = x^2 - 2x \quad (8)$$

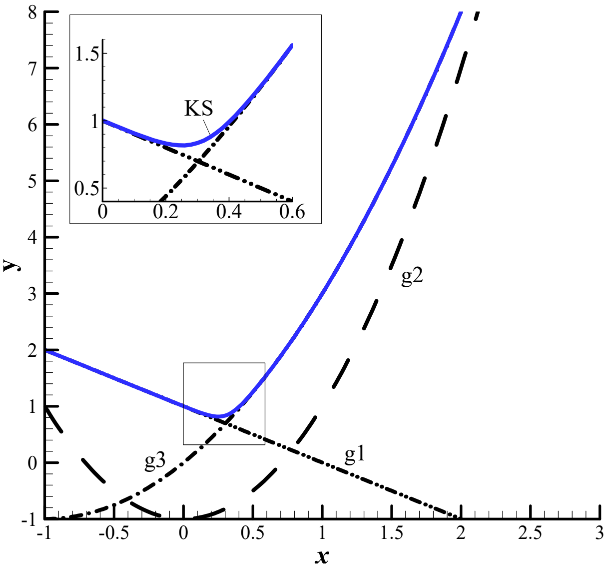

Fig. 1 KS function of three inequality constraints

2.3 Adaptive Approach for Constraint Aggregation

The original KS approach is non-adaptive as a predetermined aggregation parameter ρ keeps constant throughout the optimization. Ref [3] proposed increasing ρ as required so that a better approximation for the original constraints can be obtained. Here we will explore this approach in our surrogate-based optimization framework.

For the adaptive approach [3], ρ is increased according to the sensitivity of KS w.r.t. the aggregation parameter, KS'=dKS/dρ. At a design point, ρ is set to be the value at which KS' is relatively low (e.g. 10^{-6}), i.e., the function $KS(\rho)$ approaches it optimum value so that accuracy of the aggregation constraint cannot be improved any more by increasing ρ. This is the desired value, KS_d'.

Considering that logKS' varies linearly with the increase of logρ, the secant method is applied to logKS' in the scope of [logρ_1, logρ_d]. Then the following relationship was derived.

$$\frac{\log KS_1' - \log KS_c'}{\log \rho_1 - \log \rho_c} = \frac{\log KS_d' - \log KS_c'}{\log \rho_d - \log \rho_c} \quad (9)$$

where the subscript c denotes value at the current point and the subscript d denotes that when the desired KS' is achieved. The subscript 1 represents a value calculated at a finite step from the current point, $\rho_1 = \rho_c + \Delta\rho$.

By rearranging Eq.(9), the desired value, ρ_d, can be expressed as

$$\log \rho_d = \log\left(\frac{KS_d'}{KS_c'}\right)\left[\log\left(\frac{KS_1'}{KS_c'}\right)\right]^{-1} \log\left(\frac{\rho_1}{\rho_c}\right) + \log \rho_c \quad (10)$$

In Ref [3], KS' is calculated using complex-step derivative method. Here, we directly derive the expression of KS' by differentiating Eq.(3) on both sides.

$$\frac{dKS}{d\rho} = \frac{1}{\rho}\frac{\sum_{j=1}^{NG}(g_j - g_{\max})\exp(\rho(g_j - g_{\max}))}{\sum_{j=1}^{NG}\exp(\rho(g_j - g_{\max}))} - \frac{1}{\rho^2}\ln\left[\sum_{j=1}^{NG}\exp(\rho(g_j - g_{\max}))\right] \quad (11)$$

Assuming

$$S_1 = \sum_{j=1}^{NG}\exp(\rho(g_j - g_{\max}))$$

$$S_2 = \sum_{j=1}^{NG}(g_j - g_{\max})\exp(\rho(g_j - g_{\max})),$$

then Eq.(11) can be rearranged as

$$\frac{dKS}{d\rho} = \frac{1}{\rho}\frac{S_2}{S_1} - \frac{1}{\rho^2}\ln\left(S_1\right) \qquad (12)$$

2.4 Constraint Aggregation for Every Component

The KS function generates a conservative estimation of the constraints bound, which results in inaccuracies in the optimization results. From the right-hand side in Eq.(5), it can be concluded that these inaccuracies increase with the number of active constraints. Fig. 2 also shows such a trend of KS error increase with more active constraints.

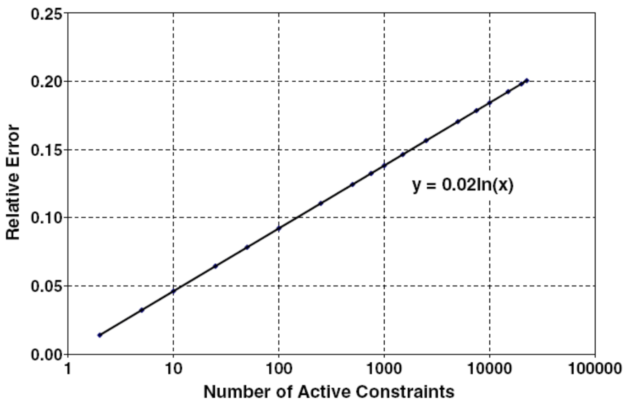

Fig. 2 Relative error in optimization result using the KS function with ρ=50[3]

If we can reduce the number of active constraints, the accuracies of KS functions might be improved. So we propose to lump the stress constraints for every component. Then the optimization model in Eq.(13) is established for wing aero-structural optimization.

$$\begin{aligned}
&\min\ f\left(\mathbf{x}\right)\\
&\text{s.t.}\ KS_{\text{upperskin}}\left\{\sigma_i\left(\mathbf{x}\right)\le\left[\sigma_b\right]\middle| i=1,\cdots,N_{\text{upperskin}}\right\}\\
&\qquad KS_{\text{lowerskin}}\left\{\sigma_i\left(\mathbf{x}\right)\le\left[\sigma_b\right]\middle| i=1,\cdots,N_{\text{lowerskin}}\right\}\\
&\qquad KS_{\text{spar}}\left\{\sigma_i\left(\mathbf{x}\right)\le\left[\sigma_b\right]\middle| i=1,\cdots,N_{\text{spar}}\right\}\\
&\qquad KS_{\text{rib}}\left\{\sigma_i\left(\mathbf{x}\right)\le\left[\sigma_b\right]\middle| i=1,\cdots,N_{\text{rib}}\right\}\\
&\qquad \mathbf{g}\left(\mathbf{x}\right)\le 0
\end{aligned} \qquad (13)$$

where the stress constraints related to the upper skin, lower skin, spar and rib are respectively aggregated.

Compared with the optimization model in Eq.(4), the number of active constraints for each aggregation constraint may be reduced, with a little penalty of optimization time as increased number of Kriging models have to be established for the constraints.

3 Results and Analysis

To evaluate different way of introducing large number of stress constraints, an aero-structural optimization study is conducted for a transport wing. The aerodynamic and structural analysis model is same as that in Ref.[6], which is alternately run to achieve a static aeroelastic convergence. The flanges of the spar and the stringers are combined with upper and lower skin as integral wallboard structure as illustrated in Fig. 3.

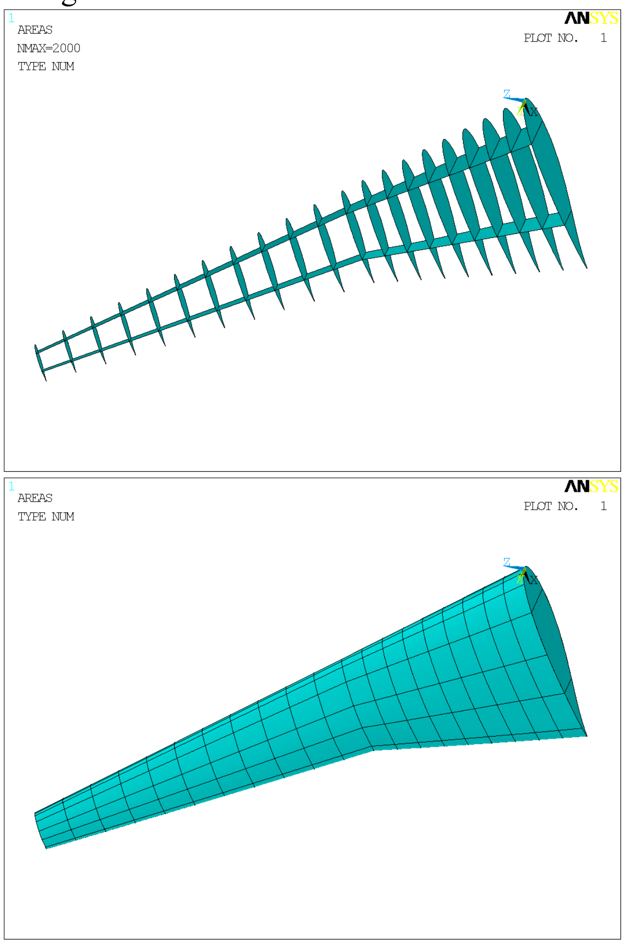

Fig. 3 Model of the wing structure (upper: model of ribs and spars, lower: model of skin)

The in-house surrogate-based optimization codes, SurroOpt [1], is used to perform the wing optimizations, the framework of which is shown in Fig. 4. SurroOpt has built-in modern DoE methods well suited for computer experiments, a variety of surrogate models (including quadratic response surface model, kriging, gradient-enhanced kriging, hierarchical kriging, radial-basis functions, etc.) and five infill sampling criteria. Numerous benchmark test problems as well as engineering design problems have been employed to test the code.

In this section, we firstly investigate how to aggregate the stress constraints only in the wing structural optimization. Then the further comparisons of different aggregation constraints will be made in the wing aero-structural optimization.

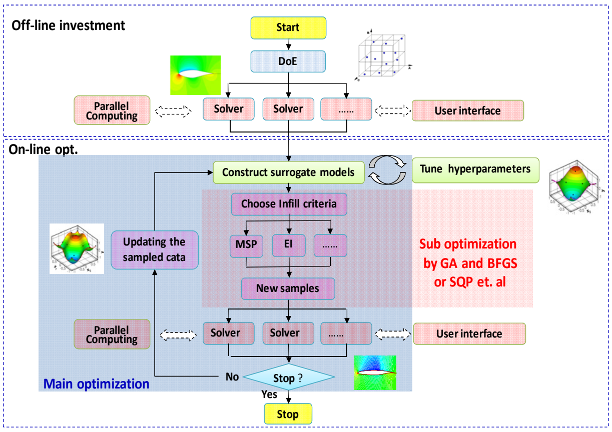

Fig. 4 Flowchart of SurroOpt, a generic optimization code

3.1 Structural Optimization

In the wing structural optimization, the design variables are sectional thickness of lower and upper skin of the wing, listed in Table 1. The objective function is the weight of the wing structure and the constraints are the stresses in each element and the wing-tip deformation. A safety factor of 1.5 is introduced into the stress constraints. The optimization problem can be stated in Eq.(14).

The optimization results are listed in Table 2. In the table, different approaches of constraint aggregation are abbreviated as numbers, e.g. #1, #2, and the explanations of each approach are given following the table.

$$\begin{aligned} &\min\ W_{\text{wing}}(\mathbf{x}) \\ &\text{s.t.}\ \ \boldsymbol{\sigma}(\mathbf{x}) \le [\sigma] \\ &\qquad \delta(\mathbf{x}) \le \delta_{\max} \\ &\qquad \mathbf{x} \in [\mathbf{x}_{\min}, \mathbf{x}_{\max}] \end{aligned} \tag{14}$$

Table 1. Design variables

Description	Number
Structural variables	
Lower skin thickness	10
Upper skin thickness	10
Total	20

For the constant-parameter KS approach, we investigated two values of the aggregation parameter: ρ=50 is a suggested reasonable value in the gradient optimization method [2][3][9], while ρ=1000 is our pre-evaluated value for the highest accuracy of constraint aggregation. Fig. 5 displays how the relative error of KS function decreases when increasing the aggregation parameter. We investigated the aggregation accuracy at three design points: x1 is the point at which the maximum of the stress constraints, g_max is close to zero; x2, at which g_max>0, i.e. the constraint is violated; x* is the optimum obtained by the approach #2.2, at which g_max<0. The figure indicates that 1000 is a reasonable value that KS function can aggregate the stress constraints at a higher accuracy for the wing structural optimization problem.

The iteration histories of optimizations using different constraint aggregation approaches are compared in Fig. 6. From Table 2 and Fig. 6 it can be concluded that:

(1) The adaptive KS constraint aggregation (#3) obtained the lowest weight. Then is the KS approach with a constant aggregation parameter. The weight obtained by the maximum constraint approach is highest, which is **22.9kg** heavier than the best result.

(2) For the constant aggregation parameter approach, an extra **6.1kg** weight reduction is achieved when increasing ρ from 50 to 1000. It indicates that the suggested value 50 by Refs.[2][3][9] is not good for our case as it produces a too conservative estimation of the constraint bound.

(3) Compare the approaches of aggregating all constraints into one constraint (#2.2 and #3) and aggregating the constraints of the same component into one constraint (#4.1, #4.2), the later approach can reduce more weight in the constant parameter case: with the

same ρ=1000, it had an extra 5.6kg weight reduction. It indicates that the approach #4.1 produces a more accurate estimate of the original active constraints envelope than the approach #2.2 with the same value of the aggregation parameter. However, with an adaptive aggregation parameter, it behaved worse than the case of aggregating all constraints into one, which might due to not so many active constraints, more approximation error incurred by increased surrogate models. From the results of the approaches #4.1 and #4.2 it can be seen that the stress constraint of the front spar and the deform constraint might become the active constraint bound near the optimum.

Table 2. Optimization results comparison

Description	Unit	#1	#2.1	#2.2	#3
W_{wing}	kg	1869.4	1862.0	1855.9	**1846.5**
$(\sigma-[\sigma])/[\sigma]$		-0.1609	-0.0221	-0.1424	-0.1121
$\delta-\delta_{\max}$	m	-0.0003	-0.0009	-0.0002	-0.0001
Description				#4.1	#4.2
W_{wing}	kg			1850.3	1856.2
$(\sigma_{\text{back spar}}-[\sigma])/[\sigma]$				-0.4232	-0.4111
$(\sigma_{\text{front spar}}-[\sigma])/[\sigma]$				-0.1064	-0.0937
$(\sigma_{\text{lower skin}}-[\sigma])/[\sigma]$				-0.2733	-0.3730
$(\sigma_{\text{upperskin}}-[\sigma])/[\sigma]$				-0.1577	-0.2165
$(\sigma_{\text{rib}}-[\sigma])/[\sigma]$				-0.7206	-0.7099
$\delta-\delta_{\max}$	m			-0.0001	-0.0180

#1: Maximum constraint approach
#2: Constraint aggregation using KS function with constant aggregation parameter, ρ. (**#2.1**: ρ=50; **#2.2**: ρ=1000)
#3: Constraint aggregation using adaptive KS function proposed by Ref.[3]
#4: Constraint aggregation w.r.t. the same component, ρ. (**#4.1**: ρ=1000; **#4.2**: adaptive ρ)

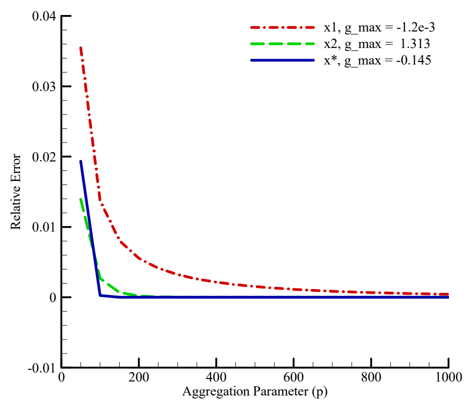

Fig. 5 Relative error of KS function

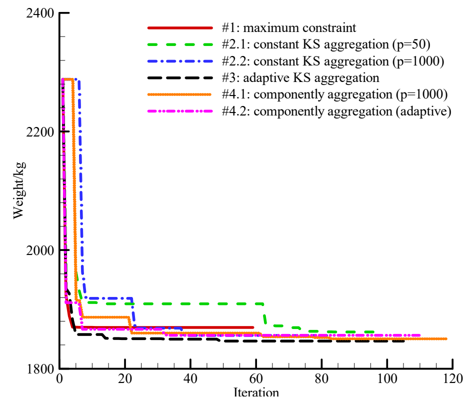

Fig. 6 Comparison of convergence histories of wing weigh for the structural optimization case

3.2 Aero-Structural Optimization

In this sub-section, different approaches of constraint aggregation are investigated in the wing aero-structural optimization. Besides the design variables for structural sizing as those in Table 1, another 4 variables defining the wing geometry and 1 variable for angle of attack are appended, which are listed in Table 3. The objective function is the weight of the wing structure and the constraints are the stresses in each element and the wing-tip deformation. In addition, the lift and lift-to-drag ratio must be no less than the prescribed value and the wing geometry is constrained to make sure enough wing area. The optimization problem is written as follows:

$$\begin{aligned}
\min\ & W_{\text{wing}}(\mathbf{x}) \\
\text{s.t.}\ & \boldsymbol{\sigma}(\mathbf{x}) \le [\sigma] \\
& \delta(\mathbf{x}) \le \delta_{\max} \\
& L(\mathbf{x}) \ge L_{\min} \\
& L/D(\mathbf{x}) \ge (L/D)_{\min} \\
& S_{\text{ref}}(\mathbf{x}) \ge S_{\min} \\
& \mathbf{x} \in [\mathbf{x}_{\min}, \mathbf{x}_{\max}]
\end{aligned} \tag{15}$$

Table 3. Design variables

Description	Number
Geometry variables	
Span	1
Taper ratio	1
Twist	1
Sweep	1

Aerodynamic variables	
Angle of attack	1
Structural variables	
Lower skin thickness	10
Upper skin thickness	10
Total	25

The optimization results are listed in Table 4. In the table, different approaches of constraint aggregation are abbreviated as numbers, e.g. #1, #2, and the explanations of each approach are given following the table. According to the preliminary studies in section 3.1, only the results of the maximum constraints approach and the constraint aggregation approach using the adaptive KS function are compared. The iteration histories are compared in Fig. **6**. It can be concluded that, compared with only considering the maximum constraint, the optimization of lumping the constraints using the adaptive KS function can reduce weight much more, i.e. **270.6kg**. That is to say, how to introduce the stress constraints will largely affect the optimization results.

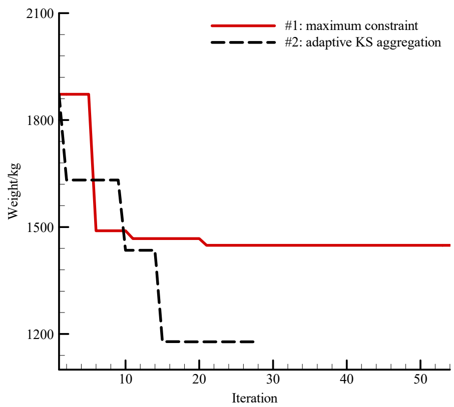

Fig. 7 Comparison of convergence histories of wing weigh for the aero-structural optimization case

Table 4. Optimization results comparison

Description	Unit	#1	#2
Span	m	31.04	29.36
Taper ratio		0.20	0.20
Twist	º	-2.99	-2.76
Sweep	º	27.84	29.67
Angle of attack	º	0.63	1.10
W_{wing}	kg	1448.6	**1178.0**
$(\sigma-[\sigma])/[\sigma]$		-0.003	-0.064
$\delta-\delta_{max}$	m	-0.035	-0.005
L-L_{min}	10^4kg	-0.042	-0.128
L/D		-0.110	-0.020
$S-S_{ref}$	10^2m^2	-0.081	-0.004

#1: Maximum constraint approach
#2: Constraint aggregation using adaptive KS function proposed by Ref.[3]

4 Conclusions

Lumping constraints into one or few constraints were investigated since aero-structural optimization of wings with large number of stress constraints will cause prohibitive computational cost and numerical difficulties. Based on our in-house surrogate-based optimization code, we investigated different constraint aggregation approaches via structural and aero-structural optimization of a transport aircraft wing. The results show that:

(1) With the adaptive constraint aggregation using KS function, the results are much better than that only considering the maximum constraint.
(2) With a constant parameter, we found that the suggested value 50 by some references is not good for the wing aero-structural optimizations to estimate the constraint bound accurate enough. Increasing the aggregation parameter can improve the optimization result.
(3) The surrogate-based optimization is very efficient and robust for wing aero-structural optimizations, even when the curvature of the aggregated constraint where the active constraints intersect is very large.

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