STATISTICAL STABILITY AND CONTROL ANALYSIS TO INVESTIGATE THE CONTROL EFFORT OF IMPROVING HANDLING QUALITY CHARACTERISTICS

Derya Aksaray , Dimitri Mavris
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Keywords: *stability analysis, stochastic root locus, handling qualities*

Abstract

This paper presents a statistical stability and control analysis that takes into account the parameter uncertainty on the state-space representation of a system. The aim of the statistical analysis is to investigate how much control effort is required to improve the handling quality characteristics of a particular system during early design stages. In this context, this study enables to design a robust system from a control perspective by introducing an expression for the control effort. In this research, the control effort of a system is quantified by an evaluation criterion including the variability of the feedback gain matrices. The gain matrices in the statistical analysis are computed by a fixed notional control architecture, which is referred to as an evaluative controller in this paper. The proposed study is implemented on the longitudinal and lateral dynamics of a notional aircraft similar in size to a Boeing 747, and the results are presented.

1 Introduction

Accomplishing a desired mission strongly depends on the handling quality (HQ) characteristics of an aircraft. These characteristics are impacted by the geometry (in particular the tail and wing sizing) and controller of an aircraft. The literature presents many studies that mention one of these two ways to evaluate HQ characteristics during design. For instance, Mavris et al. [1] proposed to create some surrogate models for the stability derivatives of a high speed civil transport aircraft with respect to its design variables. They assessed the HQ characteristics in a probabilistic manner by varying the design variables. However, they did not focus on the controller design to improve the HQ characteristics. Chudoba and Smith [2], and later Perez et al. [3] introduced a control-oriented aircraft design procedure along with a controller design; however, they did not conduct any probabilistic analysis to account for uncertainty in the system.

A stochastic root locus analysis is a statistical stability and control (S&C) analysis technique that provides information pertaining to the effects of parameter uncertainty on system stability [4]. The characteristic equation of a system is varied, and a Monte Carlo analysis is conducted to provide the density of the system roots. This analysis provides insight into the probability of instability or poor performance [9]. For instance, Stengel [10] studied the effects of aerodynamic uncertainty on the lateral-directional stability of an aircraft, conducting a Monte Carlo analysis to investigate the probability of instability.

This paper is inspired by Stengel's study [10] and presents a statistical S&C analysis to investigate the variations in the controller gains that enhance the HQ characteristics of an aircraft. The stochastic root locus analysis enables exploration of the density of system roots with a design of experiments (DOE) approach. Instead of considering the probability of instability, this paper focuses on the probability of not meeting the requirements of level 1 HQ characteristics, which provides insight into the control of the cases with poor performance. The controller gains, which enhance the HQ characteristics, are computed, and the variation of the gains helps to better understand how much control effort is required to improve the HQ characteristics of a system.

The information regarding the control effort of improving HQ characteristics can be very important for design purposes. Both control engineers and configuration designers may gain benefits from this information while they are making decisions on the design variables. It has been established that robust control techniques can design a controller by handling the parameter uncertainty inside a system. The purpose of this research, however, is to examine whether it is possible to design a system whose controller requires less effort to achieve desired dynamic characteristics. Hence, bringing the control effort information in design phases reveals the motivation of this paper.

This paper is organized as follows: Section 2 presents the studied problem, Section 3 depicts the proposed methodology step by step, Section 4 mentions the implementation of the proposed methodology, Section 5 demonstrates some quantitative results of the proposed methodology implemented to both longitudinal and lateral dynamics of an aircraft, and Section 6 concludes this paper by providing a brief summary, some comments, and future work.

2 Problem Overview

An S&C analysis of an aircraft requires the information pertaining to the S&C derivatives of a system. These can be obtained from either historical data or aerodynamic calculations [5, 6]. Despite the use of a high fidelity tool to calculate these derivatives, one can never be absolutely confident regarding the accuracy of the computed derivatives. Moreover, since the geometry of the aircraft is not fixed in the early design phases, these derivatives involve uncertain information. The presence of uncertainty motivates the probabilistic approach for S&C analysis discussed in this paper.

As a numerical example, the controller gain variation of a notional aircraft, similar in size to a Boeing 747, is investigated by conducting a DOE study for the S&C derivatives along with the flight velocity. Based on a latin hyper-cube design with 10000 cases, the longitudinal and lateral modes of the aircraft are explored. For each case, the HQ characteristics, namely phugoid and short-period stabilities for longitudinal modes, dutch-roll, spiral and roll stabilities for lateral modes are calculated [8]. For the cases with HQ characteristics of levels 2 and 3, a fixed notional controller is used to enhance them to level 1 quality. Note that using a fixed controller in a statistical analysis satisfies a consistent comparison scheme for each experiment. On the other hand, using a notional controller helps to better understand the HQ improvement from a controller perspective during the early design stages (it can even provide the nominal controller design for further design steps.) To prevent confusion, the notional controller that is used to characterize the system during conceptual design will be referred to as the *evaluative controller*. In this paper, linear quadratic regulator (LQR) is used as the evaluative controller.

The reason for using an LQR controller for gain calculation is due to the unique property that the feedback gain matrix is calculated in an optimization fashion such that the cost function introduced in Eq. 1 is minimized. In this equation, $x \in \mathbb{R}^{m\times 1}$ is the state vector, $u \in \mathbb{R}^{n\times 1}$ is the control vector, $Q \in \mathbb{R}^{m\times m}$ and $R \in \mathbb{R}^{n\times n}$ are the weighting matrices to penalize state and control, respectively. Some advantages of using the LQR controller are listed as follows: ensuring a stable closed loop system, achieving guaranteed levels of stability robustness, and having simple computation [11].

$$J = \int (x^T Qx + u^T Ru)dt \tag{1}$$

For comparison purposes, it is crucial to fix the comparison criteria of each system. If this is not satisfied, then the comparison results do not become reliable. For this research, first pole placement had been implemented to compute the gain matrices, which required the desired root location. However, the selection of a desired root is not a unique process because a particular performance corresponds to an area in the root locus diagram. Eventually, three disadvantages lead to not using pole placement as an evaluative controller. First of all, if the gains are computed by moving the system poles to the desired area, they may not become comparable with each other. Secondly, if the gains are computed by fixing the desired root location, then this calculation involves a strong assumption regarding the fixed location. Lastly, pole placement becomes complicated for problems including MIMO (multi-input multi-output) systems. On the other hand, LQR has a fixed objective function, and it can be applied to any system with the presence of the following assumptions: the system should be full state implying the entire state vector is available for feedback, (A,B) is stabilizable and (A,C) is detectable, $R = R^T > 0$ [11]. The problem considered in this research does not violate the preceding assumptions, thus an LQR design is applicable.

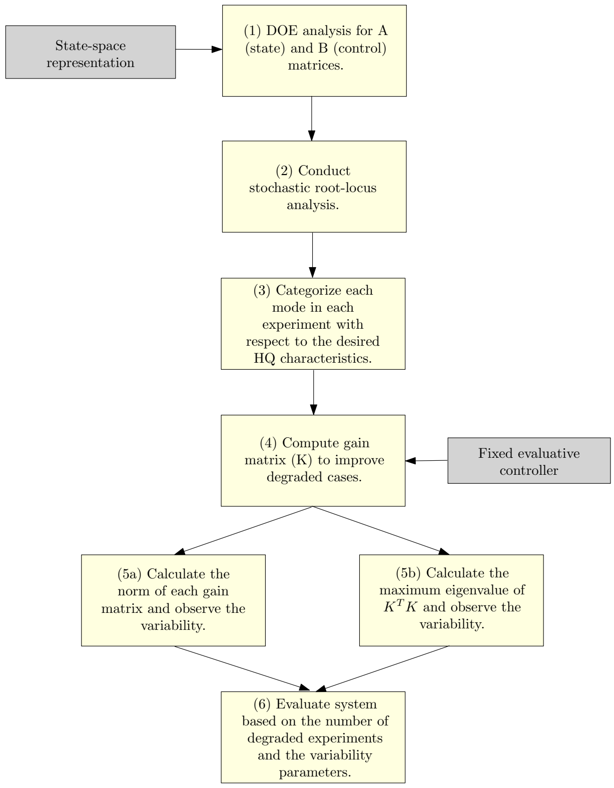
3 Proposed Methodology

The methodology presented in this paper and represented in Fig. 1 assumes a state-space representation of a linear or linearized system stated as Eq. 2. In this representation, the elements of A (state) and B (control) matrices are obtained from the dynamical equations involving both the operating conditions and the design variables of an aircraft.

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{2}$$

Furthermore, A and B matrices are used in S&C analysis. The eigenvalues of matrix A provide information regarding the stability of the system, and (A,B) pair helps to analyse the controllability of the system. In the case where (A,B) is controllable, a controller is designed to bring the system to the desired characteristic. Consequently, the state-space representation of a system is the common point of design and control issues.

Fig. 1 Overview of the methodology.



The first step of the proposed methodology is to conduct a DOE study for the state space representation of a system. This study creates experiments by varying the design variables in which uncertainty is present. In this manner, each experiment corresponds to different A and B matrices. The second step of the methodology is to conduct a root locus analysis for each experiment to investigate the density of the system roots (or modes). Then, this step is followed by characterizing each mode with respect to the desired HQ characteristics, which may be user-defined or derived from military specifications [8]. Due to the stochastic nature of the analysis, the HQ characteristics are expected to vary for a particular system among levels 1, 2 and 3.

The fourth step of the proposed methodology is to improve the modes that do not correspond to level 1 (desired) requirements of HQ characteristics. The required gain matrices are computed by a fixed evaluative controller in order to satisfy a consistent comparison technique. Note that the conducted analysis corresponds to a particular system, hence significant variabilities in HQ characteristics, as well as in the gain matrices, are undesirable. Thus, the number of cases corresponding to undesired HQ characteristics and the variability of gain matrices help to characterize the system from control effort point of view.

The fifth step in the methodology provides insight into the gain matrix analysis. In this paper, we assume a state feedback control depicted in Eq. 3, where $u \in \mathbb{R}^{n\times 1}$, $K \in \mathbb{R}^{n\times m}$, and $x \in \mathbb{R}^{m\times 1}$, and we define the control effort as a finite norm of u. We adopt two ways to interpret the control effort from the gain matrix (K) perspective as steps $5a$ and $5b$.

$$u = Kx \tag{3}$$

In Step $5a$, the norm of the i^{th} element of u is written in the light of Eq. 3 and the Cauchy-Schwarz inequality as Eq. 4. It shows that the norm of the i^{th} row of the gain matrix, $\|row_i\,(K)\|$, has an effect on the upper bound of the i^{th} control norm.

$$|u_i| = |row_i\,(K)\,x| \le \|row_i\,(K)\|\;\|x\| \tag{4}$$

In this context, if we examine each control ($u_{i=1..n}$) under the vision of Eq.4, there is a one-to-one relationship between each row of the gain matrix norm and the upper bound of each control norm. Consequently, smaller norms of each row of the gain matrix minimize the upper bound of the corresponding control, which is desirable to reduce the control effort.

To derive an upper bound for the overall control effort by using the expression of $\|row_i\,(K)\|$, one can sum all of the elements in $\|row_i\,(K)\|$ and minimize it. In the same manner, one can also minimize the second norm of the gain matrix, which satisfies a reduction for $\sum\|row_i\,(K)\|$ as well as $\|row_i\,(K)\|$, where $1 \le i \le n$. Thus we use the Frobenius norm of the gain matrix ($\|K\|_2 = \|K\|_F$) computed by a fixed evaluative controller in each experiment, and the statistical properties of $\|K\|_F$ will help to better understand the required control effort of a system.

Some studies from the literature involve various norms of gain matrix. The study mentioned in [12] adopted the idea of minimizing the $l-1$ norm of K in order to obtain sparse feedback gain matrices. Based on the sparsity of K, they interpreted the need of sensor and actuator placement in the system. To extend the results of this study, looking at the corresponding element of the gain matrix can also provide insight into the following statement: theoretically a large value in the elements of K may help to achieve a desired performance; however, in reality the actuator may not respond efficiently to the physical realization of a large value in K (e.g. a large gain may imply a large signal amplification causing the actuator to move abruptly).

Another study [13] presented an inequality on the second norm of gain matrix. Particularly, Theorem 5 in [13] states that the second norm of the gain matrix is upper bounded by some expression including the condition number of the assigned eigensystem. This paper showed that minimizing the condition number minimizes an upper bound on the feedback gain norm. In this manner, a closed loop system with smaller norms of feedback gain can retain stability even in the maximum disturbance condition. Eventually, small norms of the gain matrix are desirable because they either contribute to obtain small norms of the control, which is cost efficient from control perspective, or demonstrate more practical designs from an actuator performance point of view.

In Step $5b$, the norm of a control can be stated in the light of Eq. 3 as Eq. 5. In Eq. 5, K^TK is a square, symmetric, and non-negative definite matrix, hence it has real and non-negative eigenvalues.

$$\begin{aligned}\|u\| &= u^Tu = x^TK^TKx \\ &\le \lambda_{max}(K^TK)x^Tx\end{aligned} \tag{5}$$

As it is seen, the maximum $\lambda(K^TK)$ has an effect on the control norm such that the smaller values of $\lambda_{max}(K^T K)$ lead to a smaller upper bound on the control, which in turn implies less control effort. Based on this discussion and the assumption of a fixed evaluative controller, we can claim that a perturbed system corresponding to small values of $\lambda_{max}(K^T K)$ is favourable due to requiring less control effort.

Finally, the sixth step of the proposed methodology is creating an overall evaluation criterion that evaluates a system with respect to both the number of degrading experiments and statistical properties of the gain matrix analysis discussed in steps $(5a)$ and $(5b)$. In this manner, an evaluation criterion (EC) is created as Eq. 6 by employing the gain matrix norm and as Eq. 7 by employing the maximum eigenvalue of $K^T K$.

$$EC_{5a} = \frac{n_{imp}}{n_{total}} + \mu_{\|K\|_F} + \sigma_{\|K\|_F} \tag{6}$$

$$EC_{5b} = \frac{n_{imp}}{n_{total}} + \mu_{\lambda_{max}(K^T K)} + \sigma_{\lambda_{max}(K^T K)} \tag{7}$$

In Eqns. 6 and 7, n_{imp} is the number of cases requiring improvement (in other words cases that correspond to levels 2 and 3 for some desired HQ characteristics), n_{total} is the total number of experiments in the statistical analysis, $\mu_{(.)}$ is the mean of the Frobenius norm of the gain matrix in Eq. 6 and the mean of the maximum eigenvalue of $K^T K$ in Eq. 7, both of which are computed by a fixed evaluative controller, $\sigma_{(.)}$ is the standard deviation of the Frobenius norm of the gain matrix in Eq. 6, the standard deviation of the maximum eigenvalue of $K^T K$ in Eq. 7. Based on Eqs. 6 and 7, the smaller EC represents a system requiring less control effort to improve HQ characteristic because of the following reasons:

- In a statistical analysis, fewer n_{imp} corresponds to a more desirable system since it implies that most of the experiments do not violate the requirements of desired HQ characteristics.
- In the presence of nonzero n_{imp}, smaller norms of the gain matrix and smaller maximum eigenvalues of $K^T K$ are more desirable since they impose a smaller upper bound on the control effort (norm of control vector).
- In addition to the smaller values of the norm or eigenvalue, less variability of these values is also desirable for robustness reasons.

In summary, the proposed methodology employs a statistical $S\&C$ analysis to investigate the number of degrading experiments and the statistics of $\|K\|_F$ and $\lambda_{max}(K^T K)$ in the presence of uncertainty. The results of the statistical analysis provide insight into the HQ characteristic improvement of a vehicle from control effort perspective.

4 Implementation of the Proposed Methodology

As mentioned in the preceding sections, the proposed methodology is implemented to a notional aircraft, similar in size to a Boeing 747, flying at $M = 0.5$. A DOE study is conducted by using a latin hypercube design to create the experiments. This study assumes that variables involving uncertainty are uncorrelated with each other. In order to magnify the effects of uncertainty on the system, $\pm 30\%$ variations of the nominal values are present in the study. In this manner, we produce 10000 cases by varying 14 and 16 variables for longitudinal and lateral analyses, respectively. One of these variables is the flight velocity, which is 518 ft/s for both cases. The other nominal values are derived from [7], and illustrated in Tables 1 and 2.

Table 1 Longitudinal $S\&C$ derivatives for a notional Boeing 747 (English units).

$X_U = -4.883 * 10^1$	$Z_{\dot{w}} = 3.104 * 10^2$	$X_e = 3.994 * 10^4$
$X_w = 1.546 * 10^3$	$M_U = 8.176 * 10^3$	$Z_e = -3.341 * 10^5$
$Z_U = -1.342 * 10^3$	$M_w = -5.627 * 10^4$	$M_e = -3.608 * 10^7$
$Z_w = -8.561 * 10^3$	$M_q = -1.394 * 10^7$	
$Z_q = -1.263 * 10^5$	$M_{\dot{w}} = -4.138 * 10^3$	

Based on the dynamic equations of motion and data mentioned in Tables 1 and 2, the nominal A and B matrices for longitudinal and lateral dynamics can be computed as shown below. The

Table 2 Lateral $S\&C$ derivatives for a notional Boeing 747 (English units).

$Y_v = -1.625 * 10^3$	$Y_p = 0$	$Y_r = 0$
$L_v = -7.281 * 10^4$	$L_p = -1.180 * 10^7$	$L_r = 6.979 * 10^6$
$N_v = 4.404 * 10^4$	$N_p = -2.852 * 10^6$	$N_r = -7.323 * 10^6$
$Y_a = 0$	$L_a = -2.312 * 10^6$	$N_a = -7.555 * 10^5$
$Y_{rud} = 1.342 * 10^5$	$L_{rud} = 3.073 * 10^6$	$N_{rud} = -1.958 * 10^7$

states and the control of longitudinal and lateral dynamics are also provided in Table 3.

$$A_{long} = \begin{bmatrix} -0.0028 & 0.0899 & 0 & -32.2 \\ -0.0795 & -0.5071 & 599.17 & 0 \\ 0.0002 & -0.0013 & -0.3799 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B_{long} = \begin{bmatrix} 2.3232 \\ -19.7914 \\ -0.833 \\ 0 \end{bmatrix}$$

$$A_{lat} = \begin{bmatrix} -0.0945 & 0 & -595.7 & 32.2 \\ -0.0045 & -0.7521 & 0.4281 & 0 \\ 0.0014 & -0.1211 & -0.2419 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B_{lat} = \begin{bmatrix} 0 & 7.8062 \\ -0.1477 & 0.1581 \\ -0.0306 & -0.6756 \\ 0 & 0 \end{bmatrix}$$

Table 3 State and control descriptions for longitudinal and lateral dynamics.

Longitudinal Dynamics	
U	velocity of the aircraft along the body axis
w	velocity of the aircraft perpendicular to body axis
q	pitch rate
θ	pitch angle
δ_e	elevator deflection
Lateral Dynamics	
v	velocity of the aircraft along y-body axis
p	roll rate
r	yaw rate
ϕ	roll angle
δ_a	aileron deflection
δ_r	rudder deflection

As depicted before, the fixed evaluative controller in this study is an LQR including the objective function presented in Eq. 1. The following observation was reached from using various Q and R matrices: if the matrix R involves small numbers, the control is less penalized, which leads to moving the roots further away from the original location in the root locus diagram. In order to sustain desired HQ characteristics without moving the roots significantly from the original position, we assume $Q = I$ and $R = 1000\,I$, in which I is the identity matrix with proper dimension. Eventually, the selection of these matrices are based on trial and error.

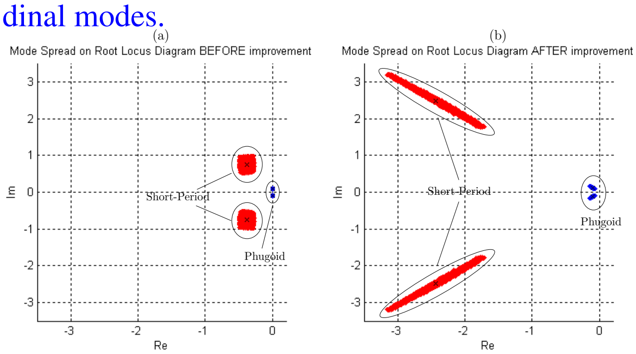
5 Results

The numerical results of this paper are obtained from the statistical $S\&C$ analysis for longitudinal and lateral dynamics of a notional aircraft. First, the results of the longitudinal $S\&C$ analysis are discussed, then the results of the lateral analysis are presented.

5.1 S&C Analysis for Longitudinal Dynamics

In the statistical $S\&C$ analysis for longitudinal dynamics, 10000 A and B matrices were generated for a notional aircraft flying at $M = 0.5$. Then, for each experiment including the nominal case, the longitudinal modes were calculated by the eigenvalue analysis. The results pertaining to the DOE study before improvement are illustrated in Fig. 2a.

Fig. 2 Stochastic root locus analysis for longitudinal modes.



The DOE study shows that the vehicle does not sustain the requirements of level 1 phugoid stability. Most of the cases correspond to level

3 requirements as presented in Table 4. On the other hand, the vehicle mostly satisfies the requirement of level 1 short period stability. (Recall that the requirements of short period and phugoid stabilities are derived from [8].) Due to the low performance of the phugoid stability characteristics, the vehicle response is enhanced by the evaluative controller. Based on the LQR design mentioned before, the unsatisfactory cases are enhanced, and the new mode spread is obtained as illustrated in Fig. 2b. Note that the cases improved by using an LQR controller satisfy the requirements of both phugoid and short period stabilities. The results of the DOE analysis are presented in Table 4.

Table 4 DOE results for longitudinal *S&C* analysis.

	Before improvement	After improvement
$L_{1,Phugoid}$	0	10001
$L_{2,Phugoid}$	46	0
$L_{3,Phugoid}$	9955	0
$< L_{3,Phugoid}$	0	0
$L_{1,Short}$	9994	10001
$L_{2,Short}$	7	0
$L_{3,Short}$	0	0
$< L_{3,Short}$	0	0
Number of experiments	*Total*	*Improved*
	10001	10001

After enhancing the HQ characteristics of the vehicle, a question arises regarding which gain matrices correspond to this improvement. Note that in a particular system, large elements in the gain matrix are not desirable due to inducing high control effort (norm of control) and large signal amplifications, which in turn cause complex actuators or unnecessary noise amplification. For this reason, the gain matrix is evaluated with respect to its Frobenius norm, and the corresponding histogram of the norms are displayed in Fig. 3. Based on the conducted statistical study, the mean and standard deviation of $\|K_{long}\|_F$ are computed as 10.7 and 1.48, respectively. Moreover, the elements of the gain matrix are also investigated to see which elements influence the Frobenius norm the most. The histograms of each element are plotted on Fig. 4, and $K_{1,3}$ and $K_{1,4}$ are observed as the elements involving large values in the matrix.

Note that the states of the longitudinal dynamics are U, the velocity of the aircraft along the body axis, w, the velocity of the aircraft perpendicular to the body axis, q, the pitch rate, and θ, the pitch angle. Moreover, the control assumed in the longitudinal dynamics is only the elevator angle. In this manner, the elevator angle is mostly derived by the variations in the pitch rate and pitch angle multiplied by the corresponding element in the gain matrix. Hence, it is important to improve the dynamics of a system with gains without violating any physical restrictions of an elevator.

As previously discussed, $\lambda_{max}(K^T K)$ also has an effect on the upper bound of the control effort. Thus, the variability of the maximum eigenvalues was investigated in each experiment. The re-

Fig. 3 Histogram of the gain matrix norm computed from longitudinal analysis.

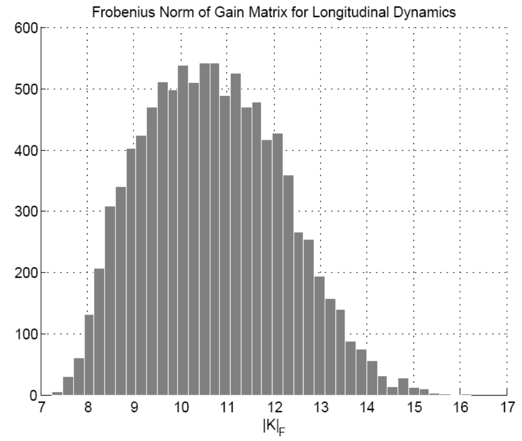


Fig. 4 Histogram of the gain matrix elements computed from longitudinal analysis.

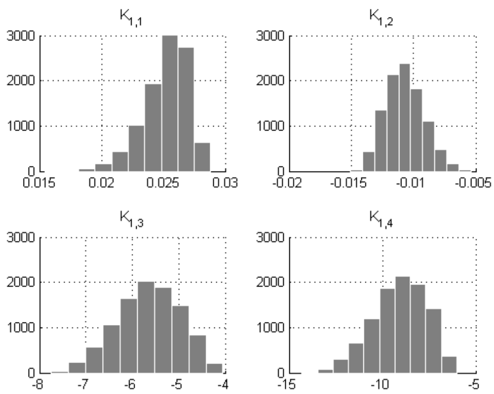


Fig. 5 Histogram of $\lambda_{max}(K^T K)$ computed from longitudinal analysis.

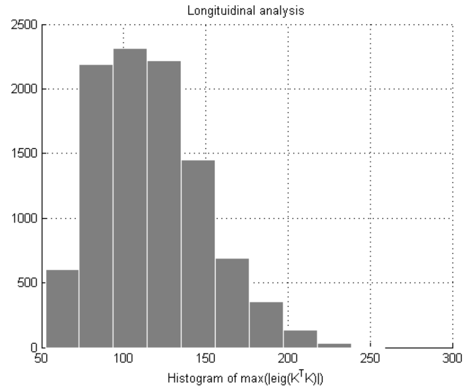
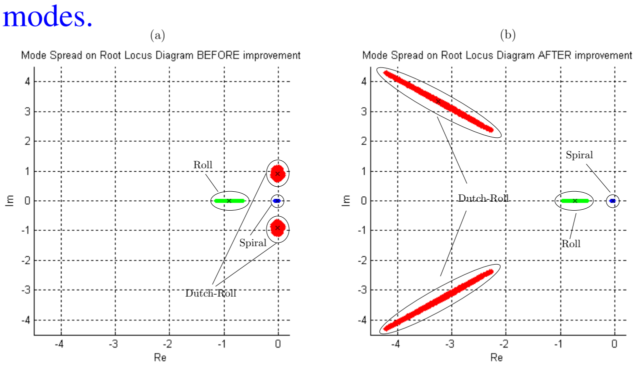


Fig. 6 Stochastic root locus analysis for lateral modes.



sults, displayed in Fig. 5, show that the mean and standard deviation of $\lambda_{max}(K^T K)$ are 116.64 and 32.42, respectively. Finally, EC involving $\|K\|_F$ and EC involving $\lambda_{max}(K^T K)$ are calculated from the statistical S&C analysis of longitudinal dynamics as Eqs. 8 and 9, respectively.

$$EC_{\|K\|_F,long} = \frac{10001}{10001} + 10.7 + 1.48 = 12.88 \quad (8)$$

$$EC_{\lambda_{max}(K^T K),long} = \frac{10001}{10001} + 116.64 + 32.42 = 150.06 \quad (9)$$

5.2 S&C Analysis for Lateral Dynamics

Similarly to the previous section, the lateral dynamic analysis was conducted for the same notional aircraft flying at $M = 0.5$. In the same manner, 10000 experiments were produced, and the lateral modes in each experiment were computed by the eigenvalue analysis.

Fig. 6a shows the mode spread of the DOE study involving 10000 cases. Each mode is evaluated with respect to the requirements of dutch-roll, spiral and roll stabilities from [8]. The results show that the requirements of level 1 dutch-roll stability are not satisfied by the vehicle. In addition, there exist many points that do not correspond to the requirements of any levels 1, 2 and 3. These points are represented as $< L_{3,Dutch-Roll}$ in Table 5. Note that the dutch-roll mode is the most critical mode in lateral dynamics, hence the presence of significant amount of unsatisfactory cases imposes to improve the response of the vehicle by designing a controller. Consequently, the evaluative controller mentioned in the previous sections is used for enhancing the dutch-roll mode characteristics.

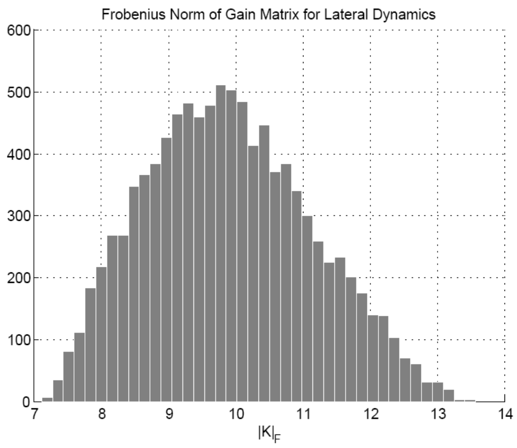
Table 5 DOE results for lateral *S&C* analysis.

	Before improvement	After improvement
$L_{1,Dutch-Roll}$	0	10001
$L_{2,Dutch-Roll}$	1057	0
$L_{3,Dutch-Roll}$	2041	0
$< L_{3,Dutch-Roll}$	6903	0
$L_{1,Spiral}$	9179	6198
$L_{2,Spiral}$	822	3803
$L_{3,Spiral}$	0	0
$< L_{3,Spiral}$	0	0
$L_{1,Roll}$	9794	5324
$L_{2,Roll}$	207	4677
$L_{3,Roll}$	0	0
$< L_{3,Roll}$	0	0
Number of experiments	*Total*	*Improved*
	10001	10001

Following this improvement, the new mode spread is illustrated in Fig. 6b, and the HQ characteristic assessment is presented on Table 5. The results show that all points achieve the requirements of level 1 dutch-roll stability after improvement; however, a degradation is observed for roll and spiral mode characteristics. This degradation can be seen as insignificant because the roll and spiral modes are non-oscillatory and slow, respectively. Even if they have unstable characteristics for a vehicle, a pilot is able to maintain them during flight. Hence, the degradation of roll and spiral mode characteristics is acceptable when the dutch-roll mode characteristics are improved significantly.

As in the previous analysis, the gain matrices corresponding to the improvement of the unsatisfactory cases are recorded. Based on the statistical study, the mean and standard deviation of $\|K_{lat}\|_F$ are computed as 9.90 and 1.23, respectively. Fig. 7 presents the histogram of the gain matrix norm. In addition, each element of the gain matrix is also investigated to identify the elements that have the most influence on the Frobenius norm. The statistical analysis shows that the

Fig. 7 Histogram of the gain matrix norm computed from lateral analysis.



matrix norm is mostly induced by $K_{2,3}$. The details of each element are also illustrated in Fig. 8. Note that the states of the lateral dynamics are v, the velocity of aircraft along the y-body axis, p, the roll rate, r, the yaw rate, and ϕ, the roll angle. Moreover, the assumed controls in lateral dynamics are the deflection angles of aileron and rudder. Based on the results, the rudder deflection is mostly derived by the variations in r multiplied by $K_{2,3}$. Eventually, it is important to improve the dynamics of a system with gains without violating any physical restrictions of the rudder.

Similarly to the previous section, the term $\lambda_{max}(K^T K)$ of each experiment is analysed for lateral study since it has an effect on the upper bound of the control effort. The results, illustrated in Fig. 9, show that the mean and standard deviation of $\lambda_{max}(K^T K)$ for lateral study are 99.27 and 24.74, respectively. Finally, EC in-

Fig. 8 Histogram of the gain matrix elements computed from lateral analysis.

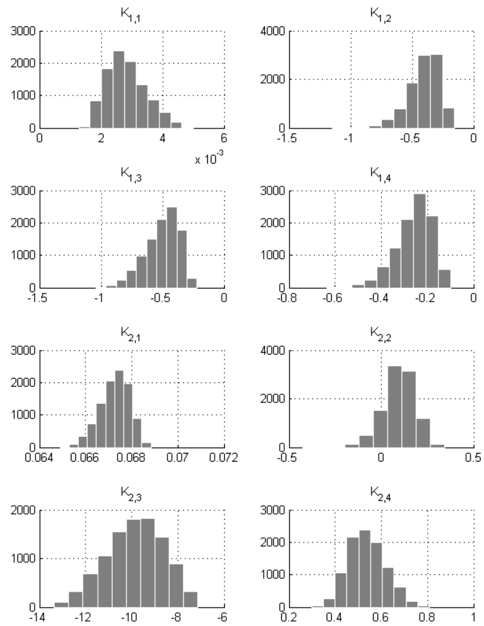
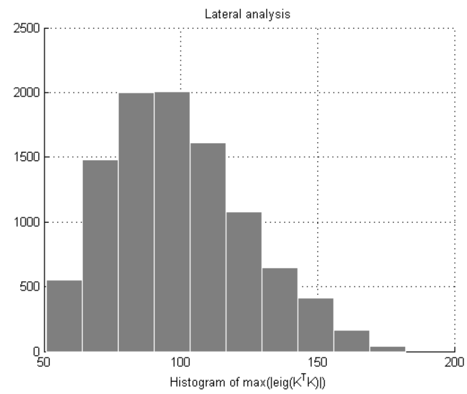


Fig. 9 Histogram of $\lambda_{max}(K^T K)$ computed from lateral analysis.



volving $\|K\|_F$ and EC involving $\lambda_{max}(K^T K)$ are calculated from the statistical S&C analysis of lateral dynamics as Eqs. 10 and 11, respectively.

$$\begin{aligned} EC_{\|K\|,lat} &= \frac{10001}{10001} + 9.9 + 1.23 \\ &= 12.13 \end{aligned} \tag{10}$$

$$\begin{aligned} EC_{\lambda_{max}(K^T K),lat} &= \frac{10001}{10001} + 99.27 + 24.74 \\ &= 125.01 \end{aligned} \tag{11}$$

6 Conclusion

This paper presents a statistical stability and control analysis to investigate how much control effort is required to improve the handling quality characteristics of a vehicle during the early design stages. The goal of the statistical analysis, as a design of experiments study, is to vary the system parameters due to the presence of uncertainty and to observe the mode spread in order to have a better understanding of system robustness. The major contribution of this paper is to introduce an overall evaluation criterion to compute the control effort in the early design stages. The evaluation criterion discussed in this paper consists of not only the number of cases with unsatisfactory handling quality characteristics but also the improvement of these unsatisfactory cases. This improvement is achieved by introducing a fixed notional controller, which is referred to as the evaluative controller. This controller, which is a linear quadratic regulator in this study, may or may not be the actual controller of the real flight; however, it provides a preliminary understanding of the required control effort in the early design stages. Hence, the system characterization from the control effort perspective is achieved by the proposed evaluation criterion involving the number of unsatisfactory cases normalized by the total number of experiments, and the statistics of $\|K\|_F$ or $\lambda_{max}(K^T K)$, where the gain matrix K is calculated by the fixed evaluative controller. Using the evaluation criterion, a system with lower value of EC implies a system requiring less control effort to achieve the desired handling quality characteristics.

Note that using the terms regarding the gain matrix in the evaluation criterion restricts one to assess systems belonging to similar families. In other words, the evaluation criterion proposed in this paper may not be useful to compare a micro vehicle and a transport aircraft because their control signals are not in the same level of magnitude. Nevertheless, the criterion discussed in this paper becomes useful for a nominal design for which the design exploration and exploitation are still in process. Consequently, the geometry of the vehicle can be modified during early design phases by satisfying minimum EC values based on the desired characteristics. Hence, the future work of this research is to use this information in design process to identify the physical geometry changes and tuning the configuration of a vehicle. In this manner, we will expect to reduce the required control effort for a vehicle that satisfies the desired handling quality characteristics.

References

[1] D.N. Mavris, D.A. DeLaurentis, and D.S. Soban, “Probabilistic Assessment of Handling Qualities Characteristics in Preliminary Aircraft Design”, AIAA-1998-492, Aerospace Sciences Meeting and Exhibit, 36th, Reno, NV, Jan. 12-15, 1998.

[2] B. Chudoba, and H. Smith, “A Generic Stability and Control Methodology for Novel Aircraft Conceptual Design”, AIAA-2003-5388, AIAA Atmospheric Flight Mechanics Conference and Exhibit, Austin, TX, Aug. 11-14, 2003.

[3] R.E. Perez, H.H.T. Liu, and K. Behdinan, “Multidisciplinary Optimization Framework for Control-Configuration Integration in Aircraft Conceptual Design”, Journal of Aircraft, Vol.43:6, pg. 1937-1948, 2006.

[4] L.R. Ray, and R.F. Stengel, “Application of Stochastic Robustness to Aircraft Control Systems”, Journal of Guidance, Control and Dynamics, Vol.14:6, pg. 1251-1259, 1991.

[5] J. Roskam, "Airplane Design", ISBN-13:978-1884885549.

[6] USAF Digital DATCOM, http://openae.org/digital-datcom-download.

[7] B. Etkin, and L.D. Reid, “Dynamics of Flight”, John Wiley and Sons, ISBN: 0-471-03418-5, 1996.

[8] Military Specifications, MIL-F-8785C, 1980.

[9] R.F. Stengel, "Flight Dynamics", Princeton University Press, ISBN: 0-691-11407-2, 2004.

[10] R.F. Stengel, "Some Effects of Parameter Variations on the Lateral-Directional Stability of an Aircraft", Journal of Guidance, Control and Dynamics, Vol.3:2, pg. 124-131, 1980.

[11] W.S. Levine, "The Control Handbook", CRC Press, ISBN: 978-0849385704, 1996.

[12] A. Hassibi, J. How, and S. Boyd, "A Path Following Method for Solving BMI Problems in Control", Proceedings of American Control Conference, San Diego, CA, June 1999.

[13] J. Kautsky, N.K. Nichols, and P. Van Dooren, "Robust Pole Assignment in Linear State Feedback", International Journal of Control, Vol.41:5, pg. 1129-1155, 1985.

Copyright Statement